The Glue Around Quarks and the Interquark Potential

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The quarks of quark models cannot be identified with the quarks of the QCD Lagrangian. We review the restrictions that gauge field theories place on any description of physical (colour) charges. A method to construct charged particles is presented. The solutions are applied to a variety of applications. Their Green's functions are shown to be free of infra-red divergences to all orders in perturbation theory. The interquark potential is analysed and it is shown that the interaction responsible for anti-screening results from the force between two separately gauge invariant constituent quarks. A fundamental limit on the applicability of quark models is identified.

1. LONG RANGE FORCES

The discovery of asymptotic freedom showed that non-abelian gauge theory could explain the observed short distance scaling behaviour. Thus QCD was able to describe partonic physics. However, it has still not been able to explain how the original constituent quarks of Gell-Mann emerge from the underlying dynamics of our theory of the strong interactions. In this talk we will review how charges should be described in quantum field theory and see what lessons can be learned from QCD for quark models.

In the standard textbook description of scattering it is assumed that at asymptotic times the coupling in some sense vanishes. This is, however, known not to hold for theories like QED and QCD where, as a consequence of the masslessness of the gauge bosons, there is a long range interactions. For Electrodynamics it has been shown [1] that at large times the interaction has the form

$$-e \int d^3 x A^\mu(x) J^{\mu \sigma}(x),$$

where the asymptotic current is

$$J^{\mu \sigma}(x) = \int d^3 p \rho(p) \frac{u^\mu}{E_p} \delta^{(3)}(x - \frac{t p}{E_p}).$$

This shows that even asymptotically we cannot in general ‘turn the interaction off’. The neglect of these interactions leads to infra-red divergences. Note though that giving the photon a small mass does cause the interaction Hamiltonian to vanish at large times — this further indicates the intimate connection between the long range nature of the interaction, gauge invariance and the infra-red problem.

What we would like to stress is that the non-vanishing of the asymptotic interaction means that Gauss' law is not trivial at large times. Gauss' law is the generator of local gauge transformations, so this then implies that the charged matter fields of the QED and QCD Lagrangians do not become physical in this limit, i.e., the matter fields are not good asymptotic fields. They must be surrounded, dressed, by an electromagnetic cloud (gluonic for colour charges).

Being physical in a gauge theory requires, at the very least, being locally gauge invariant. We thus write a physical charge generically as: $h^{-1} \psi$, where $h^{-1}$ is a field dependent element of the gauge group and we demand that under a gauge transformation, where the matter transforms as $\psi \rightarrow e^{i \vartheta} \psi$, the dressing around the charged matter transforms as $h^{-1} \rightarrow e^{-i \vartheta} h^{-1}$.

This requirement is, however, not sufficient to
specify the form of the cloud around a quark: many such dressings can be constructed (for a review see [2,3]). We need a further condition to single out physically relevant descriptions. For an asymptotic particle with four velocity $u^\mu$, we demand that $h^{-1}$ satisfies the following dressing equation: $hu^\mu \partial_\mu (h^{-1}) = -i e u^\mu A_\mu$. This equation may be motivated [2] both by seeing that in the heavy quark effective theory it yields a particle with a sharp momentum and also by the realisation that a particle dressed in this manner obeys a free asymptotic dynamics.

In QED it is possible to solve these two demands (gauge invariance and kinematical) and we then find that for an asymptotic charge the correct form of the dressing is

$$h^{-1} = e^{-wK} e^{-iex} \quad (2)$$

where

$$K(x) = -\int_\Gamma ds(\eta + v)^\mu \frac{\mathcal{G}^\nu F_{\mu\nu}}{\mathcal{G} \cdot \partial} \quad (3)$$

$$\chi(x) = \frac{\mathcal{G} \cdot A}{\mathcal{G} \cdot \partial} \quad (4)$$

and $\mathcal{G}^\mu = (\eta + v)^\mu (\eta - v) \cdot \partial - \partial^\mu$ with $\Gamma$ the past (future) trajectory of an incoming (outgoing) particle. Here we have written the four velocity $u^\mu$ as $\gamma(\eta + v)^\mu$ with $\eta$ the unit temporal vector and $v = (0,\mathbf{v})$.

This structured dressing has a gauge dependent part, $\chi$, which makes the whole charge locally gauge invariant, and is thus in some sense a minimal dressing. The additional term, $K$, is gauge invariant but is necessary if we are to fulfill the dressing equation. To understand this solution better we have studied the infra-red behaviour of the Green’s functions of these dressed charges.

In a series of detailed perturbative tests [4,5] we have verified to all orders in perturbation theory that these Green’s functions are free of on-shell infra-red divergences. The role of the two structures in the dressing is noteworthy: the minimal dressing removes the soft divergences while the additional term, $K$, kills off phase divergences. These tests are compelling evidence for the validity of our underlying requirements and for our ensuing descriptions of charges.

We also stress here that the use of an incorrect dressing does not yield infra-red finite on-shell Green’s functions — gauge invariance is not enough: one must find the physically relevant solutions and their correct interpretation.

2. COLOUR CHARGES AND CONSTITUENTS

In QCD physical quarks must be surrounded by a gluonic cloud. This requirement of gauge invariance is, however, still more urgent in Chromodynamics. This is because the colour charge operator, although not itself gauge invariant, is invariant on locally gauge invariant states [6]. Thus any description of coloured quarks must be gauge invariant. Our other kinematical requirement we still impose so as to have quarks with a sharp momentum.

We have constructed dressings for quarks and gluons in perturbation theory [7,3]. The construction of the dressing appropriate to a static quark may be performed, using a simple algorithm, to any order in perturbation theory [3]. However, the minimal requirement of gauge invariance means that there is a close link between dressings and gauge fixings. One can thus prove [3] that there is a topological obstruction to the construction of gauge invariant quarks with a well defined colour. Quarks are not true QCD observables, but rather only possess a limited domain of validity. This domain is the realm of quark models and its breakdown, the onset of confinement, needs to be precisely pinned down. This will require non-perturbative calculations of the interplay between dressings and the non-trivial topology of QCD. The remainder of this talk though will be dedicated to a perturbative study [8] ascertaining whether the quark language can actually be motivated from QCD.

3. HADRONS MADE FROM QUARKS?

It is not just that experimentalists see hadrons rather than quarks — some models of hadrons do not involve constituent quarks at all. We therefore now want to study whether the construction of colour singlet hadrons from (dressed, gauge in-
variant) quarks is energetically favoured or not. In QED it can be shown that a flux tube picture of an $e^+e^-$ system is unstable and that it decays into two separate dressed charges\(^2\). What we will now do is to construct two dressed static quarks, separated by some distance \(r\), study the potential energy of this state and compare it with the known QCD result for the potential of the lowest energy state involving two static quark (matter) fields.

This potential is known from Wilson loop calculations to have at order \(\alpha^2\) the form:

\[
V(r) = -\frac{g^2 C_F}{4\pi r} \left[ 1 + \frac{g^2 C_A}{4\pi} \left( 4 - \frac{1}{3} \right) \log(|\mu r|) \right],
\]

i.e., a Coulombic potential with an anti-screening logarithmic correction. This last we have divided up into two parts: an anti-screening term (the \(4\)) and a screening (the \(1/3\)) contribution.

The coefficient of the logarithm is proportional to the one-loop beta function. It is well known that the all-important anti-screening contribution comes from longitudinal glue, while the term which tries to screen colour charge arises from gauge invariant glue [9,10].

Our dressings, as we have seen, are built out of a minimal part, \(\chi\), constructed out of longitudinal degrees of freedom and the additional gauge invariant term. We would therefore expect that if in this perturbative region a description of hadrons in terms of constituent quarks is energetically favoured, then the minimal dressing should yield the dominant anti-screening part of the potential.

At lowest order the Coulombic potential is essentially abelian, so to test the non-abelian nature of hadrons we need to work at \(O(\alpha^2)\). We then require the minimal dressing to \(O(g^4)\). The potential will then be the separation dependent part of the expectation value of the Hamiltonian between our constituent quarks.

The abelian minimal dressing for a static charge, \(\exp(-i\epsilon x)\), with \(\epsilon = \partial_i A_i/\nabla^2\), may be extended to QCD as follows

\[
\exp(-i\epsilon x) \rightarrow \exp(g_x^a T^a) \equiv h^{-1}
\]

with

\[
g_x^a T^a = (g\chi^a_1 + g^2\chi^2_2 + g^3\chi^3_3 + \cdots) T^a
\]

where

\[
\chi^a_1 = \frac{\partial_j A^a_j}{\nabla^2} \quad \chi^a_2 = f_{abc} \frac{\partial_j}{\nabla^2} \left( \chi^b_1 A^c_j + \frac{1}{2} (\partial_j \chi^a_1) \chi^a_1 \right) \ldots
\]

Henceforth \(h^{-1}\) signifies this minimal dressing.

The expectation value of the Yang-Mills Hamiltonian, between such minimally dressed quark/antiquark states, \(\bar{\psi}(y) h(y) h^{-1}(y') \psi(y') |1\rangle\) gives the potential

\[
-\text{tr} \int d^3x \langle 0 | [E_i^a(x), h^{-1}(y)] h(y) \times [E_i^a(x), h^{-1}(y') | h(y') |1\rangle.
\]

The equal-time commutator \([E_i^a(x), A_j^a(y)] = i\delta_{ij}\delta(x-y)\) then yields the expected Coulomb potential:

\[
V^a(r) = -\frac{g^2 N C_F}{4\pi r}.
\]

To test the non-abelian nature of the dressed quarks, we first calculate \([E_i^a(x), h^{-1}(y)] h(y)\). This may rapidly be shown to reduce to

\[
g[E_i^a(x), \chi_1(y)] + g^2[E_i^a(x), \chi_2(y)] + g^3[E_i^a(x), \chi_3(y)] + O(g^4),
\]

plus many gauge dependent terms which cancel amongst themselves in physical quantities. Substituting this into (9) yields for the potential [8]

\[
V^a(r) = -\frac{3g^4 C_F C_A}{(4\pi)^3} \int d^3x \! d^3w \frac{1}{|z-w|} \int \! \frac{d^3w}{|z-w|} \langle A_i^a(w) | A_j^a(z) \rangle.
\]

At this order in \(\alpha\) we may use the free propagator

\[
\langle A_i^a(w) | A_j^a(z) \rangle = \frac{1}{2\pi^2} \frac{(z-w)(z-w)^*}{|z-w|^4}.
\]

The integrals are straightforward and we obtain

\[
V^a(r) = -\frac{g^4}{(4\pi)^2} \frac{N C_F C_A}{2\pi^2} 4\log(|\mu r|),
\]

which is just the predicted anti-screening contribution to the potential. This demonstrates that the anti-screening effect characteristic of non-abelian gauge theories results from the interaction of two constituent quarks.
4. CONCLUSIONS

Gauge theories are characterised by long ranged interactions and this results in the asymptotic fields obeying a non-trivial dynamics. This, with the help of Gauss' law, tells us in turn that physical charged particles, be they electrons or quarks, cannot be naively identified with the matter fields of the Lagrangian of our gauge theory. Rather we must use dressed fields, where the cloud surrounding any physical charge is taken into account. In this talk we have presented a scheme to construct dressed charges with well-defined velocities.

In explicit QED calculations we saw that these dressings solve the infra-red problem already at the level of Green's functions. The dressings have two components which separately cancel the two sorts of infra-red singularities in QED. We note that in the theory of massless electrodynamics there is evidence [2] that the collinear divergences typical of theories with massless charges will also be removed by the solutions to the dressing equation.

The need for dressings is most blatant in QCD. We simply cannot talk about coloured particles in any meaningful way without them. If coloured quarks obeying Pauli statistics are to emerge from the the underlying Lagrangian as a description of hadrons they will be dressed quarks.

Perturbatively we have a well-defined method for constructing dressings that yield gauge invariant quarks, but, as we saw above, there is a global obstruction (essentially the Gribov ambiguity) preventing the construction of an observable quark. This sets a fundamental limit on the applicability of quark models.

However, even when quarks can still be constructed, they might not be physically significant. It could be energetically preferred to construct colour singlet, gauge invariant hadrons in a fashion that does not involve constituent quarks. We have therefore calculated the potential felt by two gauge invariant quarks in a colour singlet state using minimally dressed quarks. This led directly to the the anti-screening contribution found in the QCD lowest energy state with two matter fields. This shows that this paradigm non-abelian effect is due to the interaction of two gauge invariant quarks. Furthermore the calculation presented here precisely identifies the gluonic configuration responsible for this force.

The immediate next steps are to identify the glue underlying the screening part of the potential (is this the additional $K$ term in the dressing?) and to study the anti-screening contribution at higher orders in perturbation theory. Beyond perturbation theory, we recall that the global limitation on constructing gauge invariant quarks means that any constituent picture of hadrons will break down once sufficient non-perturbative physics is probed. The fundamental aim of the programme of research sketched in this talk is to determine the scale at which the apparent constituent structure of hadrons is revealed to be a mirage.

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REFERENCES