We review the experimental and theoretical status of the standard electroweak theory, and its fundamental parameters. We obtain the global best fit result for the Standard Model Higgs boson of \( M_H = 107^{+42}_{-20} \) GeV, and find the 95% upper limit of 255 GeV. Parameters describing physics beyond the Standard Model are discussed as well. Particular emphasis is given to implications for supersymmetric extensions of the Standard Model.

1 The Z and the Weak Neutral Current

Weak neutral currents are a primary prediction and a direct test of electroweak unification. They were discovered in 1973 by the Gargamelle experiment using the Proton Synchrotron (PS) at CERN \(^1\), and confirmed by the HPWF detector at Fermilab \(^2\). Subsequently, neutrino-nucleon and neutrino-electron scattering experiments improved to the percent level, testing the weak interaction quantitatively. Electron-deuteron and electron-positron scattering, as well as atomic parity violation experiments, are sensitive to weak-electromagnetic interference effects, and were crucial for the confirmation of the electroweak Standard Model (SM). The W and Z bosons were finally discovered directly by the UA1 \(^3\) and UA2 \(^4\) Collaborations at the Super Proton Synchrotron (SPS) at CERN in 1982 and 1983, respectively. For a description of the history we refer the reader to Ref. \(^5\).

With the basic structure of the SM established, CERN's Large Electron Positron accelerator (LEP) and SLAC's Stanford Linear Collider (SLC) were designed to test it on the quantum level. With these machines it was possible to determine many Z properties with per mille accuracy, including the outstanding measurement of the Z mass, \( M_Z \), at LEP 1 with a relative precision of 2 parts in \( 10^6 \). Run I of Fermilab's Tevatron (CDF and DØ) and the second phase of LEP (ALEPH, DELPHI, L3, and OPAL) contribute per mille
determinations of the $W$ mass, $M_W$. With the mass of the top quark, $m_t$, as determined at the Tevatron, $M_W$ and other high precision observables can also be calculated within the SM in the framework of a quantum field theory. The agreement with the measurements establishes the SM as a spontaneously broken renormalizable gauge theory, and verifies the gauge group and representations. It predicts at least one extra state, the Higgs boson, with a mass, $M_H$, below 1 TeV (from triviality bounds and for reasons of perturbativity). Combining all direct and indirect data in a likelihood fit, it is possible to extract more precise information on $M_H$ leading to upper bounds of at most a few hundred GeV. We will review SM parameter estimation in Section 4.

The high accuracy of theory and experiment allows severe constraints on possible TeV scale physics, such as unification or compositeness. For example, the ideas of technicolor and non-supersymmetric Grand Unified Theories (GUT's) are strongly disfavored. On the other hand, supersymmetric unification, as generically predicted by string theories, is supported by the observed approximate gauge coupling unification at an energy slightly below the Planck scale. Constraints on parameters describing physics beyond the SM will be reviewed in Section 5.

2 New Data

2.1 $Z$ Pole Physics

The most important input into precision tests of electroweak theory continues to come from the $Z$ factories LEP 1\textsuperscript{6} and SLC\textsuperscript{7}. The vanguard of the physics program at LEP 1 with about 20 million recorded $Z$ events is the analysis of the $Z$ lineshape. Its parameters are $M_Z$, the total $Z$ width, $\Gamma_Z$, the hadronic peak cross section, $\sigma_{\text{had}}$, and the ratios of hadronic to leptonic decay widths, $R_\ell = \frac{\Gamma(\ell\ell)}{\Gamma(\gamma\gamma)}$, where $\ell = e, \mu, \tau$. They are determined in a common fit with the leptonic forward-backward (FB) asymmetries, $A_{FB}(\ell) = \frac{3}{4} A_e A_\ell$. Here

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}$$

(1)

is a parameter for fermion $f$, defined in terms of the vector $(v_f = I_{3,f} - 2Q_f \sin^2 \theta_{W}^f)$ and axial-vector $(a_f = I_{3,f}) Zf \gamma^\mu$ coupling; $Q_f$ and $I_{3,f}$ are the electric charge and third component of isospin, respectively, and $\sin^2 \theta_{W}^f \equiv \frac{r_f}{\gamma_f}$ is an effective mixing angle.

An average of about 73\% polarization of the electron beam at the SLC allows for a set of competitive and complementary measurements with a much smaller number of $Z$'s ($\gtrsim 500,000$). In particular, the left-right (LR) cross
section asymmetry, \( A_{LR} = A_c \), represents the most precise determination of the weak mixing angle by a single experiment (SLD)\(^7\). Mixed FB-LR asymmetries, \( A_{LR}^{FB}(f) = \frac{3}{2} A_f \), single out the final state coupling of the \( Z \) boson. This is done for leptons\(^8\), \( s \) quarks\(^9\), as well as \( b \) and \( c \) quarks.

The results for \( A_{LR}^{FB}(\tau) \) (and \( A_{LR}^{FB}(e) \)) can directly be compared with the LEP results\(^9\) on the final state \( \tau \) polarization, \( \mathcal{P}_\tau \) (and its angular distribution, \( \mathcal{P}_\tau^{FB} \)). For several years there has been an experimental discrepancy at the 2\( \sigma \) level between \( A_\ell \) from LEP and the SLC\(^10\). With the 1997/98 high statistics run at the SLC, and a revised value for \( \mathcal{P}_\tau^{FB} \), the two determinations are now consistent with each other,

\[
\begin{align*}
A_\ell(\text{LEP}) & = 0.1470 \pm 0.0027, \\
A_\ell(\text{SLD}) & = 0.1503 \pm 0.0023.
\end{align*}
\]

The LEP value is from \( A_{FB}(\ell) \) and the \( \tau \) polarization measurements, while the SLD value is from \( A_{LR} \) and \( A_{LR}^{FB}(\ell) \). The data are consistent with lepton universality, which is assumed here. There remains, however, a 2.5\( \sigma \) discrepancy between the two most precise determinations of \( \sin^2 \theta_W \), namely \( A_{LR} \) and \( A_{FB}(b) \) (assuming no new physics in the SM).

Of particular interest are the results on the heavy flavor sector\(^9\) including \( R_q = \frac{\Gamma(\tau \rightarrow q\ell\nu)}{\Gamma(\tau \rightarrow \mu\ell\nu)} \), \( A_{FB}(q) \), and \( A_{LR}^{FB}(q) \), with \( q = b \) or \( c \). In addition, results have been quoted on \( A_{FB}(s) \)\(^12,13\) and \( R_{d,s}/(R_d + R_s + R_u) \)\(^14\). There is a theoretical prejudice that the third family is the one which is most likely affected by new physics. Interestingly, the heavy flavor sector has always shown the largest deviations from the SM. E.g., \( R_b \) deviated at times by almost 4\( \sigma \). Now, however, \( R_b \) is in good agreement with the SM, and thus puts strong constraints on many types of new physics. At present, there is some discrepancy in \( A_{LR}^{FB}(b) = \frac{3}{8} A_b \), and \( A_{FB}(b) = \frac{3}{4} A_b A_u \), both at the 2\( \sigma \) level. Using the average of Eqs. (2), \( A_b = 0.1489 \pm 0.0018 \), both can be interpreted as measurements of \( A_b \). From \( A_{FB}(b) \) one would obtain \( A_b = 0.887 \pm 0.022 \), and the combination with \( A_{FB}^{LR}(b) = \frac{3}{8} (0.867 \pm 0.035) \) would yield \( A_b = 0.881 \pm 0.019 \), which is almost 3\( \sigma \) below the SM prediction. Alternatively, one could use \( A_b(\text{LEP}) \) above which is closer to the SM prediction to determine \( A_b(\text{LEP}) = 0.808 \pm 0.025 \), and \( A_b = 0.888 \pm 0.020 \) after combination with \( A_{LR}^{FB}(b) \), i.e., still a 2.3\( \sigma \) discrepancy. In order to explain this 5–6\% deviation in \( A_b \) in terms of new physics in loops, a 25–30\% radiative correction to \( \tilde{\kappa}_b \) defined through \( \tilde{\kappa}_b^2 = \tilde{\kappa}_b \sin^2 \theta_W \) would be needed. Only a new type of physics which couples at the tree level preferentially to the third generation\(^14\), and which does

\[^{14}\text{There is still some discrepancy in } A_{LR} \text{ from the SM prediction. This is mostly from upward fluctuations in the 1993 and 1996 data. The preliminary results from 1997, } A_{LR} = 0.1475 \pm 0.0012 \pm 0.0016, \text{ and 1998, } A_{LR} = 0.1487 \pm 0.0031 \pm 0.0017, \text{ are in excellent agreement.} \]
not contradict $R_b$ (including the off-peak measurements by DELPHI 16), can conceivably account for a low $A_b$. Given this and that none of the observables deviates by 2σ or more, we can presently conclude that there is no compelling evidence for new physics in the precision data.

LEP also quotes a value for the hadronic charge asymmetry, $Q_{FB}$, representing an additional determination of $\frac{\sigma}{p}(A_{FB}(q))$. Similarly, SLD measures the hadronic charge flow asymmetry 16. $A_Q = A$, which is basically given by the ratio of (weighted) FB and LR–FB asymmetries.

Table 1: Z pole precision observables from LEP and the SLC. Shown are the experimental results, the SM predictions, and the pulls. The SM errors from the uncertainties in $M_Z$, in $M, m_t, \alpha(M_Z)$, and $\alpha_s$. They have been treated as Gaussian and their correlations have been taken into account.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Group(s)</th>
<th>Value</th>
<th>Standard Model</th>
<th>pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ [GeV]</td>
<td>LEP</td>
<td>91.1907 ± 0.0021</td>
<td>91.1865 ± 0.0023</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>LEP</td>
<td>2.4830 ± 0.0024</td>
<td>2.4857 ± 0.0017</td>
<td>0.8</td>
</tr>
<tr>
<td>$\Gamma_{\text{had}}$ [GeV]</td>
<td>LEP</td>
<td>1.7423 ± 0.0023</td>
<td>1.7424 ± 0.0016</td>
<td>0.2</td>
</tr>
<tr>
<td>$\Gamma_{\text{inv}}$ [MeV]</td>
<td>LEP</td>
<td>500.1 ± 1.9</td>
<td>501.6 ± 0.2</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Gamma^{\ell^+\ell^-}$ [MeV]</td>
<td>LEP</td>
<td>83.90 ± 0.10</td>
<td>83.98 ± 0.03</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}$ [ab]</td>
<td>LEP</td>
<td>41.491 ± 0.038</td>
<td>41.473 ± 0.015</td>
<td>0.7</td>
</tr>
<tr>
<td>$R_e$</td>
<td>LEP</td>
<td>20.764 ± 0.045</td>
<td>20.742 ± 0.041</td>
<td>0.3</td>
</tr>
<tr>
<td>$R_{\mu}$</td>
<td>LEP</td>
<td>20.764 ± 0.045</td>
<td>20.742 ± 0.041</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_{FB}(e)$</td>
<td>LEP</td>
<td>0.0153 ± 0.0025</td>
<td>0.0161 ± 0.0003</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_{FB}(\mu)$</td>
<td>LEP</td>
<td>0.0164 ± 0.0013</td>
<td>0.0165 ± 0.0017</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_{FB}(r)$</td>
<td>LEP</td>
<td>0.0164 ± 0.0013</td>
<td>0.0165 ± 0.0017</td>
<td>0.3</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>LEP + SLD</td>
<td>0.21636 ± 0.00074</td>
<td>0.2158 ± 0.0002</td>
<td>0.1</td>
</tr>
<tr>
<td>$R_e$</td>
<td>LEP + SLD</td>
<td>0.1735 ± 0.0014</td>
<td>0.1723 ± 0.0001</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha_{FB}(b)$</td>
<td>OPAL</td>
<td>0.371 ± 0.023</td>
<td>0.3592 ± 0.0001</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_{FB}(c)$</td>
<td>LEP</td>
<td>0.0990 ± 0.0021</td>
<td>0.1028 ± 0.0010</td>
<td>1.8</td>
</tr>
<tr>
<td>$\alpha_{FB}(s)$</td>
<td>LEP</td>
<td>0.0709 ± 0.0044</td>
<td>0.0734 ± 0.0008</td>
<td>0.6</td>
</tr>
<tr>
<td>$\alpha_{FB}(t)$</td>
<td>OPAL</td>
<td>0.101 ± 0.005</td>
<td>0.1029 ± 0.0010</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>SLD</td>
<td>0.867 ± 0.035</td>
<td>0.9047 ± 0.0001</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>SLD</td>
<td>0.647 ± 0.040</td>
<td>0.6676 ± 0.0006</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_b$</td>
<td>SLD</td>
<td>0.647 ± 0.040</td>
<td>0.6676 ± 0.0006</td>
<td>0.5</td>
</tr>
<tr>
<td>$A_{LR}$ (hadrons)</td>
<td>SLD</td>
<td>0.1510 ± 0.0025</td>
<td>0.1466 ± 0.0015</td>
<td>1.8</td>
</tr>
<tr>
<td>$A_{LR}$ (leptons)</td>
<td>SLD</td>
<td>0.1504 ± 0.0072</td>
<td>0.1504 ± 0.0072</td>
<td>0.5</td>
</tr>
<tr>
<td>$A_b$</td>
<td>SLD</td>
<td>0.120 ± 0.019</td>
<td>0.120 ± 0.019</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_c$</td>
<td>SLD</td>
<td>0.142 ± 0.019</td>
<td>0.142 ± 0.019</td>
<td>0.2</td>
</tr>
<tr>
<td>$A_{\tau} (Q_{FB})$</td>
<td>SLD</td>
<td>0.162 ± 0.015</td>
<td>0.162 ± 0.015</td>
<td>0.1</td>
</tr>
<tr>
<td>$A_{\tau} (\bar{Q}_{FB})$</td>
<td>LEP</td>
<td>0.1431 ± 0.0045</td>
<td>0.1431 ± 0.0045</td>
<td>0.3</td>
</tr>
<tr>
<td>$A_{\tau} (\bar{Q}_{FB})$</td>
<td>LEP</td>
<td>0.1479 ± 0.0051</td>
<td>0.1479 ± 0.0051</td>
<td>0.3</td>
</tr>
</tbody>
</table>

The high precision Z pole observables are summarized in Table 1. Given for each observable is the experimental value with the total ( = statistical
+ systematic, added in quadrature) error, the SM prediction, and the pull, i.e., the deviation from the SM normalized by the total error. \( \Gamma(\text{had}), \Gamma(\text{inv}), \) and \( \Gamma(\ell^+\ell^-) \) are derived quantities and given for illustration only. Very good agreement with the SM is observed. Only \( A_{LR} \) and the two measurements sensitive to \( A_b \) discussed above, show some deviation, but even those are below \( 2\sigma \).

### 2.2 LEP 2 and Tevatron Results

LEP \( 2^n \) operating at and above the \( W^+W^- \) threshold, and the CDF \(^{17}\) and DØ \(^{18}\) Collaborations at the Tevatron, produce two completely independent determinations of \( M_W \), with presently the same accuracy. They are in very good agreement with each other, and yield the world average (including the older result by the UA2 Collaboration \(^{19}\)),

\[
M_W = 80.388 \pm 0.063 \text{ GeV.} \tag{3}
\]

The determination of \( m_t \) by CDF and DØ is dominated by the lepton + jet channels, which combine the merits of statistics (hadrons) and cleanliness (leptons). The dilepton channels (CDF and DØ) and the all hadronic channel (CDF) add extra information with to some extent different systematic uncertainties. The combined value is \(^{20}\)

\[
m_t = 173.8 \pm 3.2 \text{ (stat.)} \pm 3.9 \text{ (syst.) \text{ GeV.}} \tag{4}
\]

Fermion pair production at LEP above the \( Z \) resonance yields a number of important tests and cross checks of the SM. For example, \( \gamma-Z \) interference effects, which are suppressed at the \( Z \) resonance, become more sizable at higher energies. The LEP Collaborations have performed additional “S-matrix” fits, in which they allow these interference effects to differ from the SM expectations. They allow three extra off-resonance parameters, analogous and in addition to the on-resonance parameters \( \sigma_{\text{had}}, R_\ell, \) and \( A_{FB}(\ell) \). Including \( M_Z \) and \( \Gamma_Z \) and assuming family-universality this represents an eight parameter fit. Good agreement with the SM is found, reflecting the fact that FB-asymmetry measurements above the \( Z \) peak are also in good agreement with SM expectations.

The measurements at LEP 2 above the \( W^+W^- \) threshold and from DØ at the Tevatron are sensitive to triple-gauge-boson vertices. While there are a total of 14 independent couplings, one can use \( SU(2) \times U(1) \) gauge invariance

---

The invisible width, \( \Gamma(\text{inv}) \), constrains the number, \( N_v \), of standard neutrino flavors. A fit to all data with \( N_v \) free yields \( N_v = 2.992 \pm 0.011 \).
and LEP 1 constraints to reduce the number of triple gauge couplings to three. Each coupling is extracted from the data by setting the other two to zero (the SM value). The results are \[ \begin{align*}
\Delta \kappa & = 0.13 \pm 0.14, \\
\Delta g_1^2 & = 0.00 \pm 0.08, \\
\lambda_\gamma & = -0.03 \pm 0.07.
\end{align*} \]

2.3 Low Energy Data

Deep inelastic neutrino-hadron scattering (DIS) experiments played an important role for the establishment of the SM, and now serve as quantitative probes. This year the NuTeV Collaboration at Fermilab announced a very precise measurement of the Paschos-Wolfenstein ratio, \[ R^- = \frac{R^v - r R^p}{1 - r} = g_L^2 - g_R^2. \]

Here, \( R^v \) \( (R^p) \) is the ratio of neutral to charged current (anti)neutrino scattering cross sections, while \( r \) is the ratio of charged current \( \nu \) to \( \bar{\nu} \) cross sections. \( R^- \) has an advantage over the more traditional DIS observable \( R^p \) in that the effects of scattering from sea quarks cancel in the difference of \( \nu \) and \( \bar{\nu} \) cross sections. The actual measured quantity, \[ R^-_{\text{meas}} = R^v_{\text{meas}} - \alpha R^p_{\text{meas}}, \]

is constructed to minimize uncertainties from the so-called slow-rescaling parameters associated with the charm threshold, which have been dominant in the past. \( \alpha = 0.5136 \) has been obtained by means of a Monte Carlo simulation. This method can only be applied when a high statistics \( \bar{\nu} \) beam is available. Within the SM (but not beyond) and after fixing \( m_t \) and \( M_H \), the result, \[ R^-_{\text{meas}} = 0.2277 \pm 0.0022, \]

can be expressed as a measurement \[ M_W = 80.26 \pm 0.11 \text{ GeV}. \] This can be compared with the final result of the CCFR experiment \[ M_W = 80.35 \pm 0.21 \text{ GeV}. \] In practical numerical implementations, especially in the presence of new physics, the actual combination of \( Zq\bar{q} \) couplings should be used. In our analyses we include earlier results by the CDHS and CHARM Collaborations, as well, and take into account correlations induced by physics model uncertainties. For a recent update on the physics model parameters, see Ref. 26.

Note, that the \( R^v \) component introduces a larger \( m_t \) dependence compared to earlier DIS measurements, such as from CCFR.
Table 2: Non-Z pole precision observables from the Tevatron, LEP 2, neutrino scattering and APV. Shown are the experimental results, the SM predictions, and the pulls. The second error after the experimental value, when given, is theoretical. The SM errors are from Table 1: Non-inputs as in Table 1. The CHARM results have been adjusted to CDHS conditions, and can be directly compared.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Group(s)</th>
<th>Value</th>
<th>Standard Model</th>
<th>pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$ [GeV]</td>
<td>Tevatron</td>
<td>172.8 ± 5.0</td>
<td>171.4 ± 4.8</td>
<td>0.5</td>
</tr>
<tr>
<td>$M_W$ [GeV]</td>
<td>LEP</td>
<td>80.37 ± 0.09</td>
<td>80.362 ± 0.023</td>
<td>0.1</td>
</tr>
<tr>
<td>$R^e$</td>
<td>NaK 250</td>
<td>0.237 ± 0.0007</td>
<td>0.239 ± 0.0003</td>
<td>-0.9</td>
</tr>
<tr>
<td>$R^e$</td>
<td>CCFR</td>
<td>0.582 ± 0.0026</td>
<td>0.582 ± 0.0005</td>
<td>-0.2</td>
</tr>
<tr>
<td>$R^e$</td>
<td>CDHS</td>
<td>0.3096 ± 0.0028</td>
<td>0.3089 ± 0.0003</td>
<td>0.2</td>
</tr>
<tr>
<td>$R^e$</td>
<td>CHARM</td>
<td>0.3021 ± 0.0025</td>
<td>-0.3</td>
<td>1</td>
</tr>
<tr>
<td>$R^e$</td>
<td>CHARM</td>
<td>0.384 ± 0.0016 ± 0.007</td>
<td>0.3839 ± 0.0003</td>
<td>-0.1</td>
</tr>
<tr>
<td>$R^e$</td>
<td>CHARM</td>
<td>0.403 ± 0.0014 ± 0.007</td>
<td>0.4031 ± 0.0005</td>
<td>1</td>
</tr>
<tr>
<td>$R^e$</td>
<td>CDHS 1976</td>
<td>0.365 ± 0.0015 ± 0.007</td>
<td>0.3813 ± 0.0005</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Last year the Boulder group reported a much improved measurement of the amplitude of the parity violating transition between Cesium's 6S and 7S states. The experimental error in the extracted weak charge,

$$Q_W = -72.41 ± 0.25 \text{ (exp.) ± 0.80 (theory),}$$

(9)
decreased by a factor of 7. A slight improvement in the atomic theory calculations due to a new semi-empirical determination of the atomic polarizability shifted the central value of $Q_W$ closer to the SM prediction and reduced the theory error. In the future the total error is expected to decrease even further to about 0.5%. $Q_W$ is obtained from an average of two different hyperfine transitions. The Boulder group observes a significant difference in the 65F-4 to 75F-3 and the 65F-3 to 75F-4 transitions. Most of this effect (≈ 85%) is assigned to the nuclear anapole moment (the axial electromagnetic form factor of the nucleus), which has not been observed before. Its size is somewhat larger than expected from theory estimates, but the latter are nuclear model dependent. Measurements of atomic parity violation (APV) in thallium have also recently improved to match the theory calculations and to give useful constraints on new physics. Indeed, this type of measurement is very sensitive to the S parameter to be discussed later.
The non-Z pole precision observables are summarized in Table 2. They include the results from $\nu$-electron scattering by the CHARM II Collaboration, as well as the world averages. They are presented in terms of the vector and axial-vector couplings, $g_\nu^V$ and $g_\nu^A$. The $p\bar{p}$ value for $M_W$ assumes a common systematic error of $\pm 50$ MeV between the three experiments.

2.4 Other SM Tests

Another important neutral current process in the context of new physics, and in particular of low energy supersymmetry, is the flavor changing loop mediated transition $b \to s \gamma$. The ALEPH Collaboration studies $B$ mesons and baryons, while CLEO focuses on meson decays. Many of the hadronic and systematic uncertainties cancel when one normalizes the rate by the charged current process $b \to c\nu$. After having done so, one finds the two experiments in good agreement with each other, and the combined result,

$$R^{\text{exp}} = \frac{\Gamma(b \to s \gamma)}{\Gamma(b \to c\nu)} = (2.33 \pm 0.52) \times 10^{-3}. \quad (10)$$

A lot of effort has gone into theory calculations of $b \to s \gamma$. For references and a discussion of electroweak radiative corrections see Ref. 37, which quotes the SM prediction,

$$R^{\text{theory}} = (3.01 \pm 0.25) \times 10^{-3}. \quad (11)$$

Alternatively, by writing

$$R^{\text{theory}} = (3.00 \pm 0.25)(1 + 0.10\rho) \times 10^{-3}, \quad (12)$$

$R$ can also be interpreted as a measurement of one of the Wolfenstein parameters of the CKM mixing matrix,

$$\rho = -2.2 \pm 1.9. \quad (13)$$

The present world average of the anomalous magnetic moment of the muon is

$$a_\mu^{\text{exp}} = \frac{g_\mu - 2}{2} = (116592300 \pm 840) \times 10^{-11}. \quad (14)$$

On the other hand, the estimated SM one and two loop electroweak contribution

$$a_\mu^{\text{EW}} = (151 \pm 4) \times 10^{-11}, \quad (15)$$

is much smaller than the uncertainty. However, a new experiment at BNL is expected to reduce the experimental error to $\pm 40 \times 10^{-11}$ or better. The limiting factor will then be the uncertainty from the hadronic contribution,

$$a_\mu^{\text{had}} = (6924 \pm 62) \times 10^{-11}. \quad (16)$$
which has recently been estimated with the help of $\tau$ decay data and finite-energy QCD sum rule techniques. This result constitutes a major improvement over previous ones which had more than twice the uncertainty \cite{42}. It would be important to verify it, and reduce the error even further to meet the experimental precision.

Additional hadronic uncertainties are induced by the light-by-light scattering contribution \cite{43},

$$a_{\mu}^{\mathrm{BLS}} = (-92 \pm 32) \times 10^{-11}, \quad (17)$$

and other subleading hadronic contributions \cite{44},

$$a_{\mu}^{\mathrm{had}} \left( \frac{\alpha}{\pi} \right)^3 = (-100 \pm 6) \times 10^{-11}. \quad (18)$$

The SM prediction is

$$a_{\mu}^{\text{theory}} = (116591596 \pm 67) \times 10^{-11}. \quad (19)$$

With the anticipated accuracy at the BNL it will be possible to explore new physics up to energies of 5 TeV and more. The result of the initial run at BNL in 1997 is \cite{9}

$$a_{\mu}^{\text{BNL97}} = (1165925 \pm 15) \times 10^{-9}. \quad (20)$$

The error is expected to decrease to $\pm 1 \times 10^{-9}$ during the 1998–1999 run. A precise measurement can give important hints and constraints on new physics, specifically supersymmetry in the large tan $\beta$ region \cite{45}.

3 Theoretical Developments

3.1 Schemes

There are a variety of different renormalization schemes, reflected by various definitions of the weak mixing angle. The on-shell scheme is conceptually simple, carrying the tree level relation, $s_{\mathrm{W}}^2 \equiv 1 - M_W^2/M_Z^2$, to all orders. Due to large loop effects induced by the top-bottom doublet, $s_{\mathrm{W}}^2$ is numerically several per cent smaller than the effective mixing angles, $s_{f}^2 \equiv \kappa f s_{\mathrm{W}}^2$. These are defined through $Z$ pole asymmetries, and therefore (like $s_{\mathrm{W}}^2$) observable and scheme independent. The relatively large form factors $\kappa f$ induce large reducible higher order corrections, which frequently dominate the irreducible (genuine) corrections. Numerically much closer to the $s_{f}^2$ is the $\overline{\text{MS}}$ definition $\sin^2 \overline{\theta}_{\overline{\text{MS}}}(M_Z) \equiv s_{f}^2$. As a consequence the $\overline{\text{MS}}$ scheme has excellent
convergence properties. It is also (unlike the $s_Z^2$) flavor independent, and it is very convenient in the context of gauge coupling unification. There it is compared to the strong coupling constant, $\alpha_s$, which is traditionally treated in the $\overline{\text{MS}}$ scheme, as well. In general, calculations in the $\overline{\text{MS}}$ scheme tend to be technically simpler, and it is especially convenient in the presence of mixed QCD-electroweak corrections. One drawback of $s_Z^2$ is that it is a theoretical construct and not directly related to an observable. As a result, there exist various versions of $\overline{\text{MS}}$ mixing angles. Here we use the definition with the top quark decoupled. The scale, which has to be specified as well, can sometimes be used to eliminate spurious logarithms in coefficient functions, a freedom not available for the other definitions.

3.2 Recent Radiative Corrections

To match the accuracy of the high precision data, multi-loop perturbative calculations have to be performed. These include leading two-loop electroweak, three-loop mixed electroweak-QCD, and three-loop QCD corrections. $O(\alpha\alpha_s)$ vertex corrections to $Z$ decays have become available only recently, inducing an increase in the extracted $\alpha_s$ by about 0.001. The inclusion of top mass enhanced two-loop $O(\alpha^2 m_t^4)$ and $O(\alpha^2 m_b^2)$ effects is crucial for a reliable extraction of $M_H$.

We have collected all available results in a new radiative correction package. All $Z$ pole and low energy observables are self-consistently evaluated with common inputs. The routines are written entirely within the $\overline{\text{MS}}$ scheme, using $\overline{\text{MS}}$ definitions for all gauge couplings and quark masses. This reduces the size of higher order terms in the QCD expansion.

The largest theoretical uncertainty arises from the $M_W-M_Z-s_Z^2$ interdependence. The problem is directly related to the renormalization group (RGE) running of the electromagnetic coupling,

$$\alpha(M_Z) = \frac{\alpha}{1 - \Delta\alpha(M_Z)}. \quad (21)$$

While the contributions from leptons and bosons (and the top quark when not technically decoupled) can be computed with sufficient accuracy, the hadronic contributions from the five lighter quarks escape a first principle treatment due to strong interaction effects. If one trusts perturbative QCD down to energies below the $J/\Psi$ threshold, one can use RGE techniques for charm and bottom quarks, as well. Complications from quark confinement are then effectively moved to the actual extraction of the $c$ and $b$ quark masses, which contribute a major uncertainty in this approach. Alternatively, one
can use $J/\Psi$ and $\Upsilon$ resonance parametrizations and a dispersion relation (DR) to arrive at the charm and bottom contributions to $\Delta \alpha(M_Z)$. The latter approach is slightly more precise, while the first one allows a selfconsistent treatment of the parametric uncertainty from $\alpha_s$, an advantage in electroweak fits. Both approaches give very similar results. Finally, one might prefer to use $e^+e^-$ data and the dispersive approach for the entire energy regime up to well above the $\Upsilon$ resonances (the cutoff is typically 40 GeV)\cite{ref1,ref2,ref3}. However, since the relevant data is not very precise, one faces larger uncertainties; the results are in good agreement with the more theoretical approaches, but tend to give larger values of $\Delta \alpha(M_Z)$ and smaller resulting best fit values for $M_H$. As for the three light flavors, a DR cannot be circumvented in any of the approaches, and differences arise only due to different data or fit procedures. The calculation of $\alpha_{\text{QCD}}$ is naturally performed using an unsubtracted DR\cite{ref4}; the on-shell coupling is computed through a subtracted one\cite{ref5,ref6,ref7}. In our analysis for the Particle Data Group\cite{ref8} we relied on Ref.\cite{ref9}, while here we use Ref.\cite{ref10}. The LEP Electroweak Working Group (LEPEWWG)\cite{ref11} employs Ref.\cite{ref12}.

It is interesting that the TOPAZ collaboration at TRISTAN\cite{ref13} has actually observed the running of $\alpha$, albeit with large uncertainty. From the cross section for $e^+e^- \rightarrow e^+e^\mu^+\mu^-$ (relative to $\mu^+\mu^-$) they obtain $\alpha^{-1}(57.77\text{ GeV}) = 128.5 \pm 1.9$, compared with the theoretical expectation $129.6 \pm 0.1$.

4 Standard Model Fit Results

4.1 Overview

We use the complete data set described in Section 2, and summarized in Tables 1 and 2 for a global electroweak analysis. We carefully took into account experimental and theoretical correlations, in particular in the $Z$-lineshape sector, the heavy flavor sector from LEP and the SLC, and for the DIS experiments. Predictions within and beyond the SM were calculated by means of a new radiative correction program based on the MS renormalization scheme (see Section 3). All input and fit parameters are included in a selfconsistent way, and the correlation (present in theory evaluations of $\alpha(M_Z)$) between $\alpha_s$ and $\Delta \alpha(M_Z)$ is automatically taken care of. We find very good agreement with the results of the LEPEWWG\cite{ref11}, except for well-understood effects originating from higher orders. We would like to stress that this agreement is quite remarkable as they use the electroweak library ZFITTER\cite{ref14}, which is based on the on-shell renormalization scheme. It also demonstrates that once the most recent theoretical calculations, in particular Refs.\cite{ref15,ref16}, are taken into account,
the theoretical uncertainty becomes quite small, and is in fact presently negligible compared to the experimental errors. The relatively large theoretical uncertainties obtained in the Electroweak Working Group Report \(^{57}\) were estimated using different electroweak libraries, which did not include the full range of higher order contributions available now.

In the Standard Model analysis we use the fine structure constant, \(\alpha\), and the Fermi constant, \(G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}\), as fixed inputs. The error in \(G_F\) is now of purely experimental origin after the very recent calculation of the two-loop QED corrections to \(\mu\) decay have been completed \(^{59}\). They lower the central value by \(2 \times 10^{-10} \text{ GeV}^{-2}\) and the extracted \(M_H\) by 1.3\%.

Moreover, there are five independent fit parameters, which can be chosen to be \(M_Z\), \(M_H\), \(m_t\), \(\alpha_s\), and \(\Delta \alpha(M_Z)\). Alternatively, \(M_Z\) can be replaced by \(s_W^2\) or \(s_Z^2\). Unless explicitly stated, we do not use \(\alpha_s\) determinations from outside the \(Z\) lineshape sector. The fit to all precision data is perfect with an overall \(\chi^2 = 28.8\) for 36 degrees of freedom, and yields,

\[
\begin{align*}
M_H &= 107.4^{+0.7}_{-0.7} \text{ GeV}, \\
m_t &= 171.4 \pm 4.8 \text{ GeV}, \\
\alpha_s &= 0.1206 \pm 0.0030, \\
s_W^2 &= 0.23129 \pm 0.00019, \\
s_Z^2 &= 0.23158 \pm 0.00019, \\
s_Z^2 &= 0.22332 \pm 0.00045.
\end{align*}
\]

(22)

Moreover, none of the observables deviates from the SM best fit prediction by more than 2 standard deviations.

In Table 3 we show the results of fits to various data sets. Shown are the \(M_S\) and on-shell mixing angles, as well as \(\alpha_s\) and \(m_t\). In these fits we have fixed the Higgs mass to its global best fit value, \(M_H = 107\) GeV, so that the sensitivity to \(m_t\) becomes transparent. The first line corresponds to the fit to all data. Note the smaller errors compared to the results (22) where \(M_H\) was a free parameter. The third line corresponds to a fit to all indirect data. The extracted \(m_t = 169.4\pm4.6\) GeV is in good agreement with the direct result from the Tevatron, and is of similar accuracy. On the other hand, the \(m_t\) extracted from the LEP 1 (SLD) observables is slightly below (above) the Tevatron value. There is still a 2.2\(\sigma\) discrepancy between the \(s_Z^2\) determinations at LEP and the SLC. The last 2 lines show the results from DIS. When combined with \(M_Z\), DIS represents a measurement of \(s_Z^2 (R_{\nu})\), and also has some sensitivity to \(m_t (R_{\nu})\). The last fit is equivalent to regarding DIS as a measurement of \(s_W^2\). Even after the end of the LEP 1 era, \(\nu\)-hadron scattering experiments continue to represent competitive measurements. This is even more true in the presence of new physics.
Table 3: Fit values of $\Delta Z$, $\Delta Y$, $\alpha$, and $m_t$ for various combinations of observables. The quoted values are for the global best fit result, $M_H = 107 \text{ GeV}$. The uncertainties are from $m_t$, $\alpha$, and $\alpha(M_Z)$.

<table>
<thead>
<tr>
<th>Data</th>
<th>$\Delta Z$</th>
<th>$\Delta Y$</th>
<th>$\alpha(M_Z)$</th>
<th>$m_t$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>All indirect + $m_t$</td>
<td>0.2312(8)</td>
<td>0.2233(4)</td>
<td>0.1200(30)</td>
<td>171.4 ± 3.4</td>
</tr>
<tr>
<td>All indirect + $m_t + \alpha$</td>
<td>0.2312(6)</td>
<td>0.2233(4)</td>
<td>0.1188(18)</td>
<td>171.4 ± 3.4</td>
</tr>
<tr>
<td>Z pole</td>
<td>0.2313(15)</td>
<td>0.2236(5)</td>
<td>0.1200(30)</td>
<td>169.4 ± 4.6</td>
</tr>
<tr>
<td>LEP 1</td>
<td>0.2316(21)</td>
<td>0.2246(7)</td>
<td>0.1219(31)</td>
<td>161.2 ± 6.4</td>
</tr>
<tr>
<td>SLD + $M_Z$</td>
<td>0.2306(31)</td>
<td>0.2216(11)</td>
<td>0.1200 (fixed)</td>
<td>158.8 ± 9.0</td>
</tr>
<tr>
<td>$A_{FB}(b) + M_Z$</td>
<td>0.2310(30)</td>
<td>0.2259(15)</td>
<td>0.1200 (fixed)</td>
<td>148.5 ± 13.5</td>
</tr>
<tr>
<td>$M_W + M_Z$</td>
<td>0.2315(34)</td>
<td>0.2228(12)</td>
<td>0.1200 (fixed)</td>
<td>175.4 ± 10.0</td>
</tr>
<tr>
<td>DIS + $M_Z$</td>
<td>0.2318(78)</td>
<td>0.2255(29)</td>
<td>0.1200 (fixed)</td>
<td>152.2 ± 25.1</td>
</tr>
<tr>
<td>DIS + $m_t$</td>
<td>0.2334(22)</td>
<td>0.2253(21)</td>
<td>0.1200 (fixed)</td>
<td>173.9 ± 5.0</td>
</tr>
</tbody>
</table>

4.2 $M_H$

The data show a strong preference for a low $M_H \sim \mathcal{O}(M_Z)$. Unlike in previous analyses, the central value of the global fit to all precision data, including $m_t$ and excluding further constraints from direct searches,

$$M_H = 107^{+67}_{-45} \text{ GeV},$$

is now above the direct lower limit,

$$M_H > 90 \text{ GeV} \ [95\% \ CI],$$

from searches at LEP 2. In fact, it coincides with the 5σ discovery limit (and is 2 GeV below the 95% exclusion limit) from LEP 2 running at 200 GeV center of mass energy with 200 pb$^{-1}$ integrated luminosity per experiment. The 90% central confidence interval from precision data only is given by

$$39 \text{ GeV} < M_H < 226 \text{ GeV}.$$ (25)

These results are to be compared with the theoretical SM expectation,

$$115 \text{ GeV} \lesssim M_H \lesssim 750 \text{ GeV},$$ (26)

where the lower bound is derived from vacuum stability requirements, and the upper bound is a (lattice) triviality bound. The fit result (23) is also in agreement with the prediction for the lightest neutral Higgs boson,

$$m_{h^0} \lesssim 130 \text{ GeV},$$ (27)
within the Minimal Supersymmetric Standard Model (MSSM). In non-minimal extensions of the MSSM, the bound (27) is relaxed to about 150 GeV.

For the determination of the proper $M_H$ upper limits, we scan equidistantly over $M_H$, combining the likelihood $\chi^2$ function from the precision data with the exclusion curve (interpreted as a prior probability distribution function) from LEP 2. This curve is from Higgs searches at center of mass energies up to 183 GeV. We find the 90 (95, 99)% confidence upper limits,

$$M_H < 220 \text{ (255, 335)} \text{ GeV.}$$

Notice, that the LEP 2 exclusion curve increases the 95% upper limit by almost 30 GeV.

Indirect $M_H$ constraints from precision data are now very similar in precision to the indirect $m_t$ constraints from less than a decade ago, just before the commencement of the Z pole era at LEP 1 and the SLC. This is rather remarkable, as $M_H$ effects are, in contrast to the leading quadratic $m_t$ effects, only approximately logarithmic. Higgs hunting in precision data is further hampered by the strong correlation of $\ln M_H$ with $m_t$ (70%) and $\alpha(M_Z)$ (30%).

The strongest constraints come from the asymmetries, but $M_W$, $R_b$, and the lineshape observables are also significant. In the past, the tendency for a low $M_H$ came almost entirely from $A_{LR}$ and $R_b$, both of which were in conflict with the SM. Now they deviate much less (especially $R_b$), plus there are extra constraints. This also reduces possible confusion with new physics (to which $R_b$ is particularly sensitive). It should be noted, however, that the results on $M_H$ are strongly correlated with the $S$ and $T$ parameters, as discussed in Section 5. The increase in $\chi^2$ when shifting $M_H$ up to 1 TeV, which used to be only a few units, is now

$$\Delta \chi^2 = \chi^2(M_H = 1 \text{ TeV}) - \chi^2_{\min} = 25.7,$$

i.e., a TeV scale SM Higgs boson is now excluded at the 5$\sigma$ level.

On the other hand, the $\chi^2$ function is fairly shallow for Higgs masses up to $O(200 \text{ GeV})$. Precise predictions are difficult due to the low sensitivity in that regime. One also has to keep in mind that there are still some 2$\sigma$ level deviations, such as in $A_{LR}$ and $A_{FB}(b)$, and ambiguities in the treatment of $\alpha(M_Z)$. Unlike the central values, however, the upper limits on $M_H$ are rather insensitive to the used $\alpha(M_Z)$, this is due to compensating effects from the larger central value of $\alpha(M_Z)$ (corresponding to lower extracted Higgs masses) and the larger error bars in the data driven approach (see the discussion in Section 3.2).

One can take the point of view that the $A_{LR}$ measurement at the SLC and $A_{FB}(\tau)$ from LEP are by themselves in conflict with the lower Higgs
limit (24). For example, including the Tevatron $m_t$ one would predict from $A_{LR}$,

$$M_H(A_{LR}) = 39^{+46}_{-25} \text{ GeV}. \quad (30)$$

This may well be due to a statistical fluctuation, and to avoid a bias we treat it as such. Indeed, $A_{LR}$ and $A_{FB}(\tau)$, are both consistent with the lower direct $M_H$ limit within 1.1σ. Moreover the $\chi^2$ per d.o.f. in the SM fit is excellent, discouraging the use of scale factors to increase error bars. It should also be stressed again that our upper $M_H$ limits do take into account the direct search results in a proper Bayesian way. However, to get a sense of how sensitive our limits depend on $A_{LR}$ and $A_{FB}(\tau)$, we study the effect of repudiating both measurements completely from the fit, and find,

$$M_H(A_{LR}, A_{FB}(\tau)) = 195^{+110}_{-78} \text{ GeV}, \quad (31)$$

and the 90 (95)% upper limit,

$$M_H(A_{LR}, A_{FB}(\tau)) < 356 (419) \text{ GeV}. \quad (32)$$

Hence, even such a radical treatment increases the upper $M_H$ limit by only about 60% or $\sim 150$ GeV. We disagree here with the conclusions drawn from PDG scale factor studies (on a limited data set) in Ref. 66, in which a 95% upper limit of 750 GeV is found.

### 4.3 $\alpha_s(M_Z)$

As for the extraction of the strong coupling constant at the $Z$ scale, we find the best fit value,

$$\alpha_s = 0.1206 \pm 0.0030, \quad (33)$$

in excellent agreement with other determinations. For example, the ALEPH 67 and OPAL 68 Collaborations obtain from hadronic $\tau$ decays,

$$\alpha_s(\Gamma_{\tau}) = 0.1202 \pm 0.0027, \quad \alpha_s(\Gamma_{\tau}) = 0.1219 \pm 0.0020, \quad (34)$$

respectively. Another result of similar precision is obtained from $\Upsilon$ spectroscopy using non-relativistic QCD for lattice gauge theory 69,

$$\alpha_s(\overline{b}\overline{b} \text{ spectrum}) = 0.1174 \pm 0.0024. \quad (35)$$

This is also consistent with a preliminary result from $J/\Psi$ spectra 70,

$$\alpha_s(\overline{c}\overline{c} \text{ spectrum}) = 0.1159 \pm 0.0030. \quad (36)$$

For comparison, the LEPEWWG 9 quotes $\alpha_s = 0.119 \pm 0.003$. 

15
Measurements of the proton structure functions $F_2$ and $xF_3$ in neutrino DIS yield

$$\alpha_s(\text{DIS}) = 0.119 \pm 0.002 \pm 0.004,$$

(37)

where the first error is experimental and the second theoretical. The NuTeV Collaboration plans to reduce the total error of $\alpha_s$ from scaling violations in DIS to 0.002. Finally, there are a variety of jet event shape extractions of $\alpha_s$ from $e^+e^-$ annihilation at, below, and above the $Z$ peak, again in excellent agreement with Eq. (33). One can clearly conclude that the old low energy versus high energy controversy for $\alpha_s$ is over!

We can use these $\alpha_s$ measurements as an additional external constraint in global fits. In order to do so, we use the world average by the Particle Data Group with the $Z$ lineshape value removed,

$$\alpha_s = 0.1178 \pm 0.0023. \quad (38)$$

The result (cf., the second row in Table 3),

$$\alpha_s = 0.1188 \pm 0.0018, \quad (39)$$

can be viewed as the present world average. Inclusion of the constraint (38) reduces the 90 (95, 99)\% upper $M_H$ limit by 6 (9, 13) GeV.

The lineshape value (33) is also consistent with predictions from gauge unification in supersymmetric GUT’s and string theories. Using $\alpha(M_Z)$ and $\frac{s_Z^2}{2}$ one predicts

$$\alpha_s = 0.130 \pm 0.010. \quad (40)$$

4.4 Future Prospects

The LEP 1 results are now close to being finalized, although some work on systematic errors and correlations still needs to be done. Similarly, the uncertainty in $M_W$ from the Tevatron run I is expected to decrease somewhat. Run II at the Tevatron with its ten times larger luminosity is anticipated to measure $M_W$ to ±40 MeV per experiment and channel. The statistical error of $M_W$ from LEP 2 is likely to decrease to ±40 MeV within a year and to ±25 MeV after its completion. For a projection of the systematic error, the effects of color reconnection and Bose-Einstein correlations need to be understood more rigorously. Future $M_W$ measurements could reach a precision of ±25 MeV, including a theoretical uncertainty from uncalculated higher order radiative corrections. Moreover, run II should determine $m_t$ within 2 GeV, including the theoretical ambiguity from the conversion between pole and running mass definitions. The SLD Collaboration is seeking for additional run
time ("SLD 2000"), which would permit them to double their statistics. This would greatly improve the $A_{LR}$ and $A_{FB}^{P}$ measurements, which are still statistically limited. Given the superior three-dimensional vertexing with the SLD detector (VXD3), it would also allow competitive measurements of $R_b$ and $R_c$, with errors comparable to those from LEP.

Adding the hypothetical constraints,

\begin{align*}
  m_t \ (\text{run II}) &= 171.4 \pm 2.0 \text{ GeV}, \\
  M_W \ (\text{run II + LEP 2}) &= 80.362 \pm 0.025 \text{ GeV}, \\
  A_{FB} \ (\text{SLD 2000}) &= 0.1406 \pm 0.0020, \\
  R_b \ (\text{SLD 2000}) &= 0.2158 \pm 0.0010,
\end{align*}

(41)

to the data (where the central values are the current global best fit values), one might find around the year 2002,

$$M_H \ (\text{future}) = 107^{+136}_{-29} \text{ GeV},$$

(42)
i.e., a 30% determination. From direct searches, Higgs masses up to 94 (97) GeV can be discovered (excluded) from the present 189 GeV run at LEP 2.

After run II there might be a further luminosity upgrade at the Tevatron (TeV33) reducing the top mass error to about $\pm 1$ GeV. High precision measurements of the total width and the leptonic branching ratio of the $W$ boson would be possible. In addition, a per mille determination of the weak mixing angle through $A_{FB}(\ell)$ is conceivable. Most importantly, with 30 fb$^{-1}$ at TeV33, Higgs boson searches up to 130 GeV would be possible.

5 Beyond the Standard Model

5.1 Unification or Compositeness

The successful supersymmetric gauge coupling unification discussed in Section 4.3 could be coincidental. If taken seriously, however, it could also be taken as circumstantial evidence for supersymmetry (SUSY). Moreover, it would constrain extra matter to come either in complete standard families or singlets. And it would demand the absence of extended gauge structures below the unification scale $M_{GUT} \sim 10^{16}$ GeV, unless they commute with the SM gauge group. Of course, there is always the possibility of subtle cancellations of different effects.

In addition to the encouraging gauge unification, there is the agreement between the general prediction from perturbative low energy SUSY,

$$m_{\tilde q} \lesssim 150 \text{ GeV},$$

(43)

For a recent example, see Ref. 74.
and the precision data. The results in Section 4.2 apply strictly only in the
context of the SM, or when the extra sparticles and Higgs bosons predicted
by SUSY are decoupled, i.e., heavier than a few hundred GeV. In this case,
effects in the precision data are small, and it is a remarkable prediction of the
decoupled MSSM that no deviation in the precision data are to be expected.
Similarly, flavor changing neutral currents (FCNC) and CP violating effects
may be small, as well.

On the contrary, in scenarios of new physics involving a composite (dynamical)
Higgs sector and/or composite fermions, one expects a variety of effects.
These include in particular FCNC, which are typically predicted along with
many new 4-Fermi operators. However, (in the absence of fine tuning) FCNC
operators are already excluded up to scales of $\mathcal{O}(100 \text{ TeV})$, and APV excludes
contact operators of $eq$ type for scales up to $\mathcal{O}(10 \text{ TeV})$. These types of new
physics also tend to predict a decrease in $R_b$, the opposite of what is being
observed. Finally, they are in conflict with oblique radiative corrections, as
defined and discussed below.

5.2 Oblique Parameters

The data are precise enough to constrain additional parameters describing
physics beyond the SM. Of particular interest is the $\rho$-parameter, defined by

$$\rho_0 = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W (m_t, M_H)}$$

which is a measure of the neutral to charged current interaction strength. The
SM contributions are absorbed in $\rho$, so that in the SM $\rho_0 = 1$, by definition.
Examples for sources of $\rho \neq 1$ include non-degenerate extra fermion or boson
doublets, and non-standard Higgs representations.

In a fit to all data with $\rho_0$ as an extra fit parameter, the correlation of
$M_H$ with $m_t$ is lifted, and replaced by a strong (73%) correlation with $\rho_0$. As
a result upper limits on $M_H$ are weak when $\rho_0$ is allowed. Indeed, $\chi^2(M_H)$ is
very shallow with

$$\Delta \chi^2 = \chi^2(1 \text{ TeV}) - \chi^2(M_Z) = 4.5,$$

and its minimum is at $M_H = 46 \text{ GeV}$, which is already excluded. We obtain,

$$\rho_0 = 0.9996^{+0.0009}_{-0.0006},$$
$$m_t = 172.9 \pm 4.8 \text{ GeV},$$
$$\alpha_s = 0.1212 \pm 0.0031,$$

$\rho_0$ is also strongly anticorrelated with $\alpha_s$ ($-53\%$) and $m_t$ ($-46\%$).
in excellent agreement with the SM. The central values are for \( M_H = M_Z \), and the uncertainties are \( 1\sigma \) errors and include the range, \( M_Z \leq M_H \leq 167 \text{ GeV} \), in which the minimum \( \chi^2 \) varies within one unit. Note, that the uncertainties for \( \ln M_H \) and \( \rho_0 \) are non-Gaussian: at the \( 2\sigma \) level \( (\Delta \chi^2 \leq 4) \), Higgs masses up to 800 GeV are allowed, and we find

\[
\rho_0 = 0.9996^{+0.0031}_{-0.0013} (2\sigma). \tag{47}
\]

This implies strong constraints on the mass splittings of extra fermion and boson doublets \( 75 \),

\[
\Delta m^2 = m_1^2 + m_2^2 - \frac{4m_1^2m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1}{m_2} \geq (m_1 - m_2)^2, \tag{48}
\]

namely, at the \( 1\sigma \) and \( 2\sigma \) levels, respectively,

\[
\sum_i \frac{C_i}{3} \Delta m_i^2 < (38 \text{ GeV})^2 \text{ and } (93 \text{ GeV})^2, \tag{49}
\]

where \( C_i \) is the color factor. Due to the general condition (43) in the MSSM, stronger \( 2\sigma \) constraints result here,

\[
\rho_0 (\text{MSSM}) = 0.9996^{0.0017}_{-0.0013} (2\sigma). \tag{50}
\]

The constraints (49) would therefore change to

\[
\sum_i \frac{C_i}{3} \Delta m_i^2 < (38 \text{ GeV})^2 \text{ and } (64 \text{ GeV})^2 (\text{MSSM}). \tag{51}
\]

Similarly, constraints on heavy degenerate chiral fermions can be obtained through the \( S \) parameter \( 76 \), defined through a difference of \( Z \) boson self-energies,

\[
\frac{\alpha(M_Z)}{4\pi c_W^2} S \equiv \frac{\Pi_{ZZ}^{\text{phys}}(M_Z^2) - \Pi_{ZZ}^{\text{phys}}(0)}{M_Z^2}. \tag{52}
\]

The superscripts indicate that \( S \) includes new physics contributions only. Likewise, \( T = (1 - \rho_0^{-1})/\alpha \) and the \( U \) parameter to be discussed below, also vanish in the SM. A fit to all data with \( S \) allowed yields,

\[
S = -0.26^{+0.24}_{-0.17},
M_H = 390^{+690}_{-110} \text{ GeV},
\alpha_s = 0.1221 \pm 0.0035.
\tag{53}
\]

Thus, our definition differs somewhat from the original definition \( 76 \) which included the \( m_t \) and \( M_H \) contributions to the self-energies in \( S, T, \) and \( U \).
In the presence of $S$, constraints on $M_H$ virtually disappear. In fact, $S$ and $M_H$ are almost perfectly anticorrelated ($-92\%$). A heavy degenerate ordinary or mirror family contributes $2/3\pi$ to $S$. By requiring $M_Z \leq M_H \leq 1$ TeV, we find with $3\sigma$ confidence,

$$S = -0.20^{+0.40}_{-0.33} \ (3\sigma).$$

A fourth sequential fermion family is excluded at the $99.8\%$ CL.

New physics contributions to the third oblique parameter, $U$, which is defined through

$$\frac{\delta (M_Z)}{4 \pi^2 Z} (S + U) \equiv \frac{\Pi_{WW}^\text{new} (M_W^2) - \Pi_{WW}^\text{old} (0)}{M_W^2},$$

are usually expected to be small. A fit to all data with $U$ allowed,

$$U = 0.09 \pm 0.19,$$

$$M_H = 110^{+70}_{-46} \text{ GeV},$$

$$m_t = 171.1 \pm 4.9 \text{ GeV},$$

$$\alpha_e = 0.1207 \pm 0.0030,$$

reveals perfect agreement with the SM prediction $U = 0$. Notice, that allowing $U$ has little effect on the extracted $M_H$, as it has only small correlations with the SM parameters.

A simultaneous fit to $S$, $T$, and $U$ can be performed only relative to a specified $M_H$. If one fixes $M_H = 600$ GeV, as is appropriate in QCD-like technicolor models, one finds

$$S = -0.27 \pm 0.12,$$

$$T = 0.00 \pm 0.15,$$

$$U = 0.19 \pm 0.21.$$  \hspace{1cm} (57)

Notice, that in such a fit the $S$ parameter is significantly smaller than zero. From this an isodoublet of technifermions, assuming $N_{TC} = 4$ technicolors, is excluded by almost 6 standard deviations, and a full technigeneration by more than $15\sigma$. However, the QCD-like models are excluded on other grounds, such as FCNC. In particular, in models of walking technicolor $S$ can be smaller or even negative.

The allowed range of the oblique parameters in the context of SUSY is obtained by demanding $M_Z \leq M_H \leq 150$ GeV, which yields,

$$S = -0.17^{+0.17}_{-0.12},$$

$$T = -0.16^{+0.15}_{-0.18},$$

$$U = 0.19 \pm 0.21.$$  \hspace{1cm} (58)

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Note the $2\sigma$ upper limit $T \leq 0.14$. Allowing supersymmetric contributions to $R_b$, which can be mediated by light top squark and chargino loops, this limit would tighten further to

$$T \leq 0.12 (2\sigma).$$

These results are to be compared with the predictions of various scenarios for the mediation of SUSY breaking from the hidden to the observable sector. For example, in the minimal supergravity model with universal soft SUSY breaking terms, there are regions of parameter space in which $T$ can be as large as 0.20, so they have to be excluded. Of course, there are in general also (smaller) contributions to $S$ and $U$, as well as non-oblique corrections, so much more parameter space can be excluded than what is suggested by the constraint (59). A systematic analysis of precision data in the MSSM, and a discussion of the excluded parameter space can be found in Ref. 78.

5.3 Weaker Interactions

Many GUT’s and string models predict extra gauge symmetries and new exotic states. For example, $SO(10)$ GUT contains an extra $U(1)$ as can be seen from its maximal subgroup, $SU(5) \times U(1)_X$. The spinorial $16$ representation contains besides the known quarks and leptons one further state, the right-handed neutrino. This is highly desirable for the generation of neutrino mass through the see-saw mechanism. However, the scale at which the $U(1)_X$ is broken is not predicted, and the mass of the accompanying gauge boson, $M_{Z'}$, is arbitrary. Thus, there is no reason to expect a $Z'$ of this kind at the electroweak or TeV scales, and to look for them there is like the “search under a lamppost”.

Similarly, $E_6$ GUT contains the subgroup $SO(10) \times U(1)_\psi$, giving rise to another $Z'$. In addition, the fundamental $27$ contains besides the $16$ of $SO(10)$, a fundamental decouplet plus a singlet. The decouplet decomposes into a vector-like pair of $SU(3)$ fundamentals, containing an isoscalar down-type quark and a lepton doublet. Searches for these exotic states at LEP 2 and the Tevatron set lower limits of about 90 GeV on their masses.

The potential $Z'$ boson is in general a superposition of the SM $Z$ and the new boson associated with the extra $U(1)$,

$$Z = Z_1^0 \cos \theta + Z_2^0 \sin \theta, \quad Z' = -Z_1^0 \sin \theta + Z_2^0 \cos \theta,$$

where

$$\tan^2 \theta = \frac{M_{Z'}^2 - M_Z}{M_{Z'} - M_{Z_1^0}}.$$
and where $M_{Z'}$ is the SM value for $M_Z$ in the absence of mixing. Note that $M_Z < M_{Z'}$ (for the case that $Z'_1$ is the lighter of the states before mixing), and that the SM $Z$ couplings are changed by the mixing. $M_{Z'}$ can be calculated from $M_W$ and compared with $M_Z$ from LEP. A significant difference could be an indication for the presence of the extra boson, as would a difference between $M_Z$ and the value predicted by the other $Z$ pole observables. $Z'$ exchange is suppressed (out of phase) at the $Z$ pole. However, stringent limits can be obtained from weak neutral current data at lower energies.

One finds $M_{Z'} > 330$ GeV from precision data for arbitrary mixing, but limits can increase to the TeV scale in specific models with known mixing. Collider limits on $Z'$ masses depend on the chiral couplings of the new gauge boson to ordinary quarks and leptons, and on possible exotic decay modes. For a $Z'$ with SM couplings and a width which scales like the SM one, CDF (DØ) sets a lower limit of 690 (670) GeV. For typical GUT models the limits are $M_{Z'} > 600$–1000 GeV, with very constrained mixing angles $\theta$ of at most a few per mille. Limits on a leptophobic $Z'$ are weaker, with $M_{Z'} \sim 150$ GeV and $\theta$ a few per cent allowed. For reviews, see Refs. 81.

Perturbative string models with supergravity mediated SUSY breaking usually predict many extra $Z'$ bosons and exotic states. Unlike the GUT scenarios, these models tend to favor $Z'$ masses of $O(M_Z)$. The general idea is that the mixing of the two MSSM Higgs doublets (the $\mu$ term) which is expected to vanish at tree level (and in fact to all orders in the absence of SUSY breaking) is generated radiatively through large Yukawa couplings. The $\mu$ problem, i.e., the expectation that either $\mu = 0$ or of order the Planck scale, is solved by the introduction of a SM singlet field $S$. Its expectation value generates an effective $\mu$ term of $O(M_H^2) \sim O(M_Z^2) \sim O(M_{\text{SUSY}}^2)$. Given this kind of scenario, a TeV scale $Z'$ boson is no longer "lampport physics".

Another class of models involves symmetry breaking along D-flat directions. This can introduce $Z'$ bosons with intermediate scale masses (e.g., $10^{12}$ GeV), and can have interesting consequences for the generation of fermion mass hierarchies and neutrino masses.

6 Conclusions

The precision data confirms the validity of the SM at the electroweak loop level (i.e., within a few per mille), and there is no compelling evidence for deviations. The determination of $m_t$ from electroweak loops is consistent with the kinetic mass measurement, and the low versus high energy $\alpha_s$ problem has disappeared. The evaluation of $\alpha(M_Z)$ is due to new perturbative and non-perturbative QCD treatments more precise than in the past.
A low Higgs mass is strongly favored by the data. $M_H$ is now much more robust to changes in the data set. The precise range is rather sensitive to $\alpha(M_Z)$, but the upper limits are not. However, in the presence of non-standard contributions to the $S$ or $T$ parameters, no strong $M_H$ bounds can be found.

There are stringent constraints on parameters beyond the SM, such as $\rho$, $S$, $T$, $U$, and others. This is a serious problem for models of dynamical symmetry breaking, compositeness, and the like. Those constraints are, however, consistent with gauge unification, and also with the MSSM favoring its decoupling limit. Moreover, the low favored $M_H$ is in agreement with the expected mass range for the lightest neutral Higgs boson in the MSSM.

Perturbative string models (and also many GUT’s) suggest extra $Z'$ bosons. Rather generically they are expected at the electroweak scale and are highly predictive, with many specific models already excluded. In other words, a TeV scale $Z'$ is no longer “lanppost physics”, but is among the best motivated possibilities beyond the MSSM. In other scenarios extra $Z'$ bosons can appear at intermediate scales, and could solve the fermion mass hierarchy problem, and simultaneously generate neutrino masses.

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