On the exact conservation laws in thermal models and the analysis of AGS and SIS experimental results

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Abstract. The production of hadrons in relativistic heavy ion collisions is studied using a statistical ensemble with thermal and chemical equilibrium. Special attention is given to exact conservation laws, i.e. certain charges are treated canonically instead of using the usual grand canonical approach. For small systems, the exact conservation of baryon number, strangeness and electric charge is to be taken into account. We have derived compact, analytical expressions for particle abundances in such ensemble. As an application, the change in $K/π$ ratios in AGS experiments with different interaction system sizes is well reproduced. The canonical treatment of three charges becomes impractical very quickly with increasing system size. Thus, we draw our attention to exact conservation of strangeness, and treat baryon number and electric charge grand canonically. We present expressions for particle abundances in such ensemble as well, and apply them to reproduce the large variety of particle ratios in GSI SIS 2 A GeV Ni–Ni experiments. At the energies considered here, the exact strangeness conservation fully accounts for strange particle suppression, and no extra chemical factor is needed.

1. Introduction

The statistical models, based on the assumption of the quick natural entropy maximization before the freeze-out in the relativistic heavy-ion reaction, have been successful in describing the total hadron abundancies in various experiments, see e.g. [1]. The usual method of grand canonical ensemble is appropriate in case of very large interaction systems. However, the systems studied experimentally are rather small, so the exact conservation laws should be taken into account. In such approach, the thermal analysis agrees well even with as small systems as induced in CERN $e^+e^−$, $p−p$ and $p−\bar{p}$ experiments [2].

For larger systems, taking into account the simultaneous conservation of baryon number $B$, strangeness $S$ and electric charge $Q$ is more complicated. We review here the method developed, and the application to AGS E802 $p−p$, $p−A$ and $A−A$ experiments in the systematic manner to find if the increase in production of kaons compared to pions can be due to finite volume effect incorporated in relativistic canonical statistical mechanics [3].

If the net number of any relevant charge becomes large, the task of computing abundancies in an ensemble respecting exact conservation of $B$, $S$ and $Q$ becomes difficult. Letting $B$ and $Q$ fluctuate in the grand canonical manner, we may focus to strangeness conservation. As an application, we have shown that the particle ratios in GSI Ni+Ni experiments at 2 GeV A agree well with the statistical predictions [4].
2. The model

In the usual grand canonical partition function, the states respecting exact quantum number conservation are included, but only the conservation of expectation values is demanded. By denoting the set of conserved quantum numbers by \( \{ C_i \} \), the canonical sum of states, \( Z(\{ C_i \}) \), can be projected from grand canonical one, \( Z_G \) [5]. In case of \( N U(1) \) internal symmetries, the projection takes the form

\[
Z(\{ C_i \}) = \left[ \prod_{i=1}^{N} \frac{1}{2\pi} \int_{0}^{2\pi} d\phi_i e^{-iC_i \phi_i} \right] Z_G(T, \{ \lambda_{C_i} \}).
\] (1)

Here we have assigned a group angle \( \phi_i \) and a Wick-rotated fugacity factor \( \lambda_i = e^{i\phi_i} \) for every exactly conserved charge.

2.1. \( Z_{B,S,Q} \)

Let us quote first the expression for the canonical partition function respecting strict conservation of \( B, S \) and \( Q \). After putting in the set of overall charges, \( \{ C_i \} = \{ B, S, Q \} \), and performing some algebraic exercise [3, 6], we find

\[
Z_{B,Q,S}(T, V) = Z_0 \left( \prod_{\nu=1}^{7} \sum_{n_{-\nu}=-\infty}^{\infty} \right) \times I_{-B+n_2+n_3+n_4+n_5+n_6+n_7}(2N_n) \\
\times I_{-Q+n_1+n_2-n_3+n_5-n_6+2n_7}(2N_{\pi^\pm}) \\
\times I_{-S+n_1-n_4-n_5-n_6}(2N_{K^0}) \\
\times I_{n_1}(2N_{K^\pm})I_{n_2}(2N_{\pi^\pm})I_{n_3}(2N_{\Delta^-}) \\
\times I_{n_4}(2N_{\Lambda})I_{n_5}(2N_{\Sigma^+})I_{n_6}(2N_{\Sigma^-})I_{n_7}(2N_{\Delta^+}).
\] (2)

Here we have used the modified Bessel functions \( I_n \) with arguments \( 2N_i \), where \( N_i \) is the sum of one particle partition functions for particles carrying same set of quantum numbers, \( \{ B_i, S_i, Q_i \} \), as the hadron \( i \). \( Z_0 \) is the partition function for particles with vanishing quantum numbers considered. In the expression above, we have omitted the contribution from hadrons with \( |S_i| > 1 \), but the generalization is straightforward [3]. The mean abundance of hadron \( i \) in the system is

\[
\langle N_i \rangle = \frac{Z_{B-B_i,Q-Q_i,S-S_i}}{Z_{B,Q,S}} Z_i^1,
\] (3)

The evaluation of the canonical partition function with three simultaneously conserved quantum numbers becomes numerically very time consuming for large values of \( B \). So far, for systems with \( B > 20 \) we have been forced to resort to the grand canonical treatment.
2.2. $Z_S$

In large systems, such as Ni+Ni, the grand canonical treatment of baryon number and electric charge is justified. However, the exact strangeness conservation is still mandatory in theoretical considerations. If we only include the particles with $|S_i| \leq 1$, and require vanishing net strangeness, we end up with \[4, 7\]

\[ Z_{S=0}(T, V, \lambda_B, \lambda_Q) = Z_0 I_0(2\sqrt{\frac{N_1}{N_{-1}}}), \]  

(4)

where the $N_{S_i}$ are the sums of grand canonical one-species partition functions of particles carrying strangeness $S_i$ defined by

\[ Z_i^1 = \lambda_B^B \lambda_Q^Q g_i \frac{V}{2\pi^2} \int_0^{\infty} dp p^2 e^{\beta \sqrt{p^2+m_i^2}}. \]

(5)

The equation (4) can be generalized to include the particles with higher strangeness content as well, see \[6, 7\]. Now the mean number of hadrons $i$ is

\[ \langle N_i \rangle = Z_i^1 \left( \sqrt{\frac{N_1}{N_{-1}}} \right)^{S_i} \frac{I_S(2\sqrt{\frac{N_1}{N_{-1}}})}{I_0(2\sqrt{\frac{N_1}{N_{-1}}})}. \]

(6)

The nonlinear volume dependence of strange particle production rates is visible in the $I_S/I_0$ coefficient in equation (6). Whereas in grand canonical formalism the abundance of kaons is linear in volume ($Z_i^1$), the $I_1/I_0$ term gives an additional coefficient of first order in volume in small system limit. This nonlinearity decreases smoothly with increasing volume, until in thermodynamic limit it vanishes.

3. Applications and discussion

3.1. $K/\pi$ ratios in AGS experiment E802

We have applied the $Z_{B,S,Q}$ in comparison with experimental $K/\pi$ production ratios reported by E802 collaboration, see table 1. In figure 1 we find the theoretical curves

<table>
<thead>
<tr>
<th>Collision</th>
<th>$K^+/\pi^+$</th>
<th>Ref.</th>
<th>$K^-/\pi^-$</th>
<th>Ref.</th>
<th>$B$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p + Be$</td>
<td>7.8±0.4%</td>
<td>[8, 9]</td>
<td>2.0±0.2%</td>
<td>[8]</td>
<td>3.9</td>
<td>2.3</td>
</tr>
<tr>
<td>$p + Al$</td>
<td>9.9±0.5%</td>
<td>[9]</td>
<td></td>
<td></td>
<td>5.4</td>
<td>3.1</td>
</tr>
<tr>
<td>$p + Cu$</td>
<td>10.8±0.6%</td>
<td>[9]</td>
<td></td>
<td></td>
<td>6.9</td>
<td>3.7</td>
</tr>
<tr>
<td>$p + Au$</td>
<td>12.5±0.6%</td>
<td>[8, 9]</td>
<td>2.8±0.3%</td>
<td>[8]</td>
<td>9.7</td>
<td>4.5</td>
</tr>
<tr>
<td>$Si + Au$</td>
<td>18.2±0.9%</td>
<td>[8]</td>
<td>3.2±0.3%</td>
<td>[8]</td>
<td>102.7</td>
<td>44.0</td>
</tr>
<tr>
<td></td>
<td>19.2±3%</td>
<td>[10]</td>
<td>3.6±0.8%</td>
<td>[10]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
for $K^+/\pi^+$ and $K^-/\pi^-$ ratios in constant baryon density as functions of baryon number to be consistent with experimental results. This shows that the increase in $K/\pi$ ratios along with increasing system size can also be explained without any phase transition or the in-medium effect in kaon masses \cite{11, 12, 13}.

3.2. Production ratios in GSI Ni+Ni experiments at 2 GeV A

As an application of $Z_S$ the thermal model analysis is compared to experimental hadronic ratios in GSI Ni+Ni 2 GeV A experiments. In this case, the widths of resonances affect substantially the results, so we have applied the relativistic Breit-Wigner resonance shape in computing the phase space integrals (5).

In Ni+Ni system, the isospin asymmetry ($\frac{B}{2Q} = 1.04$) has to be taken into account by introducing chemical potential for electric charge. This parameter, however, is eliminated by the simple binding condition between baryon- and charge densities, $n_B$ and $n_Q$:

$$n_B = 2(\frac{B}{2Q})n_Q.$$ (7)

The comparison between model predictions and experimental data is collected in table 2. Only the ratio $\eta/\pi^0$ does not fit reasonably in thermal model scheme, but it serves as a clear indication of incompleteness of the thermalized ideal gas model considered. Apart from that, the other measured hadronic ratios are well described by the model. As a clarification, the equal value curves for ratios in $(T, \mu_B)$ plane in constant volume are shown in figure 2.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.2</td>
<td>3</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>$K^+/K^-$</td>
<td>25.7</td>
<td>21 $\pm$ 9</td>
</tr>
<tr>
<td>$K^+/\pi^+$</td>
<td>0.0071</td>
<td>0.0074 $\pm$ 0.0021</td>
</tr>
<tr>
<td>$\phi/K^-$</td>
<td>0.103</td>
<td>0.1 $\pm$ 0.002</td>
</tr>
<tr>
<td>$\pi^+/\pi^-$</td>
<td>0.893</td>
<td>0.89 $\pm$ 0.002</td>
</tr>
<tr>
<td>$\eta/\pi^0$</td>
<td>0.008</td>
<td>0.037 $\pm$ 0.002</td>
</tr>
<tr>
<td>$\pi^+/p$</td>
<td>0.225</td>
<td>0.195 $\pm$ 0.020</td>
</tr>
<tr>
<td>$\pi^0/B$</td>
<td>0.104</td>
<td>0.125 $\pm$ 0.007</td>
</tr>
<tr>
<td>$d/p$</td>
<td>0.129</td>
<td>0.26</td>
</tr>
</tbody>
</table>
References

Figure captions

**Figure 1.** Thermal model expectations for the production ratios $K^+/\pi^+$ and $K^-/\pi^-$ at a temperature of 100 MeV and a baryon density of 0.04 fm$^{-3}$ compared to experimental results from the Brookhaven AGS. The experimental ratios from $Si-Au$ collisions ($B \sim 103$) is moved to $B = 21$ for the sake of convenience.

**Figure 2.** Curves in the $(\mu_B,T)$ plane corresponding to the GSI Ni+Ni 2 A GeV hadronic ratios indicated. The interaction volume corresponds to the radius of 4.2 fm, and the isospin asymmetry is $B/2Q = 1.04$. 
$K^+/\pi^+ \text{ ratio}$

- $T = 100 \text{ MeV}, B/V = 0.04 \text{ fm}^3$

- Canonical, $B/2Q = 1$
- Canonical, $B/2Q = 5/4$
- TD limit, $B/2Q = 1$
- TD limit, $B/2Q = 5/4$
- Experiments

$K^-/\pi^- \text{ ratio}$

- $T = 100 \text{ MeV}, B/V = 0.04 \text{ fm}^3$

- Canonical, $B/2Q = 1$
- Canonical, $B/2Q = 5/4$
- TD limit, $B/2Q = 1$
- TD limit, $B/2Q = 5/4$
- Experiments
\[ T \text{ [GeV]} \]

\[ \frac{K^+}{K^-} = 21 \pm 9 \]

\[ \frac{K^+}{\pi^+} = 0.0074 \pm 0.0021 \]

\[ \frac{\phi}{K^-} = 0.1 \]

\[ \frac{\pi^+}{p} = 0.195 \pm 0.020 \]

\[ R = 4.2 \text{ fm} \]

\[ \frac{B}{2Q} = 1.04 \]