Quark masses and the chiral condensate with a non-perturbative renormalization procedure

V. Giménez\(^a\), L. Giusti\(^b\), F. Rapuano\(^c\), M. Talevi\(^d\), A. Vladikas\(^e\).

\(^a\)Univ. de València and IFIC, Dr. Moliner 50, E-46100, Burjassot, València, Spain
\(^b\)Scuola Normale Superiore and INFN Sezione di Pisa, Piazza dei Cavalieri 7 - I-56100 Pisa Italy
\(^c\)Università di Roma 'La Sapienza' and INFN, Sezione di Roma, P.le A. Moro 2, I-00185 Roma, Italy
\(^d\)Department of Physics and Astronomy, University of Edinburgh, Edinburgh EH9 3JZ, UK
\(^e\)INFN-Sezione di Roma II, Università di Tor Vergata, Via della Ricerca Scientifica 1 Roma, Italy

We determine the quark masses and the chiral condensate in the \(\overline{\text{MS}}\) scheme at NNLO from Lattice QCD in the quenched approximation at \(\beta = 6.0\), \(\beta = 6.2\) and \(\beta = 6.4\) using both the Wilson and the tree-level improved SW-Clover fermion action. We extract these quantities using the Vector and the Axial Ward Identities and non-perturbative values of the renormalization constants. We compare the results obtained with the two methods and we study the \(O(a)\) dependence of the quark masses for both actions.

1. INTRODUCTION

To describe the light hadron spectrum one considers massless QCD a good approximation to this world and the vacuum not symmetric under chiral transformations. The chiral condensate is the order parameter which governs the spontaneous chiral symmetry breaking. The pseudoscalar mesons are identified with the goldstone bosons and their physical masses are attributed to the mass term in the QCD Hamiltonian. Since free quarks are not physical states, quark masses cannot be measured directly in the experiments and can be determined from the meson spectrum using non-perturbative techniques. On the lattice one can compute the quark masses and the chiral condensate from first principles and it is the only procedure that can be systematically improved. The methods and the symbols we have used and all the results we have obtained are fully described in [1,2].

2. QUARK MASSES

The usual on-shell mass definition cannot be used for quark masses and their values depend on the theoretical definition adopted. In the following we will give our final results for the running quark masses defined in the \(\overline{\text{MS}}\) scheme. Quark masses can be defined from the Vector Ward Identity (VWI). Neglecting terms of \(O(a)\), the VWI between on-shell hadronic states can be written as [3]

\[
\langle \alpha | \mathcal{O}_h V_{\mu} | \beta \rangle = \frac{1}{2} \left( \frac{1}{k_2} - \frac{1}{k_1} \right) \langle \alpha | S | \beta \rangle .
\]

(1)

Eq. (1) fixes the relation between the lattice bare quark mass in lattice units and the hopping parameter, i.e. for the Wilson action \(m = 1/2(1/k - 1/k_c)\). Quark masses can also be extracted from the Axial Ward Identity (AWI). Neglecting terms of \(O(a)\), the AWI can be written as [3]

\[
Z_A(\alpha | \mathcal{O}_h A_{\mu} | \beta) = 2(m_0 - \overline{m}) |\alpha| P^\mu |\beta| ,
\]

where \(\overline{m}\) is defined in [3]. The light and strange quark masses are determined by fixing to their experimental values the masses of the \(\pi\) and \(K\) mesons. The standard perturbative approach [1,4] uses lattice and the continuum perturbation theory to connect the "bare" lattice quark mass to the renormalized \(\overline{\text{MS}}\) one. The scale \(1/a\), where \(a\) is the lattice spacing, of our simulations is \(a^{-1} \approx 2 - 4\) GeV. At these scales we expect small non-perturbative effects. However the
Table 1
Summary of the parameters of the runs analyzed in this work.

<table>
<thead>
<tr>
<th>Run</th>
<th>C60a</th>
<th>C60b</th>
<th>W60</th>
<th>C62</th>
<th>W62</th>
<th>C64</th>
<th>W64</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>6.0</td>
<td>6.0</td>
<td>6.0</td>
<td>6.2</td>
<td>6.2</td>
<td>6.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Action</td>
<td>SW</td>
<td>SW</td>
<td>Wil</td>
<td>SW</td>
<td>Wil</td>
<td>SW</td>
<td>Wil</td>
</tr>
<tr>
<td># Conf</td>
<td>400</td>
<td>600</td>
<td>320</td>
<td>250</td>
<td>250</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Volume</td>
<td>$18^3 \times 64$</td>
<td>$24^3 \times 40$</td>
<td>$18^3 \times 64$</td>
<td>$24^3 \times 64$</td>
<td>$24^3 \times 64$</td>
<td>$24^3 \times 64$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2
Quark Masses and the Chiral Condensate from the VVWI in MeV. $\overline{MS}$ values are at $\mu = 2$ GeV.

<table>
<thead>
<tr>
<th>Run</th>
<th>$m_q^{\overline{MS}}$</th>
<th>$m_s^{\overline{MS}}$</th>
<th>$-\frac{1}{N_F} \langle \bar{\psi} \psi \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W60</td>
<td>5.8(2)(1)</td>
<td>130(3)(2)</td>
<td>$247 \pm 2 \pm 1$</td>
</tr>
<tr>
<td>W62</td>
<td>5.4(2)(1)</td>
<td>124(4)(2)</td>
<td>$250 \pm 3 \pm 1$</td>
</tr>
<tr>
<td>W64</td>
<td>4.9(2)(1)</td>
<td>112(5)(2)</td>
<td>$258 \pm 4 \pm 1$</td>
</tr>
</tbody>
</table>

The $\overline{MS}$ definition of the renormalized quark masses is intrinsically perturbative and can be related to the $RI$ one through continuum perturbation theory only:

$$m_{\overline{MS}}(\mu) = U^{\overline{MS}}(\mu, \mu') \frac{Z_{\overline{MS}}^{RI}(\mu')}{Z_{\overline{MS}}^{RI}(\mu') m_{\overline{RI}}(\mu')}.$$  \hspace{1cm} (5)

where $U^{\overline{MS}}(\mu, \mu')$ is the RG evolution of the quark mass. $Z_{\overline{MS}}^{RI}/Z_{\overline{MS}}^{RI}$ is the matching factor computed in perturbation theory at scales $\mu \simeq 2-4$ GeV large enough to avoid non-perturbative effects and/or higher order corrections.

### 3. THE CHIRAL CONDENSATE

The chiral condensate can be defined using the AWI arising from the variation of the non-singlet pseudoscalar density. The integrated AWI becomes [2, 3]

$$\frac{1}{N_F} \langle \bar{\psi} \psi \rangle = \lim_{m_0 \to m} 2(m_0 - m) \int d^4 x \langle P(x) P(0) \rangle.$$
Table 3

Quark Masses and the Chiral Condensate from the AWI in MeV. $\overline{MS}$ values are at $\mu = 2$ GeV.

<table>
<thead>
<tr>
<th>Run</th>
<th>$m_t^{\overline{MS}}$</th>
<th>$m_s^{\overline{MS}}$</th>
<th>$-\frac{1}{N_f}\langle\bar{\psi}\psi\rangle_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C00a</td>
<td>6.0(2)(9)</td>
<td>136(4)(20)</td>
<td>(2.42 ± 3 ± 12)$^3$</td>
</tr>
<tr>
<td>C00b</td>
<td>5.7(2)(8)</td>
<td>132(4)(19)</td>
<td>(2.44 ± 2 ± 12)$^3$</td>
</tr>
<tr>
<td>W00</td>
<td>5.7(2)(8)</td>
<td>127(4)(17)</td>
<td>(2.48 ± 2 ± 11)$^3$</td>
</tr>
<tr>
<td>C62</td>
<td>5.8(5)(6)</td>
<td>131(7)(14)</td>
<td>(2.45 ± 4 ± 9)$^3$</td>
</tr>
<tr>
<td>W62</td>
<td>5.4(2)(5)</td>
<td>122(4)(12)</td>
<td>(2.51 ± 3 ± 8)$^3$</td>
</tr>
<tr>
<td>C64</td>
<td>4.4(3)(3)</td>
<td>104(5)(6)</td>
<td>(2.65 ± 4 ± 5)$^3$</td>
</tr>
<tr>
<td>W64</td>
<td>4.7(2)(3)</td>
<td>108(5)(8)</td>
<td>(2.62 ± 4 ± 6)$^3$</td>
</tr>
</tbody>
</table>

In the chiral limit one can show that the above expression is equivalent to

$$\frac{1}{N_f}\langle\bar{\psi}\psi\rangle = -\lim_{m_0 \to m_c} \frac{f_P^2 M_P^2}{4(m_0 - m)},$$

which is the familiar Gell-Mann–Oakes–Renner (GMOR) relation. We write the relation (6) for the renormalized condensate as

$$\frac{1}{N_f}\langle\bar{\psi}\psi\rangle_1 = -\frac{1}{2}a^{-1}f^2 Z脚步C_H^S,$$

$$\frac{1}{N_f}\langle\bar{\psi}\psi\rangle_2 = -\frac{1}{2}a^{-1}f^2 Z_A^S C^{AWI},$$

where $f_x = 0.1282$ GeV is the “experimental” value in physical units [2] and

$$M_P^2 = C_H^S \left(\frac{\kappa - \frac{1}{\kappa}}{\kappa C}\right),$$

$$2a \rho = \frac{1}{C^{AWI}} M_P^2.$$

The main advantages of the above formulas are to avoid the error amplification of the standard method because we are left with only one power of the UV cutoff $a^{-1}$ and to determine the slope $C_P C_H^S (C^{AWI})$ without extrapolation to the chiral limit. Note that since we work at $\kappa$ values typical of the strange quark mass, we are implicitly assuming that the slope will not change in the chiral region.

4. RESULTS

We have computed the renormalization constants of bilinear quark operators with the NP method proposed in [5]. The $\mu$-dependence of $Z_A$ and $Z_S$ are in excellent agreement with the RG predictions. $Z_P$ is in good agreement but the chiral symmetry breaking effects deserve a more accurate analysis. The parameters and the action used in each simulation are listed in Table 1 and the main results we have obtained are reported in Tables 2 and 3. The data at $\beta = 6.4$ have to be considered for an exploratory study only, since the physical volume and the time extension of the lattice are too small to be considered reliable. The $\overline{MS}$ values of the quark masses and the chiral condensate reported in Tables 2 and 3 show a very good agreement between the values extracted from the VWI and from the AWI using the non-perturbative determinations of the renormalization constants, while the same comparison using the perturbative values of $Z_A$, $Z_P$ and $Z_S$ fails giving inconsistent results for the two methods [1]. This pattern is also found by [7,8] for different fermion actions. In the $\beta$ range studied, there is no statistical evidence for an "a" dependence of the quark masses and the chiral condensate. We believe that the best estimates for the light and strange quark masses and the chiral condensate are $m_t^{\overline{MS}}(2 \text{ GeV}) = (5.7 \pm 5 \pm 8 \pm 8) \text{ MeV}$, $m_s^{\overline{MS}}(2 \text{ GeV}) = (130 \pm 8 \pm 15 \pm 15) \text{ MeV}$ and

$$-\frac{1}{N_f}\langle\bar{\psi}\psi\rangle^{\overline{MS}}(2 \text{ GeV}) = (245 \pm 4 \pm 9 \pm 7 \text{ MeV})^3,$$

where the first error is statistical, the second is due to the non-perturbative renormalization and the third is an estimate of the overall systematic errors on the bare quantities [1].
REFERENCES

1. V. Giménez et al. hep-lat/9801028 and hep-lat/9806006.
2. L. Giusti et al. hep-lat/9807014.
4. R.D. Kenway, these proceedings.
7. D. Becirevic et al. hep-lat/9807046.
8. N. Ishizula, these proceedings.