Heavy quark potential in the instanton liquid model

R. C. Brower\textsuperscript{a}, D. Chen\textsuperscript{b*}, J. W. Negele\textsuperscript{b} and E. Shuryak\textsuperscript{c}

\textsuperscript{a}Department of Physics, Boston University, Boston, MA 02215
\textsuperscript{b}Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139
\textsuperscript{c}Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794

We study the heavy quark potential in the instanton liquid model by carefully measuring Wilson loops out to a distance of order 3 fm. A random instanton ensemble with a fixed radius $\rho = 1/3 \text{fm}$ generates a potential $V(R)$ growing very slowly at large $R$. In contrast, a more realistic size distribution growing as $\rho^6$ at small $\rho$ and decaying as $\rho^{-5}$ at large $\rho$ leads to a potential which grows linearly with $R$. The string tension, however, is only about 1/10 of the phenomenological value.

1. INTRODUCTION

There is growing evidence that instantons play a very important role in QCD. The development of the Interacting Instanton Liquid Model \cite{1} allows one to calculate nonperturbatively to all orders in the ‘t Hooft interaction, and results show that it correctly generates the quark condensate and most salient properties of light hadrons. As reviewed at this conference \cite{2}, the essential features and parameters of this model have now been confirmed on the lattice, and there is strong lattice evidence that instantons and their associated zero modes play a significant role in hadron structure.

Motivated by a provocative result by Fukushima \textit{et al.} \cite{3} reporting a string tension in the instanton liquid model close to the phenomenological value, we have undertaken a careful calculation of the instanton induced heavy quark potential.

2. SIMULATION DETAILS

The gauge field of a single instanton in $SU(2)$ in the singular gauge (centered at the origin) is given by

\begin{equation}
A_\mu(x) = 2\pi^2 \tilde{\eta}_{\mu \nu} \frac{\rho^2}{x^2 + \rho^2},
\end{equation}

where $\tilde{\eta}_{\mu \nu}$ is the ‘t Hooft symbol. Replacement of $\tilde{\eta}_{\mu \nu}$ by $\eta_{\mu \nu}$ yields the solution for an anti-instanton. The multi-instanton gauge configuration in the dilute instanton liquid model is generated by a linear superposition of the fields from a distribution of instantons and anti-instantons. Each instanton and anti-instanton has a random center location and is embedded in $SU(3)$ with a random color orientation.

We consider two distributions of the instanton size, $\rho$, for $SU(3)$: (1) a fixed size distribution, with $\rho = 1/3 \text{fm}$; and (2) a phenomenological distribution, with $D(\rho) \sim \rho^6$ at small $\rho$ as expected from the dilute instanton gas approximation and $D(\rho) \sim \rho^{-5}$ at large $\rho$ as would occur if the perturbative running of the coupling constant $g$ were frozen at sufficiently large $\rho$. We use the parameterization by Shuryak\textsuperscript{[1]} for this variable size distribution,

\begin{equation}
D(\rho) = C \frac{\rho^6}{(\rho_0^5 + \rho^5)^{2}},
\end{equation}

We choose $\rho_0$ so that the average instanton size $\bar{\rho} = 1/3 \text{fm}$ and $C$ is a normalization constant.

We generate ensembles of multi-instanton configurations in an open box of size $(L_x, L_y, L_z)$

\textsuperscript{*}Presented by D. Chen. Work supported by the Department of Energy under cooperative research agreement DE-FC02-94 ER40818.
\begin{align*}
V(T, R) &= \langle \rho / V \rangle = 0.5 \, fm^{-4}, 1.0 \, fm^{-4} and 1.5 \, fm^{-4} for both fixed and variable instanton size distributions. The number of gauge configurations studied for each different \( n \) is 1600, 8000 and 1600, respectively.
\end{align*}

\begin{align}
W(R, T) &= \text{tr} \exp \left[ \int_{T=0}^{T=R} \mathcal{A}_\rho dx_\rho \right] \quad (3)
\end{align}

are measured in a 6.4 \, fm \times 12.8 \, fm rectangle in the central \( yz \) plane. This leaves a distance of at least 3.6 \, fm or more than 10 times the average instanton size \( \bar{\rho} \) from the measured Wilson loop to the 4-d boundary so that the edge effects from the open boundary are small. The largest Wilson loop is 3.2 \, fm \times 6.4 \, fm in size. The discretization \( \Delta x \) in the path ordered integral is 0.05 \, fm, which is about 1/6 of the average instanton size.

We study three different instanton number densities, \( n = N/V = 0.5 \, fm^{-4}, 1.0 \, fm^{-4} \) and \( 1.5 \, fm^{-4} \) for both fixed and variable instanton size distributions. The number of gauge configurations studied for each different \( n \) is 1600, 8000 and 1600, respectively.

3. RESULTS

The fundamental difficulty in this calculation is measuring with adequate statistical accuracy the large Wilson loops required to determine the potential at large distances. We determine the heavy quark potential \( V(R) \) from rectangular loops of different time extent as follows:

\begin{align}
V(T, R) &= \frac{1}{\Delta} \log \frac{W(R, T + \Delta)}{W(R, T)}, \quad (4)
V(R) &= \lim_{T \to \infty} V(T, R). \quad (5)
\end{align}

To obtain the most accurate measurement of the potential, we plot \( V(T, R) \) vs. \( T \) and find the plateau corresponding to \( V(R) \).

Figures 1 and 2 show such plots at different \( R \) for both fixed and variable instanton size distributions at an instanton number density \( n = 1.0 \, fm^{-4} \). For the fixed instanton size distribution, \( \rho = 1/3 \, fm \), plateaus for \( V(T, R) \) set in at about \( T > (1 \sim 2)R \). We have good plateaus for all \( R \leq 3.2 \, fm \) at \( n = 1.0 \, fm^{-4} \). In contrast, for
Figure 3. Heavy quark potential in the instanton liquid model for a fixed instanton size $\rho = 1/3 \text{fm}$. Three different instanton number densities are plotted.

Figure 4. Heavy quark potential for a variable instanton size distribution with $\rho = 1/3 \text{fm}$. Three different instanton number densities are plotted.

the variable instanton size distribution, not only do the plateaus set in at a larger $T > (2-3)R$, but also the statistical fluctuations are much larger at large $R$, especially for $R \geq 2.0 \text{fm}$. It is clear that this behavior of the variable size distribution arises from large instantons in the tail of the distribution. Even with an ensemble as large as 8000 configurations, we are only able to obtain a potential $V(R)$ for $R \leq 2.0 \text{fm}$ at $n = 1.0 fm^{-4}$.

Figures 3 and 4 show the heavy quark potential out to the maximum distance we can reliably measure. The salient results are the following. At short distance, the slope is identical for both the fixed and the variable size distributions at a given $n$ and is proportional to $n$. At the value $n = 1.0 fm^{-4}$ of the instanton liquid model, this slope is $\sim 0.1 \text{GeV}/\text{fm}$, corresponding to $1/10$ of the physical string tension. We note that by scaling, the physical string tension would be obtained at $n = 1.0 fm^{-4}$ by increasing the mean value of $\rho$ by $10^{1/4}$ to 0.59 $\text{fm}$. When $R$ is much greater than the largest instantons in the medium, we expect $V(R)$ to approach a constant [4]. For the distribution in Eq. (2), however, it is interesting that the potential is essentially linear in the region of $0 fm - 2 fm$. Hence, even though the random instanton liquid is not strictly confining, in the region of physical interest for hadron structure, it has a significant linear component.

4. ACKNOWLEDGEMENT

The authors are grateful to Sun Microsystems for providing the 24 Gflops E5000 SMP cluster and the Wildfire prototype system on which our calculations were performed.

REFERENCES
2. J. W. Negele, plenary talk at this conference.