Two-Higgs doublet models from TeV-scale supersymmetric extra
$U(1)$ models

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Abstract

We investigate the reduction of a general TeV-scale supersymmetric extra
$U(1)$ model to a 2HDM below the TeV-scale through the tree level non-
decoupling. Portions of the parameter space of the extra $U(1)$ model appro-
priate for obtaining a 2HDM are identified. Various properties of the resulting
2HDM are connected to the parameter space of the underlying model.
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I. INTRODUCTION

For various theoretical and phenomenological reasons various extensions of the SM have been introduced. The simplest such extension is Two-Higgs Doublet Models (2HDM) where one extends the scalar sector of the SM by adding a second doublet without modifying the gauge structure [1]. Such models are mainly motivated by MSSM [2] in which one has to introduce at least two Higgs doublets. One basic use of the 2HDM’s is that one is able to create the necessary mass splitting between the up- and down-type quarks [3]. In this respect, various high energy processes have been discussed in detail [4]. Furthermore, the famous Sakharov conditions [5] for creating the baryon asymmetry in the universe cannot be satisfied in SM due to the smallness of the CP-violation. In 2HDM, however, explicit or spontaneous breaking of CP is allowed with the self interactions of the Higgs doublets [6].

MSSM has two Higgs doublets and its Higgs sector mimics that of the 2HDM in many respects, except for the fact that supersymmetry fixes various parameters in terms of the gauge couplings [1]. However, MSSM has an hierarchy problem, namely, the scale of the supersymmetric mass term \( \mu \) is not known.

In this work, we work out supersymmetric models with an additional \( U(1) \) which are known to solve the MSSM \( \mu \) problem [7–10]. Indeed, as was already argued in [11], in a large class of string models, breaking scale of the extra \( U(1) \)’s come out to be around a TeV. Whatever the Planck scale consideration (SUSY GUT’s or Superstrings) from which these models follow, to be able to get a sensible solution to the problems mentioned above it is necessary to keep supersymmetry and the gauge symmetry exact till the TeV scale. When the SUSY is broken around the TeV scale one gets \( \mu \sim \mathcal{O}(\text{TeV}) \) so that the \( \mu \)-problem is avoided. The possible existence of a supersymmetric Abelian gauge factor which is broken at the TeV scale together with the supersymmetry affects the weak scale observables through the non-decoupling effects in the Higgs sector [12].

In this work we analyze the Higgs sector of an Abelian extended SUSY model (we call this \( Z' \) model from now on) such that after the breaking of the extra supersymmetric \( U(1) \) factor there arises an effective 2HDM at the weak scale. This problem has already been worked out in [12] where various phenomenological implications of non-decoupling effects in the Higgs sector were discussed. However, in this work we present a detailed analysis of the tree level constraints on the scalar potential of a general \( Z' \) model to be able to get a 2HDM below the TeV scale.

In Sec. 2 we first discuss the reduction of the scalar potential of a general \( Z' \) model to a 2HDM potential below the TeV scale. Next, we identify the appropriate portions of the \( Z' \) model parameter space to make this reduction process viable.

In Sec. 3 we discuss the properties of the resulting 2HDM potential in connection with several phenomenological issues.

In Sec. 4 we conclude the work.

II. 2HDM FROM A TEV SCALE SUPERSYMMETRIC EXTRA \( U(1) \)

In \( Z' \) models, the MSSM gauge group is extended to \( G = SU(3)_c \times SU(2) \times U(1)_Y \times U(1)_{Y'} \) with the coupling constants \( g_3, g_2, g_Y, g_{Y'} \), respectively. Under \( G \), the Higgs super-
fields are assigned the quantum numbers $\hat{H}_1 \sim (1, 2, -1/2, Q'_1)$, $\hat{H}_2 \sim (1, 2, 1/2, Q'_2)$, $\hat{S} \sim (1, 1, 0, Q'_S)$. Here $\hat{S}$ is an SM gauge singlet whose vacuum expectation value (VEV) breaks the extra $U(1)$. Part of the superpotential containing exclusively the Higgs fields is given by

$$W \ni h_S \hat{S} \hat{H}_1 \hat{H}_2$$

In addition to this, the complete superpotential contains fermion trilinear mass terms. We discard such terms from the analysis because whenever the sfermions develop non-vanishing VEV’s color and/or charge symmetries are broken. As long as the parameter spaces we work in do not imply non-vanishing sfermion VEV’s, analysis of the Higgs dependent part of the potential suffices. We further note that due to the $U(1)$ symmetry a bare $\mu$ term is forbidden; thus form of the superpotential in (1) is unique.

The full scalar potential in the $Z'$ models is given by

$$V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 |S|^2 + \lambda_0^0 |H_1|^4 + \lambda_0^0 |H_2|^4 + \lambda_0^0 |S|^4 + \lambda_{12}^0 |H_1|^2 |H_2|^2 + \lambda_{12}^0 |H_1|^2 |S|^2 + \lambda_{2S}^0 |H_2|^2 |S|^2 + \lambda_{2S}^0 |H_2|^2 |S|^2 + \tilde{\lambda}_{12}^0 |H_1 \cdot H_2|^2 - h_s A^0_s (SH_1 H_2 + h.c.)$$

where we discarded the hat on the superfields to denote their scalar components. In (2), $m_1^2$, $m_2^2$ and $m_3^2$ are the soft mass-squareds of $H_1$, $H_2$ and $S$, respectively. $A^0_s$ is the Higgs trilinear coupling. While these quantities of mass dimension come from the soft supersymmetry breaking part of the potential, those terms involving the adimensional parameters $\lambda_i^0$ come from the supersymmetric part of the Lagrangian consisting of $F$ and $D$ terms:

$$\lambda_0^0 = \frac{1}{8} G^2 + \frac{1}{2} g_{Y'}^2 Q'^2_1$$
$$\lambda_2^0 = \frac{1}{8} G^2 + \frac{1}{2} g_{Y'}^2 Q'^2_2$$
$$\lambda_3^0 = \frac{1}{2} g_{Y'}^2 Q'^2_2$$
$$\lambda_{12}^0 = \frac{1}{4} (g_2^2 - g_Y^2) + g_{Y'}^2 Q'_1 Q'_2$$
$$\lambda_{1S}^0 = g_{Y'}^2 Q'_1 Q'_S + h_s^2$$
$$\lambda_{2S}^0 = g_{Y'}^2 Q'_2 Q'_S + h_s^2$$
$$\tilde{\lambda}_{12}^0 = h_s^2 - g_2^2/2$$

where $G = \sqrt{g_2^2 + g_Y^2}$.

In this work, we shall not investigate the CP-violating or charge breaking properties of the vacuum state so we allow only the CP-even components of the Higgs fields to develop non-vanishing VEV’s. Thus, VEV’s of the Higgs fields in (2) are subjected to the following parametrization

$$<H_1> = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v_1 \\ 0 \end{array} \right), \quad <H_2> = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2 \end{array} \right), \quad <S> = \frac{v_s}{\sqrt{2}}$$

$$3$$
with real $v_1, v_2$ and $v_s$. Furthermore by appropriate choice of the signs of the potential parameters they can always be taken positive [8].

To be able to obtain an effective 2HDM below the TeV-scale, one should realize $v_s \sim \mathcal{O}$(TeV) and $v_1 = 0 = v_2$ so that below the TeV scale SM- gauge group remains unbroken. Before an attempt to determine the portion of the parameter space for which only $U(1)Y'$ is broken, let us assume such a splitting between the SM-singlet and the doublet VEV’s indeed exist and derive the effective 2HDM potential below the TeV-scale. For convenience we apply the parametrization

$$S = \frac{1}{\sqrt{2}} (v_s + \phi_s + i\eta_s).$$

(5)

When $v_s$ is non-zero, $U(1)_{Y'}$ symmetry is broken and $\phi_s$ becomes a massive CP-even scalar with mass-squared $m^2_{\phi} = m^0_S + 3\lambda^0_1v_s^2$. $\eta_s$, on the other hand, becomes the pseudoscalar Goldstone boson to be swallowed by the gauge boson of $U(1)_{Y'}$, $Z'$, to acquire a mass. Inserting the parametrization of $S$ in (5) into the scalar potential (2), one reads off the Feynman rules relating the three Higgs fields:

$$H_iH_i\phi_s : \lambda^0_i v_s (i = 1, 2)$$

$$H_i^*H_j\phi_s : -h_s A^0 S \sqrt{2} (i \neq j = 1, 2)$$

(6)

where $H^c_i = i\sigma_2 H_i^*$. With the help of these Feynman rules one obtains the tree level diagrams representing the scattering processes $H_iH_j \rightarrow H_kH_l$ ($i, j, k, l = 1, 2$) mediated by $\phi_s$. We shall not reproduce these diagrams here as they are similar to ones given in [12]. For external momenta much smaller than $m_\phi$, the scattering amplitudes mentioned above merely result in modification of the original quartic couplings of the Higgs doublets producing the effective 2HDM quartic parameters. In addition to these quartic couplings, the mass-squareds of the Higgs doublets do also change as can be seen through the replacement of the parametrization in (5) into the quadratic terms in the potential (2). Defining, $\Phi_2 = H_2$ and $\Phi_1 = H_1^*$, the 2HDM potential below $v_s$ reads as follows:

$$V(\Phi_1, \Phi_2) = m^2_1\Phi^\dagger_1\Phi_1 + m^2_2\Phi^\dagger_2\Phi_2 + m^2_3(\Phi^\dagger_1\Phi_2 + \Phi^\dagger_2\Phi_1) + \frac{\lambda_1}{2}(\Phi^\dagger_1\Phi_1)^2$$

$$+ \frac{\lambda_2}{2}(\Phi^\dagger_2\Phi_2)^2 + \lambda_3(\Phi^\dagger_1\Phi_1)(\Phi^\dagger_2\Phi_2) + \lambda_4 |\Phi^\dagger_1\Phi_2|^2$$

$$+ \frac{\lambda_5}{2}((\Phi^\dagger_1\Phi_2)^2 + (\Phi^\dagger_2\Phi_1)^2)$$

(7)

where the new parameters here can be expressed in terms of those of the underlying $Z'$ model as follows:

$$m^2_1 = m^0_1 + \frac{1}{2}\lambda^0_1 v_s^2$$

$$m^2_2 = m^0_2 + \frac{1}{2}\lambda^0_2 v_s^2$$

$$m^2_3 = h_s A^0 S \sqrt{2} v_s$$

4
\[\lambda_1 = 2\lambda_0^0 - 4\lambda_1^0 S^2 v_s^2 / m_\phi^2\]
\[\lambda_2 = 2\lambda_2^0 - 4\lambda_2^0 S^2 v_s^2 / m_\phi^2\]
\[\lambda_3 = \lambda_{12}^0 - 2\lambda_{12}^0 S^2 v_s^2 / m_\phi^2\]
\[\lambda_4 = \tilde{\lambda}_{12}^0 - 2\left(\frac{m_3^2}{v_s}\right)^2 m_\phi^2\]
\[\lambda_5 = -4\left(\frac{m_3^2}{v_s}\right)^2 m_\phi^2\]

(8)

In obtaining these parameters we have used only tree level diagrams, a more sophisticated
derivation of which would include at least the one-loop effects contributed by the supraysia metric particle spectrum of the $Z'$ model. However, we neglect all such loop contributions by assuming that these tree level results give the most important part of the information we need. In fact, for large enough soft masses (above TeV scale) one expects SUSY spectrum to have negligible effects at the weak scale.

The reduction of the scalar potential (2) to the 2HDM potential rests on the assumption of an ordered breaking of the gauge symmetry, that is, only $U(1)_Y'$ symmetry is broken above the TeV scale while the SM-gauge gauge symmetry survives down to the weak scale at which it is broken in the usual way to reproduce the phenomenologically well-established electroweak data. This two-stage breaking of the gauge symmetry necessitates $v_s$, the SUSY breaking scale, be given by

\[v_s = \sqrt{-m_S^0^2 / \lambda_s}\]

(9)

while doublets still have vanishing VEV's. With this expression for $v_s$, the Higgs boson $\phi_s$ and the $Z'$ boson can be shown to have identical masses

\[m_\phi = M_{Z'} = \sqrt{-2m_S^0^2}\]

(10)

which has a meaning only when $m_S^0^2 < 0$. Obtaining the pattern of VEV's $v_1 = 0 = v_2$ and $v_s \neq 0$ requires a certain hierarchy between $m_S^0^2$ and the other mass parameters pertaining the Higgs doublets, that is, doublet soft mass-squareds and the Higgs trilinear coupling. We now turn to a detailed discussion of the constraints imposed on the parameters of the potential coming from the two-stage breaking of the gauge symmetry.

Our aim is to set limits on the doublet soft mass-squareds and the Higgs trilinear coupling for a given $m_S^0^2$ such that SM gauge symmetry remains unbroken at the TeV scale. For the sake of clarity it seems convenient to discuss the effects of these two mass scales separately. Hence, we first turn off $A_s^0$ and investigate the ranges of $m_1^0^2$ and $m_2^0^2$ . Next we shall turn to the discussion of $A_s^0$.

A. Effects of the soft masses

After setting $A_s^0 = 0$, all of the VEV's can be solved analytically by requiring the potential (2) to have vanishing gradients in all directions in the Higgs background. In addition to the
solution $v_1 = v_2 = v_s = 0$ characterizing the symmetric phase, we have for the broken phase

$$
v_1^2 = C_0 \left( (\lambda_{2S}^0)^2 - 4\lambda_2^0\lambda_S^0 \right) m_1^0 + \left( 2\lambda_{12}^0 - \lambda_{1S}^0 \lambda_{2S}^0 \right) m_2^2 \\
v_2^2 = C_0 \left( (\lambda_{1S}^0)^2 - 4\lambda_2^0\lambda_S^0 \right) m_2^0 + \left( 2\lambda_{12}^0 - \lambda_{1S}^0 \lambda_{2S}^0 \right) m_1^2 \\
v_s^2 = C_0 \left( 2\lambda_{1S}^0 - \lambda_{12}^0\lambda_{2S}^0 \right) m_1^0 + \left( 2\lambda_{12}^0 - \lambda_{1S}^0 \lambda_{2S}^0 \right) m_2^0 + (\lambda_{12}^0 - 4\lambda_1^0\lambda_S^0) m_s^0 \tag{11}
$$

where $1/C_0 = 4\lambda_2^0\lambda_S^0 + \lambda_{2S}^0\lambda_{1S}^0 - \lambda_{1S}^0\lambda_S^0 - \lambda_{12}^0\lambda_{1S}^0 - \lambda_{12}^0\lambda_{2S}^0$ is a common factor for all three VEV’s. From (9) it is easy to find the critical values of $m_1^{02}$ and $m_2^{02}$ at which $v_1^2$ and $v_2^2$ vanishes:

$$m_{1, crit}^2 = \frac{\lambda_{1S}^0}{2\lambda_S^0} m_s^0 \tag{12}$$

$$m_{2, crit}^2 = \frac{\lambda_{2S}^0}{2\lambda_S^0} m_s^0 .$$

Since $v_{1,2}$ change sign at these critical points they seperate two kinds of minima; while in one of which all VEV’s in (11) are non-vanishing (that is, symmetry is completely broken) and in the other one only $U(1)_{Y'}$ is broken (that is, $v_1 = 0 = v_2$ and $v_s \neq 0$). Since $m_s^2/\lambda_s^0$ must be negative for (10) be meaningful, the sign of the critical masses in (12) depends on the signs of $\lambda_s^0$ defined in (3). This requires the knowledge of $h_s, g_Y$, and the $U(1)_{Y'}$ charges, which would be possible only in a specific string or SUSY-GUT model. In addition to this observation, one expects Higgs doublets to have vanishing VEV’s when the absolute values of the doublet soft mass-squareds become much smaller than $-m_S^0$. This last observation enables one to impose certain conditions on the Higgs VEV’s (11), namely, the coefficients of $m_s^2$ must be negative for $v_2^2$, and positive for $v_1^2$ and $v_s^2$ so that, for large enough $-m_s^2$, doublet VEV’s obtained from (11) will be imaginary. Using the explicit expressions of $\lambda_i^0$ in (3), coefficients of $m_s^2$ in (11) can be shown to depend on three free parameters; $g_Y / Q'_1 / g_Y, h_s / G$ and $Q'_2 / Q'_1$. The requirements about the coefficients of $m_s^2$ in (11) produce a certain constraint on these free parameters. In fact, in Fig. 1, depicted is the allowed region in the $g_Y / g_Y - h_s / G$ plane for which the coefficients of $m_s^2$ in (11) take the required values mentioned above. When drawing this graph $|Q'_1|$ is absorbed into $g_Y$, and $Q'_2/Q'_1$ is set to unity. One recalls that, for $Q'_2/Q'_1 = 1$, $\lambda_1^0 = \lambda_2^0$ and $\lambda_0^0 = \lambda_0^0$ as is seen from (3). Although we consider a specific choice for $Q'_2/Q'_1$ here, with general formulae for VEV’s in (11), one can investigate other possibilities for $Q'_2/Q'_1$ as well. In forming Fig.1 we let $g_Y / g_Y$ and $h_s / G$ vary from 0.5 to 1.5 which is a symmetric interval around unity. We see that in this interval allowed $h_s / G$ values increase in approximate proportion with the allowed $g_Y / g_Y$ values.

Now we turn to the main question of how large doublet soft mass-squareds, in comparison with $m_s^2$, can be to have SM gauge symmetry unbroken at the TeV scale. We illustrate the ranges of the doublet soft mass-squareds with the help of the information provided by Fig. 1. Out of all the candidate points shown in Fig. 1 we take the one for which $h_s / G = 1$ and
$g_{Y'}/g_Y = 1$. This choice is in agreement with the usual prescriptions about the low-energy $Z'$ models, that is $g_{Y'} \sim g_Y$ [13] and $h_s \sim 0.7$ [8]. As mentioned before, for $Q_2'/Q_1' = 1$ used in forming Fig. 1, $\lambda_{1S}^0 = \lambda_{2S}^0$ for which the critical values of the doublet soft mass-squareds in (12) are equal. As the critical points are identical, for simplicity of the illustration, we take $m_1^{02} = m_2^{02} = m_0^2$ and show the $m_0^2/|m_S^{02}|$ dependence of the squared VEV’s (11) in units of $|m_S^{02}|$, in Fig. 2. As is seen from this figure, $v_s^2$ (solid line) increases monotonically with increasing $m_0^2/|m_S^{02}|$, never going through zero. $v_1^2$ and $v_2^2$ (dashed line), however, remain identical and drop below zero at approximately $m_0^2/|m_S^{02}| = -0.58$ as exactly predicted by the critical points in (12). Thus, Fig. 2 can be concluded by saying that SM gauge symmetry remains unbroken as long as $m_0^2/|m_S^{02}| > -0.58$ provided the parameter set used for Fig. 1 and $m_1^{02} = m_2^{02}$ are assumed.

The more general case of $Q_2'/Q_1' \neq 1$ and $m_1^{02} \neq m_2^{02}$ can be analyzed in a similar way by using the expressions of VEV’s in (11), and the critical points in (12). In this case there will be more free variables ($v_1 \neq v_2$) and their behaviour will be complicated for graphical presentation. Despite this, when the coefficient of $m_S^{02}$ is positive for doublet VEV’s and negative for SM -singlet VEV, and when the doublet mass-squareds exceed their respective critical points given in (12), one has a minimum of the potential for which SM gauge symmetry remains unbroken at the scale $v_s \sim \sqrt{m_S^{02}} \sim \mathcal{O}(\text{TeV})$. This last statement summarizes the results about the minimum of the potential leading to a 2HDM at the weak scale when the Higgs trilinear coupling mass parameter $A_s^0$ is much smaller than the quadratic mass parameters of the Higgs doublets.

### B. Effects of the Higgs trilinear coupling

The basic property of the Higgs trilinear coupling $A_s^0$ is that it forces all fields $H_1$, $H_2$, and $S$ to have identical VEV’s, depending on its strength compared to the other mass parameters in the potential [8]. This happens especially when all mass-squareds are of the same order and much smaller than $A_s^{02}$, and type of the transition from the symmetric to the broken minimum depends the sign of the sum $m^2 = m_1^{02} + m_2^{02} + m_S^{02}$. The critical point at which the transition occurs is $A_s^{\text{crit}} = (8/3)m^2$, and for positive (negative) $m^2$ passage to the broken minimum is a first (second) order phase transition [10]. In the present case, where $-m_S^{02}$ is much larger than all other mass parameters in the potential (2), one is to analyze the doublet VEV’s $v_{1,2}$ for a given $v_s$ to find the critical value of $A_s^0$ at which the transition from SM- symmetric minimum to SM-broken minimum occurs. This requires the solution of the minimization equations for the potential (2) in $v_{1,2}$ direction. Here one faces with two coupled third order algebraic equations the analytic solution of which is hard to construct, and even if constructed, the results will not be transparent. Thus, we consider now a special but an important limit, that is, $v_1 = v_2$, which can easily be obtained for certain set of parameters as illustrated in the last section. In this case it is not hard to determine the critical point $A_s^{\text{crit}}$ at which the transition to an SM- broken phase occurs:

$$A_s^{\text{crit}} = \frac{1}{\sqrt{2h_Sv_s}}(m_1^{02} + m_2^{02} + (\lambda_{1S}^0 + \lambda_{2S}^0) v_s^2)$$  \hspace{1cm} (13)
where \( v_s \) is given by (9). An inspection on this equation reveals that \( A_s^{\text{crit}} \) is large (small) when \( m_1^2 + m_2^2 \) is positive (negative). Moreover, when \( |m_1^2 + m_2^2| \) is small \( A_s^{\text{crit}} \) is mainly determined by \( v_s \). Moreover, the construction of \( A_s^{\text{crit}} \) from the minimization conditions reveals that transition is always second order. We now illustrate the range of \( A_s^0 \) for two distinct choices for the doublet soft mass-squareds. In Fig. 3, depicted are the \( A_s^0/m_0 \) dependence of \( v_s/m_0 \) (solid curve) and \( v_{1,2}/m_0 \) (dashed curve) for \( m_S^0 = -25 m_3^2 \), and other parameters are the same as ones used for Fig. 2. Here the choice of \( m_S^0 \) is arbitrary and sufficient as long as it is large negative compared to other parameters of mass dimension. As is seen from Fig. 3 and Fig. 4, the Higgs doublets have vanishing VEV’s until \( A_s^{\text{crit}} \) reaches the transition point \( A_s^{\text{crit}} = 3 m_0 \). Meanwhile \( v_s \) remains to have the value given by (9). The critical point is exactly the one predicted by (13). To see the dependence of \( A_s^{\text{crit}} \) in (13) on the doublet mass parameters we now illustrate the case of \( m_S^0 = -25 m_0^2 \) and \( m_1^0 = m_2^0 = 25 m_0^2 \) with other parameters being as in Fig. 1. In this case, as is seen from Fig. 4, critical point is large, \( A_s^{\text{crit}} = 7.65 m_0 \). And situated at the point predicted by (13).

In the light of the above observations, we conclude that when the Higgs trilinear coupling \( A_s^0 \) is below a certain critical value \( A_s^{\text{crit}} \) the Higgs doublets have vanishing VEV’s. This critical point depends on the doublet soft mass-squareds and \( v_s \). When the parameters of the potential allow for \( v_1 = v_2 \), the critical value of \( A_s^0 \) has the expression given in (13). It should be noted that, when \( m_1^2 \) and \( m_2^2 \) are positive, the Higgs doublets are unable to develop VEV’s with the help of their masses, leaving \( A_s^0 \) as the only remedy. This requires \( A_s^0 \) to be large enough (\( \sim \sqrt{m_S^0} \)) to help all fields acquire asymptotically nearly equal VEV’s as is clearly seen from Fig. 3 and Fig. 4.

### III. Properties of the Resulting 2HDM

In the last section we obtained an effective 2HDM potential after breaking the extra \( U(1) \) at the TeV scale. Equation (7) represents the most general 2HDM potential with the parameters in (8). We now discuss some important properties of (7).

- There are three parameters of mass dimension in (7): \( m_1^2 \), \( m_2^2 \) and \( m_3^2 \) each of which has its own characteristic minimum. As it is convenient to discuss their effects separately, let us first consider the case of \( |m_3^2| << |m_{1,2}^2| \). Needless to say this case corresponds to the small trilinear coupling regime of the potential (2), discussed in Sec. 2.1. In this case the neutral component of the Higgs doublet \( \Phi_i \) develops a VEV \( <\Phi_i^0> \sim \sqrt{-m_i^2/\lambda_i} \), where the mixed quartic terms are neglected. Clearly, if \( m_i^2 > 0 \), VEV becomes imaginary and potential prefers the symmetric minimum; \( <\Phi_i^0> = 0 \). There are two ways of making \( m_i^2 \) negative. First, the soft mass \( m_i^2 \) can be negative. Second \( \lambda_i^0 \) can be negative depending on the relative magnitudes of its D-term and F-term components, as listed in (3). Fig. 2, which is obtained for a special set of parameters, shows a typical case applicable when \( |m_3^2| \) is small. When soft masses of the Higgs doublets are negative, as is necessary to have \( <\Phi_i^0> \neq 0 \), one can vary \( m_{1,2}^2 \) up to \( \sim 0.5 v_s^2 \), in accordance with (12) and Fig. 2. On the other hand, the case of negative \( \lambda_i^0 \) can be obtained when \( -g_i^2 Q_i^i Q_S \) is larger than \( h_S^2 \). This latter alternative should be taken with care because perturbative nature of the model at the TeV scale can be
disturbed. Of course, having at least one of \( m_1^2 \) and \( m_2^2 \) is sufficient, thanks to the mixed quartic couplings in (7), one can make both VEV’s non zero. If \( |m_1^2| << -m_2^2 \) one gets a large \( \tan \beta \) minimum because \( |\Phi_1^0| \) will be much smaller than \( |\Phi_2^0| \).

In the opposite case, \( |m_2^2| << -m_1^2 \) one has a small \( \tan \beta \) by the similar arguments. While in former case the huge \( m_t/m_b \) ratio can be obtained for nearly equal top \( (h_t) \) and bottom \( (h_b) \) Yukawa couplings, in the latter case one would need \( h_t << h_b \). Finally, \( m_1^2 \sim m_2^2 \) implies \( \tan \beta \sim 1 \) when both mass-squareds are negative, and in this case \( h_t \sim 1 >> h_b \) is sufficient.

When \( |m_3^2| >> |m_{1,2}^2| \), \( m_3^2 \) makes up almost a unique mass scale for the potential. Then \( <\Phi_1^0> \sim <\Phi_2^0> \) and consequently \( \tan \beta \sim 1 \) for which \( h_t \sim 1 >> h_b \) as mentioned above. This large \( |m_3^2| \) case correspond to the large \( A_0^0 \) regime analyzed in Sec. 2.2.

These naive tree level considerations are not valid when the loop effects on the parameters of the potential (7) are taken into account. The largest of such corrections come to the quartic coupling of \( H_2 \), \( \lambda_3 \) due to large \( h_t \) [10]. This change in \( \lambda_3 \) directly affects \( v_2 \) whereby modifying the value of \( \tan \beta \). However, in the large \( |m_3^2| \) regime one does not expect too big changes, as long as it still dominates over the quadratic mass parameters.

- Depending on the way \( U(1)_Y \) is broken, the various operators in the 2HDM potential (7) get suppressed or amplified, as are seen from (8). When \( A_{1,2}^0 \) is vanishingly small, as discussed in Sec. 2.1, \( \lambda_5 \) becomes negligible and \( \lambda_4 \) approaches \( \lambda_4^0 \). In addition, the quadratic mass parameters \( m_{1,2}^2 \) approach to the corresponding soft mass-squareds in (2). Finally, the Higgs mixing mass \( m_3^2 \) becomes negligibly small in this limit.

When \( A_0^0 \) is the dominant mass parameter, as analyzed in Sec. 2.2, one has \( m_3^2 \sim m_1^2 \sim m_2^2 \sim A_0^{20} \). Unlike the previous case, here \( \lambda_5 \) and \( m_3^2 \) are no way negligible. However, it is worthy of noting that the values of \( \lambda_1 \), \( \lambda_2 \) and \( \lambda_3 \) is independent of the way \( U(1)_Y \) symmetry is broken. This is because \( v_3^2/m_3^2 = 1/2\lambda_3^0 \) in both cases.

- Throughout the discussions in the last section all parameters of the potential (2) have been assumed real. In addition to this, the Higgs fields were assigned real eigenvalues. If the Higgs trilinear coupling \( A_0^0 \) in the potential (2) were complex so are \( \lambda_5 \) and \( m_3^2 \) in (7). After writing the relevant parts of the potential (7) appropriately, one observes that under a CP transformation \( \Phi_i \rightarrow e^{i \theta_i} \Phi_i^* \), \( \lambda_5 \) and \( m_5^2 \) dependent terms do not transform trivially. However, since \( \lambda_5/\lambda_5^* = (m_5^2/m_5^{2*})^2 \), CP- invariance of the potential is guaranteed [14]. In fact, there are strong experimental limits, though in MSSM, forcing \( m_3^2 \) to be nearly real [15]. Thus, the resulting 2HDM potential (7) does not support the explicit violation of CP. As is seen from (8), \( \lambda_5 < 0 \), so that the 2HDM potential (7) cannot accomodate spontaneous CP violation too [16]. Thus, the resulting 2HDM does break CP neither explicitly nor spontaneously.

- As has already been discussed in [8,10], the parameters of the \( Z' \) model (2) can be connected to the unification level (strings or GUT’s) initial conditions via the RGE’s. In this sense, better the determinations at low-energies, better the knowledge one has about unification level parameters. Thus, it would be desirable to constrain the
parameters of the 2HDM potential (7) using the low energy data as much as possible. For example, the precisely measured Z-pole observables can be used to constrain these parameters [14,17].

IV. CONCLUSIONS

In this work, we have presented the reduction of a supersymmetric TeV-scale extra $U(1)$ to an effective 2HDM at the weak scale. We have identified the appropriate portions of the parameter space of the extra $U(1)$ model for obtaining a 2HDM below the TeV scale. Moreover, properties of the resulting 2HDM is connected to the mechanism of extra $U(1)$ breaking. In addition to these, we have discussed various properties of this 2HDM in connection with weak scale phenomenology, $Z$-pole data and CP-violation. Other than these $Z'$ models, reduction of NMSSM to an effective 2HDM can also be worked out. In spite of the cosmological problems due to the broken $Z_3$ symmetry [18], from the particle physics point of view such models can provide us with an effective 2HDM at the weak scale, when the SM-singlet picks up a VEV around a TeV. In this case, mass of the pseudoscalar boson breaking the unwanted Peccei-Quinn symmetry will be an important constraint. Whatever the TeV scale model we start with, the weak scale 2HDM can constrain the parameter space of the original model through the experimental data. This low-energy determination can eventually be useful in constraining the unification scale initial conditions.

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REFERENCES

Figure Captions

Fig. 1. The region in the $g_Y'/g_Y - h_s/G$ plane for which the coefficient of $m_S^2$ in (9) is negative for SM-singlet VEV and positive for the doublet VEV’s.

Fig. 2. $m_0^2/|m_S^2|$ dependence of the squared VEV’s (9) in units of $|m_S^2|$ for $m_1^0 = m_2^0 = m_0^2$ and $h_s/G = 1$ and $g_Y'/g_Y = 1$. Here solid curve is for $v_s$, and dashed curve fo $v_{1,2}$.

Fig. 3. $A_0/v_s$ dependence of $v_s$ (solid curve) and $v_{1,2}$ (dashed curve). Here $m_S^2 = -25m_0^2$ and other parameters are as in Fig. 2.

Fig. 4. $A_0/v_s$ dependence of $v_s$ (solid curve) and $v_{1,2}$ (dashed curve). Here $m_S^2 = -25m_0^2$, $m_1^0 = m_2^0 = 25m_0^2$, and other parameters are as in Fig. 1.
Figure 1
Figure 2
Figure 3

VEV/m_0

$\frac{A_s}{m_0}$
Figure 4