News on Disconnected Diagrams

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We present evidence for disconnected contributions to the $\sigma_{\pi N}$ term and the flavor singlet axial coupling $g_A^0$ of the proton on full QCD configurations, which are obtained by means of improved stochastic estimator techniques. Furthermore we discuss results from the fermionic determination of the topological charge of the QCD vacuum (in the spirit of the Atiyah-Singer theorem) again achieved with stochastic estimator methods. It turns out that this approach provides an monitor for the tunneling efficiency of the HMC on QCD with dynamical Wilson fermions which is independent on the pure gluonic method that implies cooling.

1. Introduction

Flavor singlet matrix elements, like the $\sigma_{\pi N}$ term or the axial coupling of the proton $g_A^0$, have been widely discussed in the past. Recently first lattice calculations worked out the connected as well as the disconnected diagrams contributing to $\sigma_{\pi N}$ and $g_A^0$\textsuperscript{1,2}. It turned out that the disconnected diagrams define a severe computational problem whereas the methods for the connected amplitudes work reasonable good. Two different approaches to tackle the pure vacuum fluctuations of disconnected diagrams were suggested in the literature, namely the wall-source method\textsuperscript{3} and stochastic estimator techniques (SET)\textsuperscript{4}. In the following we focus on SET with Z2 noise, coming back to the wall-source later on.

2. Stochastic Estimator Techniques

2.1. Standard Z2 Noise

Stochastic matrix inversions are based on a set of appropriate source vectors $\eta$ where all components carry a Z2 noise and fulfill

$$\langle \eta \rangle = 0, \quad \langle \eta_i \eta_j \rangle = \delta_{ij}$$

in the stochastic average. For infinite number of estimates $M^{-1}$ can then be obtained from

$$M^{-1}_{ij} = \langle \eta_j x_i \rangle = \sum_k M^{-1}_{ik} \langle \eta_j \eta_k \rangle$$

where $x$ solves $Ax = \eta$ for different source vectors $\eta$. The number of estimates equals the number of matrix inversions and the accuracy is bounded according to the computer facilities available.

2.2. Improving SET

The stochastic errors for off-diagonal traces, i.e. $Tr(\gamma_5 M^{-1})$, $Tr(\gamma_1 \gamma_5 M^{-1})$, increase drastically and improved SET methods become inevitable. Hence, we worked out the effective noise reduction of exact inversion in the spin subspace of Wilson’s fermion matrix $M$\textsuperscript{5}. This method reduces the number of estimates for a constant number of inversions by a factor 4, and could influence the accuracy of $Tr(M^{-1})$ and related observables. To cope with this problem diagonal-improved SET was invented. The error of $M^{-1}$ for finite numbers of estimates,

$$\Delta(M^{-1}_{ij}) \propto \sum_{k \neq j} M^{-1}_{ik} \langle \eta_i \eta_k \rangle$$

collects a large contribution when $k = i$ due to $M^{-1}_{ii} \gg M^{-1}_{ij}$ for the Wilson fermion matrix. The error $\Delta$ can be suppressed by subtracting the current estimate for $M^{-1}_{ii}$ after each inversion:

$$E[M^{-1}_{ij}] = M^{-1}_{ij} + \sum_{k \neq j} M^{-1}_{ik} \langle \eta_i \eta_k \rangle$$

$$-E[M^{-1}_{ii}] \langle \eta_i \eta_k \rangle$$

where $i \neq j$. This diagonal-improved SET reduces the standard error for $Tr(\gamma_5 M^{-1})$, $Tr(\gamma_1 \gamma_5 M^{-1})$.
and $Tr(\gamma_2 \gamma_5 M^{-1})$ about 40% without additional matrix inversions and does not reflect on $Tr(M^{-1})$.

3. Flavor Singlet Matrix Elements

We look at matrix elements of the proton $\langle p | q T q | p \rangle = \Delta q$ at zero momentum if $q T q$ is a flavor singlet bi-quark operator, i.e. $\Gamma = 1$ for the $\sigma_{\pi N}$ term and $\Gamma = \gamma_5 \gamma_\mu$ for the axial coupling $g_A$. The ratio of the three-point function and the proton propagator

$$R(t) = \frac{\langle p(t) | \sum_n q T q | p(0) \rangle}{\langle p(t) | p(0) \rangle}$$

is for large $t$ proportional to $\Delta q$ [6]. Connected diagrams are calculated with the conventional source method [7] whereas for the disconnected amplitudes we applied improved SET to estimate $Tr(\Gamma M^{-1})$ which is required in eq. 5. The SESAM gauge-field configurations are generated with 2 flavors of dynamical fermions on $16^3 \times 32$ lattices with the standard Wilson action ($\beta = 5.6$, $\kappa = 0.1560 - 0.1575$). Figure 1 shows $R(t)$ on 200 configurations at $\kappa = 0.156$ for $\sigma_{\pi N}$ and $g_A$, respectively. For scalar insertions the disconnected amplitudes are compatible to the connected ones, $\Delta u_{con}$, $\Delta d_{con}$, and $\Delta q_{dis} = \Delta u_{dis}$, $\Delta d_{dis}$ are positive. Axial connected insertions have opposite sign and $\Delta q_{dis}$ is rather small and negative. Even with improved SET the errors for $\Delta q_{dis}$ are still around 30%. The outstanding calculation of $\Delta s_{dis}$ for the strange quark will complete the analysis for $\sigma_{\pi N}$ and $g_A$ and is in advanced progress. Finally we should point out that the wall-source method is too noisy for the axial disconnected insertion and improved SET allows the determination of $g_A$ on our set of full QCD configurations.

4. Topology and SET

Topological properties of the QCD vacuum are essential for the understanding of flavor singlet physics, e.g. the unexpectedly large $Q$ mass or the proton spin. Here the topological charge $Q$ plays a crucial role since the Witten-Veneziano formula relates the topological susceptibility, $\chi = \langle Q^2 \rangle/V$, directly to the $Q$ mass. Instead of using the more usual field-theoretic [8] or geometric definitions [9] of $Q$, we look at the Athiya-Singer index theorem in the continuum

$$Q = n_+ - n_- = m_q Tr(\gamma_5 G(x,x'))$$

where $n_+ - n_-$ are the numbers of left and right handed chiral eigen-modes, $m_q$ the quark mass of the propagator $G(x,x')$. On the lattice this turns into

$$Q_L = m_q \kappa_p Tr(\gamma_5 M^{-1})$$

Here $M^{-1}$ is the inverse fermion matrix and $\kappa_p$ a renormalization constant [10]. Again we used improved SET to estimate the right hand side of eq. 7. In figure 2 the results for $Q_L$ on a set of HMC
trajectories from SESAM lattices ($\kappa = 0.1575$) are displayed. The topological charge via the index theorem is compared to $Q_L$ when we use cooling and the field-theoretical definition [8]. The best overlap of both data is achieved by setting $m_q\kappa_P = 2m_q$ in eq. 7 and a further determination of $\kappa_P$ is imperative. The strong correlation of the data indicates that the index theorem remains valid on the lattice. Finally two completely different methods display the tunneling efficiency of the HMC algorithm throughout various topological sectors and justify the ongoing calculation of $\chi$, the $\gamma'$ mass and the proton spin.

Furthermore we achieved values for the topological charge $Q_L$ with stochastic matrix inversion. These data are in good agreement with the results for $Q_L$ when the field-theoretical definition and cooling is applied. Both methods demonstrate the tunneling efficiency of the HMC on SESAM lattices throughout different topological sectors.

REFERENCES


5. Conclusion and Outlook

We presented an improved stochastic estimator technique and found evidence that disconnected diagrams contribute to the $\sigma_{\pi N}$ term and to the flavor singlet axial coupling of the proton, $g^{\pi A}_A$. It turns out that the disconnected contribution $\Delta u, d_{dis}$ to $\sigma_{\pi N}$ is comparable to the connected amplitude, whereas $\Delta d_{dis}$ for $g^{d}_{A}$ is rather small but non-vanishing. The final analysis for both flavor singlet operators, $\sigma_{\pi N}$ and $g^{d}_{A}$, will be published soon.