A SOLUTION TO THE HORIZON PROBLEM: 
A DELAYED BIG-BANG SINGULARITY

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Abstract

One of the main drawbacks of standard cosmology, known as the horizon problem, was until now thought to be only solvable in an inflationary scenario. A delayed Big-Bang in an inhomogeneous universe is shown to solve this problem while leaving unimpaired the main successful features of the standard model.

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1 Introduction

Standard cosmology is known to rest on three observational pillars: the expansion of the universe following Hubble law, the nearly isotropic black body cosmic microwave background radiation (CMBR) and the abundances of light elements produced during nucleosynthesis.

Besides these successful predictions, it leaves misunderstood other peculiar features of the observed universe.

In the present letter, a large class of initial singularity surfaces, the study of which has been initiated in a previous work [1], will be used to address one of the drawbacks of standard cosmology: the horizon problem.

The problem is the following: in hot Big Bang (BB) universes, the comoving region over which the CMBR is observed to be homogeneous to better than one part in $10^5$ is much larger than the comoving future light cone from the BB to the last scattering surface. The latter provides the maximal distance over which causal processes could have propagated since a given point on the BB surface. Hence, the observed quasi-isotropy of the CMBR remains unexplained.

Solving this problem was one of the main purposes of the inflationary paradigm as it was first put forward by Brout, Englert and Gunzig [2] in 1979 and independently by Guth [3] in 1981. But, as inflationary scenarios inflated, some self-produced undesirable features came into the way: for instance, reheating is not actually well understood and important details are still under study [4].

But despite these drawbacks, inflation has by now become a quasi-standard
paradigm as it was thought to be the only way to deal with the major horizon problem.

Following a suggestion by Hu, Turner and Weinberg [5], Liddle [6] has even proposed a proof that inflation is the only possible causal mechanism capable of generating density perturbations on scales well in excess of the Hubble radius, and hence the only way of solving the horizon problem.

As it was stressed by these authors, this problem involves the homogeneity and isotropy of the Freedmann-Robertson-Walker (FRW) model, proceeding from the Cosmological Principle upon which rests standard cosmology. Liddle’s entire argument depends only on the properties of the FRW metric.

The so-called Cosmological Principle is in fact not an a priori principle, but at most a simplifying working hypothesis: the universe being as it is, all astrophysics can do is to build models compatible with observation, should they contradict the Cosmological Principle. We come back to this point in the discussion.

The purpose of this letter is to solve the horizon problem by means of a delayed BB singularity in an inhomogeneous model of universe, thus discarding this Principle. For simplicity, we use a Tolman-Bondi model, since it allows a fully analytical exact reasoning.

This model will be described in section 2. Calculations and arguments will be developed in section 3 and some examples given in section 4. Section 5 will be devoted to a brief discussion of the results and to the conclusion.
2 An inhomogeneous delayed Big Bang model

In an expanding universe, going backward along the parameter called the cosmic time $t$ means going to growing energy densities and temperatures.

As one goes down the past, from our present matter dominated age defined by the constant temperature hypersurface $T \sim 2.73^\circ K$, one reaches an epoch when the radiation energy density overcomes the matter one. This radiation dominated area lasts until Planck time, $T_{Pl} \sim 10^{19} GeV$, which marks the limit beyond which quantum gravitational effects are expected to confuse our understanding of the laws of physics.

To deal with the horizon problem, one has to compute light cones. As will be further shown, a large class of models can be found for which the horizon problem is solved by means of light cones never leaving the matter dominated area. We will thus retain the Tolman-Bondi model for dust, an ideal non zero rest mass pressureless gas.

2.1 The class of Tolman-Bondi models retained

The Bondi line-element [7], in comoving coordinates $(r, \theta, \varphi)$ and proper time $t$, is:

$$ds^2 = -c^2 dt^2 + S^2(r,t) dr^2 + R^2(r,t)(d\theta^2 + \sin^2 \theta d\varphi^2)$$  \hspace{1cm} (1)

Solving Einstein’s equation for this metric with the dust stress-energy tensor gives:

$$S^2(r,t) = \frac{R^2(r,t)}{1 + 2E(r)/c^2}$$  \hspace{1cm} (2)
\[
\frac{1}{2} \dot{R}^2(r, t) - \frac{GM(r)}{R(r, t)} = E(r) \quad (3)
\]

\[
4\pi \rho(r, t) = \frac{M'(r)}{R'(r, t) R^2(r, t)} \quad (4)
\]

where a dot denotes differentiation with respect to \( t \) and a prime with respect to \( r \). \( \rho(r, t) \) is the energy density of the matter.

\( E(r) \) and \( M(r) \) are arbitrary functions of \( r \). \( E(r) \) can be interpreted as the total energy per unit mass and \( M(r) \) as the mass within the sphere of comoving radial coordinate \( r \).

\( M(r) \) remaining constant with time, it is used to define a radial coordinate \( r \): 
\[
M(r) \equiv M_0 r^3, \text{ where } M_0 \text{ is a constant.}
\]

Equation (3) can be solved and gives a parametric expression for \( R(r, t) \) for \( E(r) \neq 0 \) and an analytic one for \( E(r) = 0 \).

As there are evidences that the observed universe does not present appreciable spatial curvature, it can be reliably approximated by a flat \( E(r) = 0 \) Tolman-Bondi model.

With the above definition for the radial coordinate \( r \), \( R(r, t) \) possesses thus an analytical expression, which we write:
\[
R(r, t) = \left( \frac{9GM_0}{2} \right)^{1/3} r[t - t_0(r)]^{2/3} \quad (5)
\]

\( t_0(r) \) is another arbitrary function of \( r \), representing the BB singularity surface for which \( R(r, t) = 0 \). One can always choose \( t_0(r) = 0 \) at the center \( (r = 0) \) of the universe by an appropriate translation of the \( t = \) const. surfaces and describe our universe by the \( t > t_0(r) \) part of the \( (r, t) \) plane, increasing \( t \) corresponding to going from the past to the future.
Equation (5) substituted into equation (4) gives:

\[ \rho(r, t) = \frac{1}{2\pi G[3t - 3t_0(r) - 2rt'_0(r)][t - t_0(r)]} \] (6)

### 2.2 Shell-crossing

The above expression for \( \rho \) leads to two undesirable consequences:

1) The energy density goes to infinity not only on the BB surface \( t = t_0(r) \), but also on the shell-crossing surface:

\[ t = t_0(r) + \frac{2}{3}rt'_0(r) \] (7)

2) This energy density presents negative values in the region of the universe located between the shell-crossing surface (7) and the BB singularity, corresponding to \( 3t - 3t_0(r) - 2rt'_0(r) < 0 \) and \( t - t_0(r) > 0 \). One can wonder what does physically mean a negative energy density for dust.

Shell-crossing is thus generally considered as a mischief of Tolman-Bondi models and physicists usually try to avoid it [8], e.g. by assuming \( t'_0(r) \leq 0 \) for all \( r \).

But, as will be developed in next section, we need an increasing BB function \( t_0(r) \) to solve the horizon problem. Let us hence briefly show how to circumvent these two difficulties while keeping \( t'_0(r) > 0 \).

1) A way out the shell-crossing surface problem is to consider that, as the energy density increases while reaching its neighbourhood from higher values of \( t \), radiation becomes the dominant component of the universe, pressure can no more be neglected and the Tolman-Bondi model does no longer hold.
2) A negative value of $\rho$ proceeds from a negative value of $R'$ in equation (4).

The physical definition of energy density is:

$$\rho \equiv \frac{\delta M}{\delta V}$$ \hspace{1cm} (8)

$\delta M$ being the element of mass in an element of volume $\delta V$.

The element of 3-volume corresponding to the flat Tolman-Bondi metric, i.e. metric (1) with $S^2(r, t) = R'^2(r, t)$, is:

$$\delta V = R'R^2 \sin \theta drd\theta d\varphi$$ \hspace{1cm} (9)

which, when integrated over $\theta$ and $\varphi$, becomes:

$$\delta V = 4\pi R'R^2 dr$$ \hspace{1cm} (10)

As the physical volume $\delta V$ is by convention always positive, equation (10) possesses a physical meaning only if it is written:

$$\delta V = 4\pi |R'| R^2 dr$$ \hspace{1cm} (11)

And thus, in equation (4), one has to replace $R'$ by $|R'|$, which gives in equation (6):

$$\rho(r, t) = \frac{1}{2\pi G[3t - 3t_0(r) - 2rt'_0(r)][t - t_0(r)]}$$ \hspace{1cm} (12)

However, as, in the following, the light cones of interest never leave the region situated above the shell-crossing surface in the $(r, t)$ plane, $3t - 3t_0(r) - 2rt'_0(r)$ remains positive and equation (6) holds.
2.3 Definition of the temperature

In the course of this letter, we shall be led to use surfaces of constant temperature \( T \). Since the universe is not homogeneous, there is, at a given \( t \), no global thermodynamical equilibrium, and \( T \) is not readily defined. We assume that the characteristic scale of the \( \rho \) inhomogeneity is much larger than the characteristic length of the photon-baryon interaction and that there is always a local thermodynamical equilibrium. This enables us to define a local specific entropy \( S \) by:

\[
S(r) \equiv \frac{k_B n_\gamma(r,t)m_b}{\rho(r,t)}
\]

where \( m_b \) is the baryon mass and \( k_B \) the Boltzmann constant.

We then define \( T \) by:

\[
n_\gamma = a_n T^3 \]

where \( a_n = \frac{2\zeta(3)k_B^3}{\pi^2(hc)^3} \)

The following expression for \( T \) can then be obtained from equations (12) to (14):

\[
T(r,t) = \left( \frac{S(r)}{2\pi Gk_B m_b a_n [3t - 3t_0(r) - 2r t'_0(r)]^3} \right)^{1/3}
\]

The equation of the \( T = \text{const.} \) surfaces located after the shell-crossing surface, i.e. with \( 3t - 3t_0(r) - 2r t'_0(r) > 0 \), is thus the positive solution of the second order in \( t \) equation derived from equation (15):

\[
t = t_0(r) + \frac{r}{3} t'_0(r) + \frac{1}{3} \sqrt{r^2 t''_0(r) + \frac{3S(r)}{2\pi Gk_B a_n m_b T^3}}
\]
2.4 The “centered Earth” assumption

In this first approach of a delayed BB solution, the Earth will be assumed situated sufficiently close to the “center” of the universe, so as to justify the approximation \( r_p = 0 \), the subscript \( p \) refering to our actual location at the present time. We shall comment on this “center” of the universe in the final discussion.

The value \( S_p \) of the entropy function at \((r_p, t_p)\) is \( S_p = k_B \eta_p \), \( \eta_p \) being the present local photon to baryon density ratio, which is taken to be of order \( 10^8 \).

Remembering that \( t_0(r=0) \) has been chosen to be zero, one can add to the specifications of the \( t_0(r) \) function:

\[
rt'_0|_{r=0} = 0
\]

to get at \( (r = 0)\):

\[
t_p \sim 3 \times 10^{17} \text{s} \quad \text{for} \quad T_p = 2.73^\circ \text{K}
\]

\[
t_{ls} \sim 6 \times 10^{12} \text{s} \quad \text{for} \quad T_{ls} = 4000^\circ \text{K}
\]

These values are of the same order of magnitude as in the standard hot BB model. The nucleosynthesis scenario would thus approximately be the standard one for the here described universe in the vinicity of \( r = 0 \), provided the characteristic length of the density inhomogeneities is much larger than the mean free path of the nucleons. This latter condition will be discussed in section 5.

As the light elements abundances predicted by standard cosmology fit rather well the data observed in our neighbourhood, the choice of the “center” of the universe for the location of the Earth seems justified, as far as the above cited condition obtains.
This good agreement between the observed data and the abundances predicted by the standard model led Liddle [6] to adopt the Cosmological Principle and thus assert that the FRW metric obtains for the whole universe. This was a key-assumption for his tentative proof discussed in our Introduction.

But this assumption seems exceedingly narrowing as far as the present available data have been measured in our direct neighbourhood as compared to cosmological distances - the today most remote measured abundances are for deuterium at redshifts $z < 5$ [9]. The “centered Earth” assumption seems thus enough to complete the game.

3 Solving the horizon problem

Light travels from the last scattering surface to a present local observer on a light cone going from $(r_p = 0, t_p)$ to a 2-sphere $(r_{ls}, t_{ls})$ on the last scattering 3-sphere defined by $T = 4000^\circ$ K.

To solve the horizon problem, it is sufficient to show that this 2-sphere can be contained inside the future light cone of any $(r = 0, t > 0)$ point of space-time.

One of the key-points of the reasoning here proposed is a shell-crossing surface situated above the BB surface and monotonously increasing with increasing $r$, which is always verified if:

$$t'_0(r) > 0 \quad \text{for all} \quad r$$

$$5t'_0(r) + 2rt''_0(r) > 0 \quad \text{for all} \quad r$$

A $t_0(r)$ function increasing with $r$ implies that the BB “occured” at later $t$
for larger $r$, hence the evocative “delayed Big-Bang” we chose to qualify this singularity.

The model being spherically symmetrical and the null geodesics being radial, the relevant light cones are obtained for $\theta = \text{const.}$ and $\varphi = \text{const.}$ in equation (1).

Writing $ds^2 = 0$ in this equation, one gets a differential equation for the null cones:

$$\frac{dt}{dr} = \pm \frac{R'}{c}$$ \hspace{1cm} (17)

Substituting above the expression of $R'$ obtained from equation (5), one finds:

$$\frac{dt}{dr} = \pm \frac{1}{3c} \left( \frac{9GM_0}{2} \right)^{1/3} \frac{3t - 3t_0(r) - 2rt'_0(r)}{[t - t_0(r)]^{1/3}}$$ \hspace{1cm} (18)

Comparing to equation (7), one immediately sees that the curves representing the light cones in the $(r,t)$ plane possess an horizontal tangent on and only on the shell-crossing surface, where $3t - 3t_0(r) - 2rt'_0(r)$ goes to zero.

The curve $t(r)$ for the past light cone from $(r_p, t_p)$ verifies equation (18) with the minus sign.

As far as one considers the part of this light cone located after the shell-crossing surface, and thus after the BB singularity, $3t - 3t_0(r) - 2rt'_0(r)$ and $t - t_0(r)$ remain positive and $\frac{dt}{dr}$ is always negative. $t(r)$ is a strictly decreasing function of $r$ and the light cone will have to cross the strictly increasing shell-crossing surface at a finite point where the derivative of $t(r)$ goes to zero.

On its way to shell-crossing, the null geodesic will cross in turn each $T = \text{const.}$ surface at a finite point.

Let $(r_{ls1}, t_{ls1})$ be the coordinates of the crossing point on the last scattering
surface $T = 4000^\circ$ K.

Now consider a backward null radial geodesic starting from any point above the shell-crossing surface and directed towards $r = 0$. Its equation is a solution of differential equation (18) with the plus sign.

Its derivative remains positive as long as it does not reach the shell-crossing surface. If it was to reach this surface before the “center” of the universe, its derivative would go directly from a positive value to zero, which would imply for the curve of the light cone an horizontal tangent in the $(r, t)$ plane.

Since we consider models for which the curve representing the shell-crossing surface is strictly increasing with $r$, it cannot be horizontally crossed from upper values of $r$ and $t$ by a strictly increasing curve. And thus one is led to an inconsistency.

This implies that the backward light cone starting from any point above the shell-crossing surface reaches $r = 0$ at $t_c$ without crossing this surface, and thus with $t_c > 0$.

This statement holds for every light cone issued from any point on the last scattering surface.

There is thus an infinite number of points $(r = 0, t_c > 0)$ of which the future light cone contains the sphere on the last scattering surface seen today in the CMBR.

Every point on this sphere can be causally connected and the horizon problem is solved.
4 Examples of appropriate Big Bang functions

In previous sections, conditions have been imposed upon the BB function $t_0(r)$. They can be summarized as follows:

\[
\begin{align*}
t_0(r = 0) &= 0 \\
t'_0(r) &= 0 \quad \text{for all} \quad r \\
5t'_0(r) + 2rt''_0(r) &= 0 \quad \text{for all} \quad r \\
rt'_0 |_{r=0} &= 0
\end{align*}
\]

It is easy to verify that the class of functions:

\[
t_0(r) = br^n \quad b > 0 \quad n > 0
\] (19)

fulfills these conditions.

Another feature imposed upon the model to justify the dust approximation is that the light cones, for the Tolman-Bondi metric, never leave the matter dominated area. This prescription has been tested upon peculiar models of the above class with $S(r) = \text{const.} = k_B\eta_p$.

A number of light cones were numerically integrated with different values for $n$ and $b$, in units $c = 1$, $t_p = \frac{9GM_0}{2}$.

It has been in particular found that:

(1) For $n = 1$, the backward null geodesics starting from $(r_{ls1}, t_{ls1})$ reach $r = 0$ without leaving the matter dominated area (approximately delimited near $r = 0$ by $T = T_{eq} = 10^5 \circ K$) provided $b$ is kept larger than about $10^{12}s$. 

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(2) for $n = 2$, an analogous condition holds, the limiting value for $b$ being about $10^{14}$s.

Figures 1 and 2 show the case $n = 2$, $b = 5 \times 10^{14}$s for which $t_c = 2.18 \times 10^{12}$s.

5 Discussion and conclusion

Using a delayed BB universe, the horizon problem has been solved for a class of simple models fulfilling some restricting conditions. Further work will be necessary to discriminate between these conditions which are generic and which are only generated by the assumptions made for simplification purpose.

For instance, the dust approximation used in this letter has been retained to allow analytical calculations. It was shown in section 4 that this dust choice provided constraints upon the BB function $t_0(r)$.

The behaviour of light cones in the radiation dominated region is thus an appealing issue for future work. If it could be proven that the geometry of this region does not bend the light cones such as to have them reach the BB surface before $r = 0$, then the above constraints could be discarded.

One could therefore consider the a priori interesting case of a BB function, fulfilling conditions summarized at the beginning of section 4, but arbitrarily close to the FRW $t_0(r) = 0$.

In this case, the “unnatural” prescription “centered Earth” would no more be needed as the conditions for standard nucleosynthesis could be verified for an observer located at arbitrary values of $r$. 

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This would imply that the characteristic length of the density inhomogeneities, written for the radiation dominated model, at the Earth location, is much smaller than the mean free path of the nucleons. A constraint would thus appear on $t_0(r)$, i.e. limiting $b$ to small values compatible with the “close to FRW” assumption.

The authors are well aware of a potential difficulty of the present model, namely to put the observer near the “center” of the universe. In addition to the fact that such a location is not forbidden by scientific but only by philosophical principles (which they do not accept), they want to stress that the present model is only a first “toy-model”. They hope to build, in the future, less simple models, getting rid of this prescription.

In a recent work, they have shown that a delayed BB of type $t_0(r) = br$ can reproduce the observed dipole and quadrupole in the CMBR anisotropies - a first version of this work has been submitted for publication [1] with a decreasing BB function (negative values for $b$), but it is easy to see that the same results hold for $b > 0$.

It comes out from this work that to any given value of the location $r_p$ of the observer corresponds a value of $b$ for which the observed data are reproduced. $b$ is all the smaller as $r_p$ is larger.

A reliable model of universe of this kind could thus get rid of the “centered Earth” assumption, provided $b$ should be sufficiently small. This implies that the null geodesics, if causally connected, should be so in the radiation dominated region.

Now, why should we feel uncomfortable with the idea that we could be located near the “center” of the universe? Following Ellis, Maartens and Nel [10, 11], who
also dared assume such an “unnatural” prescription in their Static Spherically Sym-
metric (SSS) model of universe, one can claim that this is no more (un)reasonable
than the belief in a Cosmological Principle. The purpose is not to put the observer
at the “center” a priori, but to answer the question as it was put forward by these
authors in their cited papers: “Given a universe model of the type proposed, where
would one be likely to find life like that we know on Earth?” The answer of Ellis
is: “where conditions are favorable for life of this kind ... near the center, where the
universe is cool.”

The flat universe approximation, even if it seems more physically justified, can
also be discussed. If our universe would be proved not so flat as it seems to be, the
study of the open (closed ?) case would become necessary.

However, even if the here proposed class of models appears as a restrictive answer
to the horizon problem, it might, with some easy to conceive improvements, equally
account for structure formation and all scales anisotropies of the CMBR, which
could otherwise proceed from a topological defects like mechanism. This will be the
purpose of other work to come.

Inflation was, from the beginning of its success story, equally aimed at solving
the flatness and monopole problems.

As it is a mere product of Friedmann’s equations, the flatness problem only per-
tains to FRW universes and is thus irrelevant for the class of models here proposed.

As for the monopole problem, delayed BB without inflation only implies that
topological defects theories leading to a production of local stable monopoles are
ruled out.
This letter is a first attempt to show that the delayed BB scenario, as it is a natural and simple way to solve the problems of standard cosmology while keeping its best successful predictions, is worth spending time and efforts to bring it to at least as worthy a paradigm as any other on the market place.

The last point to emphasize is the following. In the years to come, two satellite boarded missions, MAP to be launched by NASA and Planck to be launched by ESA, will be dedicated to a high-resolution mapping of the CMBR anisotropies. One of their main purposes is to provide a test of cosmological theories and an estimation of cosmological parameters.

Number of recent papers attempt to show how the values of these parameters could be determined by an analysis of the data thus obtained, see e.g. [12], [13] and [14]. In these papers, the BB function $t_0(r)$ is always implicitly or explicitly set to a constant value over the spatial coordinate $r$, and the cosmological parameters considered are those pertaining to a universe with FRW background.

To be complete, the analysis of these future data will also have to be performed in the light of the present results.

References


Figure 1. case $n = 2$, $b = 5 \times 10^{14}$. The solid lines are the light cones, the upper one being the null geodesic issued from $(r_p, t_p)$ ending at $(r_{ls1}, t_{ls1})$ on the last scattering surface, the lower one being the null cone from $(r_{ls1}, t_{ls1})$ to $(r = 0, t_c)$. The dashed line represents the Big-Bang surface, and the plotted one, the last scattering and the shell-crossing surfaces which cannot be resolved at the scale of the figure.
**Figure 2.** Figure 1 zoomed on small values of $r$ and $t$. The solid line is the light cone from $(r_{\text{ba}}, t_{\text{ba}})$ to $(r = 0, t_c)$. The dashed line represents the Big-Bang surface, the dotted one, the last scattering surface and the dash-dot-dashed one, the shell-crossing and the radiation-matter equality surfaces which cannot be resolved at the scale of the figure.