Exclusive electroproduction of vector mesons and transversity distributions

Markus Diehl\textsuperscript{1}, Thierry Gousset\textsuperscript{2} and Bernard Pire\textsuperscript{3}

1. DAPNIA/SPhN, CEA/Saclay, 91191 Gif sur Yvette, France
2. SUBATECH\textsuperscript{1}, B.P. 20722, 44307 Nantes, France
3. CPhT\textsuperscript{2}, Ecole Polytechnique, 91128 Palaiseau, France

Abstract
We show that the leading twist contribution to exclusive electroproduction of transversely polarized vector mesons vanishes at all orders in perturbation theory. Therefore one cannot extract information on the quark transversity distribution of the nucleon from this reaction. In turn one has that the produced vector meson is purely longitudinal in the large-$Q^2$ limit, which provides a new test of the dominance of leading twist at finite $Q^2$.

1 Introduction
Exclusive electroproduction of photons or mesons off the nucleon provides a powerful way to extract off-diagonal (off-forward, nonforward) parton distributions [1, 2], making use of the factorization properties of these processes at large $Q^2$ [3, 4]. In the diagonal limit these quantities are related to the usual spin averaged, helicity or transversity dependent parton distributions. The experimental difficulty of accessing the chiral-odd (transversity) quark distribution

$$h_1(x) \bar{a}(p, s') \sigma^{+i} u(p, s) = \frac{p^+}{2\pi} \int dz^- e^{ix(p^+z^-)} \langle p, s' | \bar{\psi}(0) \sigma^{+i} \psi(z^-) | p, s \rangle$$

in inclusive or semi-inclusive processes [5] has triggered a strong interest in the related off-diagonal quantities [3, 6] which involve the matrix element $\langle p', s' | \bar{\psi}(0) \sigma^{+i} \psi(z^-) | p, s \rangle$, where the proton momentum in the bra and the ket is not the same.

\textsuperscript{1}Unité mixte 6457 de l’Université de Nantes, de l’Ecole des Mines de Nantes et de l’IN2P3/CNRS
\textsuperscript{2}Unité mixte 7644 du CNRS
Figure 1: Factorization of $\gamma^* p \rightarrow V p$ into an off-diagonal parton distribution, a hard photon-parton scattering $H$, and the distribution amplitude of the produced meson.

In this paper we are concerned with vector meson production $\gamma^* p \rightarrow V p$ in the kinematical regime where the photon virtuality $Q^2$ and the squared c.m. energy $W^2$ are large while the squared momentum transfer $t$ is small. In this Bjorken regime and under the condition that the virtual photon has longitudinal polarization the amplitude factorizes into an off-diagonal parton distribution, a hard photon-parton scattering $H$, and the distribution amplitude of the produced meson, cf. Fig. 1.

In reference [3] it has been noted that the production of a transversely polarized vector meson involves the chiral-odd off-diagonal parton distributions in the proton because the distribution amplitude for a transversely polarized vector meson is chiral-odd [7, 8] and because the trace of an odd number of $\gamma$-matrices vanishes. It has however been observed [9] that this contribution is zero at leading order in $\alpha_s$. In the present short paper we confirm and generalize this result. That this contribution vanishes is in fact due to angular momentum and chirality conservation in the hard scattering and holds at leading power in $1/Q$ to all orders in the strong coupling. In the large-$Q^2$ limit the produced meson must therefore have longitudinal polarization; an analysis of its decay angular distribution thus allows for stringent tests of the dominance of the leading contribution in $1/Q$, which contains off-diagonal parton distributions one would like to extract.

2 The physical picture

In the Bjorken regime we are interested in it is suitable to work in the c.m. frame, where the initial and final state protons move fast to the right and the produced meson fast to the left; we remark that this kinematic situation is essential for the power counting arguments in the factorization proof [3].

Our argument has two main ingredients. The first is that the leading term in $1/Q$ of the loop integral in Fig. 1 is obtained by approximating the hard scattering $H$ by its value with all partons being on shell and collinear along a suitably chosen $z$-axis. For our purpose it is convenient to define this axis such that the outgoing meson has zero transverse momentum, and to quantize its spin along $z$. From rotation invariance the helicities of the $q\bar{q}$-pair from the hard scattering must then add up to the helicity of the meson. We remark in passing that on the proton side a corresponding condition on the difference of initial and final proton helicities only holds when the protons are strictly collinear.
Figure 2: Space time structure of the hard scattering $\gamma^* q \to q\bar{q} + q$. The figure gives an example of the hard scattering kernel at lowest order in $\alpha_s$ and of the alignment of parton helicities along the $z$-axis.

The second ingredient is that the quark masses can be neglected in the hard scattering so that the quark chirality is conserved in $H$; here we make the usual assumption that in the limit of zero quark mass there are no chirality breaking contributions to $H$ which would be due to quantum effects. Since the helicities of the quark and the antiquark forming the transverse meson must be equal, the quark charge flow in $H$ cannot connect the lines $\alpha$ and $\beta$ in Fig. 1. This excludes gluon exchange at the proton side and we are left with a hard scattering $H$ with four external quark legs and with the charge flow connecting line $\alpha$ with $\delta$ and line $\beta$ with $\gamma$.

Depending on the parton momentum fractions, the hard scattering can now be

\begin{align*}
\gamma^* + q & \to q\bar{q} + q , \\
\gamma^* + \bar{q} & \to q\bar{q} + \bar{q} , \\
\gamma^* + q\bar{q} & \to q\bar{q} ,
\end{align*}

where we have omitted flavor indices. While cases (2a) and (2b) are familiar from the usual diagonal parton distributions, case (2c) is characteristic of the kinematical asymmetry between the two proton lines [2, 10].

In case (2a) the hard scattering has a space-time structure as shown in Fig. 2. The right-moving quark from the proton is scattered to become a left-moving quark in the meson, along with the creation of a right-moving quark and a left-moving antiquark. For a given helicity of the initial state quark all other parton helicities are fixed from chirality conservation and the constraint to form a transverse meson. Since the scattering is collinear the photon would have to transfer two units of angular momentum as can be seen from Fig. 2. This is of course not possible and we conclude that the process cannot take place to leading order in $1/Q$. In case (2b) the argument is completely analogous.

Along the same lines one can see that in case (2c) the photon would again have to transfer two units of angular momentum. We remark that now the quark configuration in the hard scattering is the same as in the reaction $\gamma^* V \to V$. From hadron helicity conservation in the hard scattering mechanism [11] it has long been known that the elastic meson form factor for transverse polarization is suppressed in powers of $1/Q$ compared with the longitudinal one [8].
3 An algebraic proof

The hard amplitude for the process is a tensor with one Lorentz index and four Dirac indices (cf. Fig. 1) and can be decomposed as

\[ H^\mu_{\alpha\beta\gamma\delta} = t^\mu_{mn} \Gamma^m_{\alpha\beta} \Gamma^n_{\gamma\delta}, \]  

(3)

where the \( \Gamma^m \) \((m = 1, \ldots, 16)\) are the standard 4\( \times \)4 basis matrices in spinor space, \( \Gamma^m = 1, \gamma^\rho, \gamma^\lambda, \gamma^\rho \gamma^\lambda, \sigma^\rho, \) and where a sum over \( m \) and \( n \) is understood.

This expression is general but we want to focus on the leading term in \( 1/Q \) of the amplitude (while retaining all orders in \( \alpha_s \)), for which we have already noticed that the hard process is collinear along the \( z \)-axis. Calling \( t^\mu_{Lmn} \), the leading term in a \( 1/Q \)-expansion of \( t^\mu_{mn} \) in Eq. (3) and introducing two lightlike vectors \( v = (1, 0, 0, 1)/\sqrt{2} \) and \( v' = (1, 0, 0, -1)/\sqrt{2} \), which respectively define a plus and a minus direction, we thus have that \( t^\mu_{Lmn} \) is a function of \( v \) and \( v' \),

\[ t^\mu_{Lmn} = f^\mu_{mn}(v, v'), \]

(4)

but not of any transverse vector.

The next step is to impose the combination of quark helicities in which we are interested. One way of finding which matrices \( \Gamma^m \) this selects on the meson side in Eq. (3) is to contract \( H \) with massless spinor solutions of the Dirac equation whose momenta point in the minus direction and whose helicities are aligned, i.e. both positive or both negative. Since the combination of these spinors is

\[ v_\beta \bar{u}_\alpha = a_i \sigma^i_{\beta\alpha}, \]

(5)

where \( a_i \) is a vector in the transverse plane, we find that the selected \( \Gamma^m \) are \( \sigma^{-i} \) with \( i = 1, 2 \). This corresponds to the fact that the leading twist distribution amplitude for a transversely polarized vector meson is constructed from the matrix element

\[ \langle V | \bar{\psi}(z^+) \sigma^{-i} \psi(0)|0 \rangle. \]

(6)

We remark that this Dirac matrix structure is due to a selection on helicity and not particular to a vector meson final state. Thus the leading twist \( q\bar{q} \)-amplitude also goes with \( \sigma^{-i} \) if we have, for instance, a left moving tensor meson with helicity \( \pm 1 \). In Sect. 2 we have seen that we must route the parton chiralities from the meson to the proton side, and therefore we have to carry out the same exercise for the proton, with the interchange of \( + \) and \( - \). For \( \Gamma^m \) this selects \( \sigma^{+j} \), where \( j = 1, 2 \) is again an index in the transverse direction. This is precisely the Dirac matrix structure entering in the definition of the transversity distribution in Eq. (1).

Among the 16\( \times \)16 combinations of \( m \) and \( n \) in \( t^\mu_{Lmn} \) we thus only need the 2\( \times \)2 combinations \( t^\mu_{Lij} \) going with \( \sigma^{-i} \) and \( \sigma^{+j} \). Lorentz invariance requires the tensor \( t^\mu_{Lij} \) to be constructed from \( v^\mu, v'^\mu \) and the transverse tensors \( g_{Tij} \) and \( \epsilon_{Tij} \) defined as

\[ g^\lambda_{ij} = g_{\lambda} - v^\lambda v'^\lambda - v'^\lambda v^\lambda, \quad \epsilon^\lambda_{ij} = \epsilon_{\lambda \sigma \tau} v^\sigma v'^\tau. \]

(7)

Parity invariance would further restrict \( t^\mu_{Lij} \) to be proportional to \( g_{Tij} \), but we do not need this for our argument.

Now we want to impose chirality conservation. To this end we perform a Fierz transformation,

\[ H^\mu_{\alpha\beta\gamma\delta} = \bar{t}^\mu_{mn} \Gamma^m_{\alpha\delta} \Gamma^n_{\gamma\beta}, \]

(8)
and notice that chirality conservation means that both fermion lines $\alpha\beta$ and $\gamma\beta$ consist of an odd number of $\gamma$-matrices.\(^3\) In the decomposition Eq.~(8) $\Gamma^m$ and $\Gamma^n$ are thus either $\gamma^\lambda$ or $\gamma^5\gamma^\lambda$. To leading power in $1/Q$ and for aligned helicities of the final $q\bar{q}$-pair we then have

$$\bar{t}^\mu_{L,mn} = t^\mu_{L,ij} \text{tr} \left( \Gamma^m\sigma^{-i}\Gamma^n\sigma^{+j} \right),$$

with $\Gamma_m$ and $\Gamma_n$ restricted to $\gamma^\lambda$ and $\gamma^5\gamma^\lambda$. Here $\Gamma_m$ with a lower index is defined by $\text{tr}(\Gamma^m\Gamma^n) = \delta^m_n$, where $\delta^m_n$ is the Kronecker symbol. Up to factors the traces in (9) are given by

$$\text{tr}(\gamma^\lambda\sigma^{-i}\gamma^\rho\sigma^{+j}) \propto g_T^\rho g_T^{ij} - g_T^\lambda g_T^{ij} - g_T^\lambda g_T^\rho g_T^\rho,$$
$$\text{tr}(\gamma^\lambda\sigma^{-i}\gamma^\rho\sigma^{+j}\gamma^5) \propto g_T^\lambda g_T^{ij} + g_T^\rho g_T^{ij},$$

(10)

where we have used that $i, j = 1, 2$. They vanish when contracted with $t^\mu_{L,ij}$ so that all in all we have

$$\bar{t}^\mu_{L,mn} = 0$$

(11)

for $m, n = 1, \ldots, 16$.

We remark that the expressions (10) are nonzero for any fixed pair $i, j$, and that zero is only obtained by the contraction with $g_T^{ij}$ or $e_T^{ij}$. To get to (11) we are thus using the invariance of the hard scattering under rotations around the $z$-axis; this corresponds to angular momentum conservation along $z$ which went into our argument in Sect. 2.

### 4 Phenomenology

Let us first stress that the above derivation is valid for any spin state of the incoming virtual photon. Phenomenological tests for the suppression of transverse meson polarization are thus different from and complementary to those for the dominance of the longitudinal component of the virtual photon, which is part of the factorization theorem [3]. The photon polarization can be tested using the distribution in the angle between the hadronic and leptonic planes in the electroproduction process $e p \to e V p$, which to leading order in $1/Q$ should be flat [12].

The fact that the production amplitude for a transverse vector meson vanishes to leading power in $1/Q$ translates into the $Q^2$-dependence of its helicity density matrix elements $\rho_{\lambda\lambda'}$ and thus of its decay angular distribution. The general expression [13] for the decays $\rho \to \pi\pi, \phi \to KK$,

$$\frac{1}{N} \frac{dN}{d\cos \theta d\phi} = \frac{3}{4\pi} \left[ \frac{1}{2}(1 - \rho_{00}) + \frac{1}{2}\cos^2 \theta (3\rho_{00} - 1) \right. \left. - \frac{1}{\sqrt{2}}\sin 2\theta \cos \phi (\text{Re}\rho_{10} - \text{Re}\rho_{-10}) + \frac{1}{\sqrt{2}}\sin 2\theta \sin \phi (\text{Im}\rho_{10} + \text{Im}\rho_{-10}) \right. \left. - \sin^2 \theta \cos 2\phi \text{Re}\rho_{1-1} + \sin^2 \theta \sin 2\phi \text{Im}\rho_{1-1} \right],$$

(12)

will thus become

$$\frac{1}{N} \frac{dN}{d\cos \theta d\phi} = \frac{3}{4\pi} \cos^2 \theta + O\left(\frac{1}{Q}\right).$$

(13)

---

3This is readily seen diagrammatically: attaching $n$ gluons (or $n - 1$ gluons and a photon) to a quark line one obtains $n$ $\gamma$-matrices from the vertices and $n - 1$ from the massless quark propagators in between.
in the scaling regime.

As remarked in Sect. 3 our derivation also holds for a transversely polarized meson with higher spin. The \( q\bar{q} \)-pair produced in the hard scattering cannot have total helicity \( \pm 1 \) at leading order in \( 1/Q \) and thus must have helicity zero. In the scaling limit the meson is therefore again produced with longitudinal polarization and its decay angular distribution completely determined.

Going a step further we can also apply our arguments to the production of a nonresonant state of two spinless mesons such as \( \gamma p \to \pi\pi p \) or \( \gamma^* p \to KKp \) when the invariant mass of the two mesons is much smaller than the photon virtuality, provided one replaces the meson distribution amplitude by the generalized distribution amplitude introduced in [14]. While the nonresonant meson pair is no longer produced in a single partial wave its angular momentum along the \( z \)-axis is still zero in the scaling limit: the distribution in the polar angle \( \theta \) is then no longer given generically, but the dependence on the azimuth \( \phi \) is flat. This test is the precise analog of the one for the dominant photon helicity mentioned above.

There is an important program of studying exclusive meson production in ongoing or planned experiments [15, 16]. We stress that the polarization structure of this reaction in the Bjorken regime offers a valuable tool to test the dominance of the leading mechanism in \( 1/Q \), which is essential before leading-twist quantities such as off-diagonal parton distributions can be extracted.

Acknowledgments

We gratefully acknowledge discussions with J.P. Ralston and O.V. Teryaev. This work has been partially funded through the European TMR Contract No. FMRX-CT96-0008: Hadronic Physics with High Energy Electromagnetic Probes.

References

X. Artru and M. Mekhfe, Z. Phys. C45, 669 (1990);
J.L. Cortes, B. Pire and J.P. Ralston, Z. Phys. C55, 409 (1992);


K. Schilling and G. Wolf, Nucl. Phys. B61, 381 (1973);  


G. Baum et al., COMPASS: A proposal for a common muon and proton apparatus for structure and spectroscopy, CERN-SPSLC-96-14 (1996);  