ELECTROWEAK RADIATIVE CORRECTIONS TO W BOSON PRODUCTION AT THE TEVATRON$^a$

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We discuss the $\mathcal{O}(\alpha)$ electroweak radiative corrections to $W$ boson production at the Tevatron and their effect on the $W$ boson mass extracted by experiment. The results of a new calculation of the $\mathcal{O}(\alpha)$ corrections are presented and compared with those of a previous calculation. We also briefly discuss the $\mathcal{O}(\alpha)$ corrections to $Z$ boson production at the Tevatron and two-photon radiation in $W$ and $Z$ events.

1 Introduction

The Standard Model of electroweak interactions (SM) so far has met all experimental challenges and is now tested at the 0.1% level $^1$. However, there is little direct experimental information on the mechanism which generates the masses of the weak gauge bosons. In the SM, spontaneous symmetry breaking is responsible for mass generation. The existence of a Higgs boson is a direct consequence of this mechanism. At present the negative result of direct searches performed at LEP2 imposes a lower bound of $M_H > 89.8$ GeV $^2$ on the Higgs boson mass. Indirect information on the mass of the Higgs boson can be extracted from the $M_H$ dependence of radiative corrections to the $W$ boson mass, $M_W$, and the effective weak mixing angle, $\sin^2 \theta_{\text{eff}} \alpha$. Assuming the SM to be valid, a global $\chi^2$-fit to all available electroweak precision data yields a 95% confidence level upper limit on $M_H$ of 280 GeV $^1$.

The current estimate of $M_H$ strongly depends $^3$ on the world average for the weak mixing angle, $\sin^2 \theta_{\text{eff}} \alpha = 0.23155 \pm 0.00019^1$. It results from a combination of LEP and SLC data which currently are not in good agreement $^1$. Furthermore, $\sin^2 \theta_{\text{eff}} \alpha$ is quite sensitive to the hadronic contribution to $\alpha(M_Z^2)$, $\Delta \alpha_{\text{had}}(M_Z^2)$. The accuracy of $\Delta \alpha_{\text{had}}(M_Z^2)$ has been the subject of a number of publications during the last four years $^4$. Error estimates range between $\delta(\Delta \alpha_{\text{had}}(M_Z^2)) = 0.0007^5$ and $\delta(\Delta \alpha_{\text{had}}(M_Z^2)) = 0.0016^6$. A smaller error for $\Delta \alpha_{\text{had}}(M_Z^2)$ implies that $\sin^2 \theta_{\text{eff}} \alpha$ receives more weight in the $M_H$ fit, i.e. the discrepancy between the LEP and SLC data becomes a limiting factor in the estimate of the Higgs boson mass from electroweak data.

A more precise measurement of $M_W$ is, therefore, very important in order to extract more accurate information on $M_H$ from electroweak data. Furthermore, in contrast to $\sin^2 \theta_{\text{eff}} \alpha$, the $W$ mass depends only mildly on $\Delta \alpha_{\text{had}}(M_Z^2)^3$. A more precise measurement of $M_W$ thus automatically reduces the sensitivity of the extracted Higgs boson mass to $\Delta \alpha_{\text{had}}(M_Z^2)$. Currently, the $W$ boson mass is known to $\pm 0.06$ GeV $^1$ from direct measurements. A significant improvement in the $W$ mass uncertainty is expected in the near future from measurements at LEP2 $^7$ and the Tevatron $^8$. The ultimate precision expected for $M_W$ from the combined LEP2 experiments is approximately 40 MeV $^7$. At the Tevatron, integrated luminosities of order 2 fb$^{-1}$ are foreseen for Run II, and one expects to measure the $W$ mass with a precision of approximately 40 MeV$^8$ per experiment and decay channel.

In order to measure the $W$ boson mass with high precision in a hadron collider environment, it is necessary to fully understand and control higher order QCD and electroweak (EW) corrections to $W$ production. The determination of the $W$ mass in a hadron collider environment requires a simultaneous precision measurement of the $Z$ boson mass, $M_Z$, and width, $\Gamma_Z$. When compared to the value measured at LEP, the two quantities help to accurately determine the energy scale and resolution of the electromagnetic calorimeter, and to constrain the muon momentum resolution $^5$. It is therefore also necessary to understand the higher order EW corrections to $Z$ boson production in hadronic collisions.

Recently, new and more accurate calculations of the $\mathcal{O}(\alpha)$ EW corrections to $W$ $^9$ and $Z$ boson production in hadronic collisions $^10$ became available. In a previous calculation, only the final state photonic corrections were correctly included $^11$. The sum of the soft and virtual parts was estimated from the inclusive $\mathcal{O}(\alpha^2)$ $W \rightarrow \ell \nu (\gamma)$ and $Z \rightarrow \ell^+\ell^- (\gamma)$ ($\ell = e, \mu$) width and the hard photon bremsstrahlung contribution. Initial state, interference, and weak contributions to the $\mathcal{O}(\alpha)$ corrections were ignored altogether. The unknown part of the $\mathcal{O}(\alpha)$ EW

corrections in Ref. [11], combined with effects of multiple photon emission, have been estimated to contribute a systematic uncertainty of $\delta M_W = 15 - 20$ MeV to the measurement of the $W$ mass.\(^8\)

In Section 2, we briefly describe the technical details of the calculation of the $O(\alpha)$ corrections to $W$ boson production presented in Ref. [9], and compare the results with those of Ref. [11]. In Section 3 we summarize the calculation of the $O(\alpha)$ QED corrections to $Z$ boson production reported in Ref. [10], and in Section 4 some preliminary results of a new calculation\(^{12}\) of two-photon radiation in $W$ and $Z$ production in hadronic collisions are presented.

2 Electroweak Corrections to $W$ Boson Production at the Tevatron

The calculation of the $O(\alpha)$ corrections to $p\bar{p} \to W \to \ell\nu$ is based on the full set of $O(\alpha^3)$ Feynman diagrams, and includes both initial and final state radiative corrections, as well as the contributions from their interference. Final state charged lepton mass effects are included in the following approximation. The lepton mass regularizes the collinear singularity associated with final state photon radiation. The associated mass singular logarithms of the form $\ln(\hat{s}/m_\ell^2)$, where $\hat{s}$ is the squared parton center of mass energy and $m_\ell$ is the charged lepton mass, are included in our calculation, but the very small terms of $O(m_\ell^2/\hat{s})$ are neglected.

To perform the calculation, a Monte Carlo method for next-to-leading-order (NLO) calculations similar to that described in Ref. [13] was used. With the Monte Carlo method, it is easy to calculate a variety of observables simultaneously and to simulate detector response. Calculating the EW radiative corrections to $W$ boson production, the problem arises how an unstable charged gauge boson can be treated consistently in the framework of perturbation theory. This problem has been studied in Ref. [14] with particular emphasis on finding a gauge invariant decomposition of the EW $O(\alpha)$ corrections into a QED-like and a modified weak part. In $W$ production, the Feynman diagrams which involve a virtual photon do not represent a gauge invariant subset. In Ref. [14] it was demonstrated how gauge invariant contributions that contain the infrared (IR) singular terms can be extracted from the virtual photonic corrections. These contributions can be combined with the also IR-singular real photon corrections in the soft photon region to form IR-finite gauge invariant QED-like contributions corresponding to initial state, final state and interference corrections. The IR finite remainder of the virtual photonic corrections and the pure weak one-loop corrections can be combined to separately gauge invariant modified weak contributions to the $W$ boson production and decay processes.

The collinear singularities associated with initial state photon radiation can be removed by universal collinear counter terms generated by “renormalizing” the parton distribution functions (PDF’s)\(^{15}\), in complete analogy to gluon emission in QCD. In addition to the collinear counterterms, finite terms can be absorbed into the PDF’s, introducing a QED factorization scheme dependence. We have carried out our calculation in the QED DIS and QED $\overline{M}\Sigma$ scheme. In order to treat the $O(\alpha)$ initial state QED-like corrections to $W$ production in hadronic collisions in a consistent way, QED corrections should be incorporated in the global fitting of the PDF’s using the same factorization scheme which has been employed to calculate the cross section. Current fits to the PDF’s do not include QED corrections. A study of the effect of QED corrections on the evolution of the parton distribution functions indicates\(^{15}\) that the modification of the PDF’s is small. The missing QED corrections to the PDF introduce an uncertainty which, however, is likely to be smaller than the present uncertainties on the parton distribution functions.

Since hadron collider detectors cannot directly detect the neutrinos produced in the leptonic $W$ boson decays, $W \to \ell\nu$, and cannot measure the longitudinal component of the recoil momentum, there is insufficient information to reconstruct the invariant mass of the $W$ boson. Instead, the transverse mass ($M_T$) distribution of the $\ell\nu$ system is used to extract $M_W$. The various individual contributions to the EW $O(\alpha)$ corrections of the $M_T$ distribution are shown in Fig. 1. To compute the

- \(^8\)\cite{ref_11}
- \(^9\)\cite{ref_8}
- \(^12\)\cite{ref_12}
- \(^15\)\cite{ref_15}
cross section, we have used here the MRSA set of parton
distribution functions\textsuperscript{16}. The detector acceptance is sim-
ulated by imposing the following transverse momentum
($p_T$) and pseudo-rapidity ($\eta$) cuts:

\begin{equation}
    p_T(\ell) > 25 \text{ GeV}, \quad |\eta(\ell)| < 1.2, \quad \ell = e, \mu, \tag{1}
\end{equation}

\begin{equation}
    p_T^\mu > 25 \text{ GeV}. \tag{2}
\end{equation}

These cuts approximately model the acceptance cuts
used by the CDF and DØ collaborations in their $W$ mass
analyses. Uncertainties in the energy and momentum
measurements of the charged leptons in the detector are
simulated in the calculation by Gaussian smearing of the
particle four-momentum vector using the specifications
for the upgraded Run II DØ detector\textsuperscript{17}.

The initial state QED-like contribution uniformly
increases the cross section by about 1\% for electron
(Fig. 1a) and muon (Fig. 1b) final states. It is largely
canceled by the modified weak initial state contribution.
The interference contribution is very small. It decreases
the cross section by about 0.01\% for transverse masses
below $M_W$, and by up to 0.5\% for $M_T > M_W$. The final
state QED-like contribution significantly changes the
shape of the transverse mass distribution and reaches
its maximum effect in the region of the Jacobian peak,
$M_T \approx M_W$. Since the final state QED-like contribution
is proportional to $\ln(s/m_\gamma^2)$, its size for muons is
considerably smaller than for electrons. As for the initial
state, the modified weak final state contribution reduces
the cross section by about 1\%, and has no effect on the
shape of the transverse mass distribution.

In Fig. 1, we have not taken into account realistic
lepton identification requirements. When these require-
ments are included, the mass singular logarithmic terms
are eliminated in the electron case because the electron
and photon momentum four vectors are combined for
small opening angles where it is difficult to resolve the
two particles\textsuperscript{8}. This significantly reduces the size of the
EW corrections. On the other hand, in order to experi-
mentally identify muons, the energy of the photon is
required to be smaller than a critical value if the $\mu - \gamma$
separation is small, and mass singular terms survive.
Removing energetic photons thus enhances the effect of the
$O(\alpha)$ corrections, and the effect of the EW corrections
in the muon case is larger than in the electron case once
lepton identification requirements are included.

As we have seen, final state bremsstrahlung has a
non-negligible effect on the shape of the $M_T$ distribution
in the Jacobian peak region. It is well known that EW
corrections must be included when the $W$ boson mass is
extracted from data, otherwise the mass is shifted to a
lower value. In the approximate treatment of the electroweak corrections used so far by the Tevatron exper-
iments, only final state QED corrections are taken into
account; initial state, interference, and weak correction
terms are ignored. Furthermore, the effect of the final
state soft and virtual photonic corrections is estimated
from the inclusive $O(\alpha^2) W \rightarrow \ell\nu(\gamma)$ width and the hard
photon bremsstrahlung contribution\textsuperscript{11}. When detector
effects are included, the approximate calculation leads
to a shift of about $-50 \text{ MeV}$ in the electron case, and
approximately $-160 \text{ MeV}$ in the muon case\textsuperscript{8}.

Initial state and interference contributions do not change
the shape of the $M_T$ distribution significantly
(see Fig. 1) and therefore have little effect on the ex-
tracted mass. However, correctly incorporating the final
state virtual and soft photonic corrections results in a
non-negligible modification of the shape of the transverse
mass distribution for $M_T > M_W$. This is demonstrated in
Fig. 2, which shows the ratio of the $M_T$ distribution
obtained with the QED-like final state correction part of
our calculation to the one obtained using the approxima-
tion of Ref. \textsuperscript{11}.

The difference in the line shape of the $M_T$ distribu-
tion between the $O(\alpha^3)$ calculation of Ref. \textsuperscript{9} and the
approximation used so far occurs in a region which is
important for both the determination of the $W$ mass,
and the direct measurement of the $W$ width. The pre-
cision which can be achieved in a measurement of $M_W$
using the transverse mass distribution strongly depends
on how steeply the $M_T$ distribution falls in the region
$M_T \approx M_W$. Any change in the theoretical prediction
of the line shape thus directly influences the $W$ mass
measurement. From a maximum likelihood analysis the
shift in the measured $W$ mass due to the correct treat-
ment of the final state virtual and soft photonic correc-
tions is found to be $\Delta M_W \approx O(10 \text{ MeV})$. This shift
is much smaller than the present uncertainty for $M_W$
from hadron collider experiments\textsuperscript{8}. However, for future
precision experiments, a difference of $O(10$ MeV) in the extracted value of $M_W$ can no longer be ignored, and the complete $O(\alpha^3)$ calculation should be used.

3 Electroweak Corrections to Z Boson Production at the Tevatron

The calculation of the $O(\alpha^3)$ corrections to Z boson production\textsuperscript{10} employs the same Monte Carlo method which was used in the $W$ case. The collinear singularities originating from initial state photon radiation are again removed by counter terms generated by renormalizing the PDF’s. However, in contrast to $W$ production, the Feynman diagrams contributing to the $O(\alpha)\,$QED corrections can be separated into gauge invariant subsets corresponding to initial and final state corrections. Furthermore, the purely weak corrections form a separately gauge invariant set of diagrams. The weak corrections are expected to be very small and are therefore ignored in our calculation. Both $Z$ and photon exchange diagrams with all $\gamma - Z$ interference effects are incorporated.

In Fig. 3 we display the ratio of the $O(\alpha^3)$ and the Born cross section as a function of the $\ell^+\ell^-$ invariant mass in $p\bar{p} \to Z \to \ell^+\ell^-$. For $40\,\text{GeV} < m(\ell^+\ell^-) < 110\,\text{GeV}$, the cross section ratio is seen to vary rapidly. Below the $Z$ peak, QED corrections enhance the cross section by up to a factor 2.7 (1.9) for electrons (muons). The maximum enhancement of the cross section occurs at $m(\ell^+\ell^-) \approx 75\,\text{GeV}$. At the $Z$ peak, the differential cross section is reduced by about 30% (20%). For $m(\ell^+\ell^-) > 130\,\text{GeV}$, the $O(\alpha)$ QED corrections uniformly reduce the differential cross section by about 12% in the electron case, and $\approx 7\%$ in the muon case. No lepton identification requirements are taken into account in Fig. 3. When these are included, the maximum enhancement is reduced to a factor 1.6 for both electrons and muons. As for the $W$, initial state corrections are uniform and small. Final state radiative corrections dominate over the entire di-lepton invariant mass range.

From Fig. 3 it is clear that final state bremsstrahlung severely distorts the Breit-Wigner shape of the $Z$ resonance curve. As a result, QED corrections must be included when the $Z$ boson mass is extracted from data, otherwise the mass extracted is shifted to a lower value. As in the $W$ case, in the approximate treatment of the QED corrections to $Z$ boson production used so far by the Tevatron experiments, only final state corrections are taken into account, and the effects of soft and virtual corrections are estimated from the inclusive $O(\alpha^3)$ $Z \to \ell^+\ell^- (\gamma)$ width and the hard photon bremsstrahlung contribution\textsuperscript{11}. When detector effects are taken into account, the approximate calculation leads to a shift of the $Z$ mass of about $-150\,\text{MeV}$ in the electron case, and approximately $-300\,\text{MeV}$ in the muon case\textsuperscript{8}. The $Z$ boson mass extracted from our $O(\alpha^3)$ $\ell^+\ell^-$ invariant mass distribution is found to be about 10 MeV smaller than that obtained using the approximate calculation of Ref. [11]. This translates into an additional shift of several MeV in $M_W$ through the dependence of the energy scale and the momentum resolution on the $Z$ boson mass measured.

4 Two Photon Radiation in $W$ and $Z$ Boson Production at the Tevatron

The $O(\alpha)$ EW corrections have a significant effect on the $W$ and $Z$ masses extracted by DØ and CDF. In particular, the large shift in the masses caused by the emission of a photon from the final state lepton line raises the question of how strongly multiple photon radiation influences the measured weak boson masses. At $O(\alpha^3)$, $W$ or $Z$ decay with collinear emission of photons from a final state charged lepton gives rise to terms which are proportional to $(\alpha/\pi^n) \ln^n(M_V^2/m_\ell^2)$ ($V = W, Z$) in $n$-photon exclusive rates.

In order to find out how important multi photon radiation in $W$ and $Z$ production is for the measurement of $M_W$ at the Tevatron, it is instructive to carry out a calculation of the two photon processes, $p\bar{p} \to \ell\nu\gamma\gamma$ and $p\bar{p} \to \ell^+\ell^-\gamma\gamma$. So far no calculation of these processes which is based on the full set of tree level $O(\alpha^3)$ Feynman diagrams, and which is valid for arbitrary lepton-photon opening angles, has been carried out. For example, the calculation of Ref. [18] assumes that $m_\ell = 0$. A non-zero $\Delta R_{\ell\gamma}$ cut, therefore, has to be imposed in order to avoid the collinear singularities. The Monte Carlo generator PHOTOS\textsuperscript{15}, on the other hand, treats final state photon radiation in the leading-log approximation and thus
leads to results which can only be trusted in the collinear region. PHOTOS ignores initial state photon radiation altogether.

In order to correctly take into account the effects of two photon radiation in W and Z production, a calculation which gives correct results for small as well as large lepton-photon opening angles is required. Here we report some preliminary results of such a calculation which is presently carried out\textsuperscript{12}. Due to the collinear singularities associated with photon radiation from the charged lepton lines, there are many different peaks in the multidifferential cross section. For an accurate evaluation of the cross section we therefore use a multiconfiguration Monte Carlo integration routine which automatically maps the peaks in the differential cross section to a uniform function according to the pole structure of the contributing Feynman diagrams\textsuperscript{20}. The matrix elements, taking into account finite lepton masses, are calculated using the MADGRAPH package\textsuperscript{21}, which automatically generates matrix elements in HELAS format\textsuperscript{22}. In order to maintain electromagnetic gauge invariance for e\nu\gamma production in presence of finite W width effects, the W propagator and the WW and W\gamma\gamma vertex functions in the amplitudes generated by MADGRAPH are modified, using the prescription of Ref. [18].

In Table 1, we display the fraction of W \rightarrow e\nu and Z \rightarrow e^+e^- events (in percent) containing one or two photons at the Tevatron as a function of the minimum photon transverse energy, E_T^{\text{min}} for E_T^{\text{min}} \geq 0.1 \text{ GeV}, the approximate tower threshold of the electromagnetic calorimeters of CDF and DØ. To simulate detector response, we have imposed the following acceptance cuts:

\[ p_T(e) > 20 \text{ GeV}, \, |\eta(e)| < 2.5, \, \text{and} \, |\eta(\gamma)| < 3.6. \]  

In the W case, we require in addition that

\[ p_T > 20 \text{ GeV} \]  

and

\[ 65 \text{ GeV} < M_T(e + n\gamma; \nu) < 100 \text{ GeV}, \]  

where \( M_T(e + n\gamma; \nu) \) is the cluster transverse mass of the \( (e + n\gamma)\nu \) system \( (n = 0, 1, 2) \). For Z events we require

\[ m(e^+e^-) > 20 \text{ GeV} \]  

and

\[ 75 \text{ GeV} < m(ee + n\gamma) < 105 \text{ GeV}, \]  

where \( m(e^+e^-) \) \( (m(ee + n\gamma)) \) is the e^+e^- (ee + n\gamma) invariant mass. For muon final states, the fraction of events containing one (two) photons is roughly a factor 2 (4) smaller than the results shown for W \rightarrow e\nu and Z \rightarrow e^+e^-.

Table 1 demonstrates that a significant fraction of weak boson events contains two photons. Multiple photon bremsstrahlung thus is expected to have a non-negligible effect on the W mass extracted from experiment.

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Acknowledgements

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References

1. D. Karlen, these proceedings; M. Grünewald, these proceedings.
2. P. McNamara, these proceedings.
4. A. Höcker, these proceedings.
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a) $p\bar{p} \rightarrow e^+\nu(\gamma)$  
$\sqrt{s} = 1.8$ TeV  

$\frac{[d\sigma/dM_T]}{[d\sigma_{\text{Born}}/dM_T]}$  

- initial state  
- interference  
- final state  

$M_T$ (GeV)  

b) $p\bar{p} \rightarrow \mu^+\nu(\gamma)$  
$\sqrt{s} = 1.8$ TeV  

$\frac{[d\sigma/dM_T]}{[d\sigma_{\text{Born}}/dM_T]}$  

- initial state  
- interference  
- final state  

$M_T$ (GeV)
a) $p\bar{p} \rightarrow l^+\nu(\gamma)$ FSR only
\[ \sqrt{s} = 1.8 \text{ TeV} \]
no lept. id. req. included

solid: electrons
dash: muons $\delta_s = 0.01$

b) $p\bar{p} \rightarrow l^+\nu(\gamma)$ FSR only
\[ \sqrt{s} = 1.8 \text{ TeV} \]
with lept. id. req. included

solid: electrons
dash: muons $\delta_s = 0.01$
$p\bar{p} \rightarrow l^+l^-(\gamma)$

$\sqrt{s} = 1.8 \text{ TeV}$

solid: $l = e$

dash: $l = \mu$