Charmless Hadronic Two-body Decays of $B_s$ Mesons

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Abstract

Two-body charmless nonleptonic decays of the $B_s$ meson are studied within the framework of generalized factorization in which factorization is applied to the tree level matrix elements while the effective Wilson coefficients are $\mu$ and renormalization scheme independent, and nonfactorizable effects are parametrized in terms of $N_{c}^{\text{eff}}(LL)$ and $N_{c}^{\text{eff}}(LR)$, the effective numbers of colors arising from $(V - A)(V - A)$ and $(V - A)(V + A)$ four-quark operators, respectively. Branching ratios of $B_s \to PP, PV, VV$ decays ($P$: pseudoscalar meson, $V$: vector meson) are calculated as a function of $N_{c}^{\text{eff}}(LR)$ with two different considerations for $N_{c}^{\text{eff}}(LL)$: (a) $N_{c}^{\text{eff}}(LL)$ being fixed at the value of 2, and (b) $N_{c}^{\text{eff}}(LL) = N_{c}^{\text{eff}}(LR)$. Tree and penguin transitions are classified into six different classes. We find that (i) the electroweak penguin contributions account for about 85% (for $N_{c}^{\text{eff}}(LL) = 2$) of the decay rates of $B_s \to \eta\pi, \eta'\pi, \eta\rho, \eta'\rho, \phi\pi, \phi\rho$, which receive contributions only from tree and electroweak penguin diagrams; a measurement of them will provide a clean determination of the electroweak penguin coefficient $a_9$, (ii) electroweak penguin corrections to $B_s \to \omega\eta'$, $\phi\eta, \omega\phi, K^{(*)}\phi, \phi\phi$ are in general as significant as QCD penguin effects and even play a dominant role; their decay rates depend strongly on $N_{c}^{\text{eff}}(LR)$, (iii) the branching ratio of $B_s \to \eta'\eta'$, the analogue of $B_d \to \eta'K$, is of order $2 \times 10^{-5}$, which is only slightly larger than that of $\eta'\eta', K^{(*)}\rho, K^{(*)}K, K^{(*)}\bar{K}$ decay modes, (iv) the contribution from the $\eta'$ charm content is important for $B_s \to \eta'\eta'$, but less significant for $B_s \to \eta\eta'$, and (v) the decay rates for the final states $K^{(*)}K^{(*)}$ follow the pattern: $\Gamma(B_s \to K^+K^-) > \Gamma(B_s \to K^+\bar{K}^-) \approx \Gamma(B_s \to K^{**}K^{**}) > \Gamma(B_s \to K^{**}K^{**})$ and likewise for $K^{0(*)}\bar{K}^{0(*)}$, as a consequence of various interference effects between the penguin amplitudes governed by the effective QCD penguin coefficients $a_4$ and $a_6$. 
I. INTRODUCTION

Recently there has been a remarkable progress in the study of exclusive charmless $B$ decays, both experimentally and theoretically. On the experimental side, CLEO has discovered many new two-body decay modes [1]:

$$B \rightarrow \eta' K^+, \eta' K_S^0, \pi^\pm K_S^0, \pi^\pm K^\pm, \pi^0 K^\pm, \omega K^\pm,$$  \hspace{1cm} (1.1)

and a possible evidence for $B \rightarrow \phi K^*$. Moreover, CLEO has improved upper limits for many other channels. Therefore, it is a field whose time has finally arrived. On the theoretical aspect, many important issues have been studied in past years, such as the effective Wilson coefficients that are renormalization scale and scheme independent, nonfactorizable effects in hadronic matrix elements, the QCD anomaly effect in the matrix element of pseudoscalar densities, running light quark masses at the scale $m_b$, and the $q^2$ dependence of form factors.

In the present paper, we plan to extend previous studies of charmless hadronic decays of $B^-$, $B_d$ mesons to the $B_s$ mesons. In principle, the physics for the $B_s$ two-body hadronic decays is very similar to that for the $B_d$ meson except that the spectator $d$ quark is replaced by the $s$ quark. Experimentally, it is known that $B^\pm \rightarrow \eta' K^\pm$ and $B_d \rightarrow \eta' K$ have abnormally large branching ratios, several times larger than previous predictions. It would be very interesting to see if the analogue of $B_d \rightarrow \eta' K$, namely $B_s \rightarrow \eta' \eta'$ or $B_s \rightarrow \eta' \eta'\eta'$ still has the largest branching ratio in two-body $B_s$ charmless decays. Another point of interest is concerned with the electroweak penguin corrections. It is naively believed that in charmless $B$ decays, the contributions from the electroweak penguin diagrams are negligible compared to the QCD penguin corrections because of smallness of electroweak penguin Wilson coefficients. As pointed out in [2], some $B_s$ decay modes receive contributions only from the tree and electroweak penguin diagrams and moreover they are dominated by the latter. Therefore, electroweak penguins do play a dominant role in some of $B_s$ decays. There also exist several penguin-dominated $B_s$ decay modes in which electroweak penguin corrections to the decay rate are comparable to that of QCD penguin contributions. In this paper, we will study this in details.

Experimentally, only upper limits on the branching ratios have been established for a few $B_s$ rare decay modes (see [3] or Table 7 of [1]) and most of them are far beyond the theoretical expectations. Nevertheless, it is conceivable that many of the $B_s$ charmless decays can be seen at the future hadron colliders with large $b$ production. Theoretically, early systematical studies can be found in [4,5]. More recently, one of us (B.T.) [6] has analyzed the exclusive charmless $B_s$ decays involving the $\eta$ or $\eta'$ within the framework of generalized factorization.

This paper is organized as follows. A calculational framework is set up in Sec. II in which we discuss the scale and scheme independent Wilson coefficient functions, parametrization of nonfactorizable effects, classification of factorizable amplitudes,..., etc. The numerical results and discussions are presented in Sec. III. Conclusions are summarized in Sec. IV. The factorizable amplitudes for all the charmless two-body $B_s$ decays are given in Appendices.
II. CALCULATIONAL FRAMEWORK

A. Effective Hamiltonian

The relevant effective $\Delta B = 1$ weak Hamiltonian for hadronic charmless $B$ decays is

$$\mathcal{H}_{\text{eff}}(\Delta B = 1) = \frac{G_F}{\sqrt{2}} \left[ V_{td} V_{ts}^* c_1 O^t_1 + c_2 O^t_2 + V_{tb} V_{ts}^* (c_1 O^t_2 + c_2 O^t_8) - V_{tb} \sum_{i=3}^{10} c_i O_i \right] + \text{h.c.}, \quad (2.1)$$

where $q = d, s$, and

$$O^u_i = (\bar{u} b)_{V-A} (\bar{q} u)_{V-A}, \quad O^u_2 = (\bar{q} b)_{V-A} (\bar{u} u)_{V-A},$$

$$O^{3(5)} = (\bar{q} b)_{V-A} \sum_q (\bar{q}' q')_{V-A(V+A)}, \quad O^{4(6)} = (\bar{q}_\alpha b_\beta)_{V-A} \sum_q (\bar{q}'_\beta q'_\alpha)_{V-A(V+A)}, \quad (2.2)$$

$$O^{7(9)} = \frac{3}{2} (\bar{q} b)_{V-A} \sum_{q'} e_{q'} (\bar{q}' q')_{V+A(V-A)}, \quad O^{8(10)} = \frac{3}{2} (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A(V-A)},$$

with $O_3-O_6$ being the QCD penguin operators, $O_7-O_{10}$ the electroweak penguin operators, and $(\bar{q}_1 q_2)_{V+A} \equiv \bar{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$. In order to ensure the renormalization-scale and -scheme independence for the physical amplitude, the matrix element of 4-quark operators has to be evaluated in the same renormalization scheme as that for Wilson coefficients $c_i(\mu)$ and renormalized at the same scale $\mu$. Generically, the hadronic matrix element is related to the tree level one via

$$\langle O(\mu) \rangle = g(\mu) \langle O \rangle_{\text{tree}}, \quad (2.3)$$

with $g(\mu)$ being the perturbative corrections to the four-quark operators renormalized at the scale $\mu$. We employ the relation (2.3) to write $\langle \mathcal{H}_{\text{eff}} \rangle = c_{\text{eff}} \langle O \rangle_{\text{tree}}$. Schematically, the effective Wilson coefficients are given by $c_{\text{eff}} = c(\mu) g(\mu)$. Formally, one can show that $c_{\text{eff}}$ are $\mu$ and renormalization scheme independent. It is at this stage that the factorization approximation is applied to the hadronic matrix elements of the operator $O$ at the tree level. The physical amplitude obtained in this manner is guaranteed to be renormalization scheme and scale independent. *

Perturbative QCD and electroweak corrections to $g(\mu)$ from vertex diagrams and penguin diagrams have been calculated in [8–11]. The penguin-type corrections depend on $k^2$, the gluon’s momentum squared, so are the effective Wilson coefficient functions. To the next-to-leading order, we obtain [12]

$$c^\text{eff}_1 = 1.149, \quad c^\text{eff}_2 = -0.325,$$
$$c^\text{eff}_3 = 0.0211 + i0.0045, \quad c^\text{eff}_4 = -0.0450 - i0.0136,$$

*This formulation is different from the one advocated in [7] in which the $\mu$ dependence of the Wilson coefficients $c_i(\mu)$ are assumed to be canceled out by that of the nonfactorization parameters $\varepsilon_8(\mu)$ and $\varepsilon_1(\mu)$ so that the effective parameters $a^\text{eff}_i$ are $\mu$ independent.
\[ c_5^{\text{eff}} = 0.0134 + i0.0045, \quad c_6^{\text{eff}} = -0.0560 - i0.0136, \]
\[ c_7^{\text{eff}} = -(0.0276 + i0.0369)\alpha, \quad c_8^{\text{eff}} = 0.054\alpha, \]
\[ c_9^{\text{eff}} = -(1.318 + i0.0369)\alpha, \quad c_{10}^{\text{eff}} = 0.263\alpha, \]  
(2.4)

at \( k^2 = m_{\pi}^2/2 \). It is interesting to note that \( c_{1,2}^{\text{eff}} \) are very close to the leading order Wilson coefficients: \( c_{1}^{\text{LO}} = 1.144 \) and \( c_{2}^{\text{LO}} = -0.308 \) at \( \mu = m_b(m_b) \) [13] and that \( \text{Re}(c_{3,4}^{\text{eff}}) \approx \frac{3}{2}c_{3,4}^{\text{LO}}(\mu) \). Therefore, the decay rates of charmless \( B \) decay modes dominated by QCD penguin diagrams will be too small by a factor of \( \sim (1.5)^2 = 2.3 \) if only leading-order penguin coefficients are employed for the calculation.

B. Parametrization of nonfactorizable effects

Because there is only one single form factor (or Lorentz scalar) involved in the class-I or class-II decay amplitude of \( B \rightarrow PP, PV \) decays (see Sec. II.C for the classification of factorizable amplitudes), the effects of nonfactorization can be lumped into the effective parameters \( a_1 \) and \( a_2 \) [14]:

\[ a_1^{\text{eff}} = c_1^{\text{eff}} + c_2^{\text{eff}} \left( \frac{1}{N_c} + \chi_1 \right), \quad a_2^{\text{eff}} = c_2^{\text{eff}} + c_1^{\text{eff}} \left( \frac{1}{N_c} + \chi_2 \right), \]
(2.5)

where \( \chi_i \) are nonfactorizable terms and receive main contributions from color-octet current operators. Since \( |c_1^{\text{eff}}/c_2^{\text{eff}}| \gg 1 \), it is evident from Eq. (2.5) that even a small amount of nonfactorizable contributions will have a significant effect on the color-suppressed class-II amplitude. If \( \chi_{1,2} \) are universal (i.e. process independent) in charm or bottom decays, then we have a generalized factorization scheme in which the decay amplitude is expressed in terms of factorizable contributions multiplied by the universal effective parameters \( a_{1,2}^{\text{eff}} \). For \( B \rightarrow VV \) decays, this new factorization implies that nonfactorizable terms contribute in equal weight to all partial wave amplitudes so that \( a_{1,2}^{\text{eff}} \) can be defined. It should be stressed that, contrary to the naive one, the improved factorization does incorporate nonfactorizable effects in a process independent form. For example, \( \chi_1 = \chi_2 = -\frac{1}{3} \) in the large-\( N_c \) approximation of factorization. Phenomenological analyses of the two-body decay data of \( D \) and \( B \) mesons indicate that while the generalized factorization hypothesis in general works reasonably well, the effective parameters \( a_{1,2}^{\text{eff}} \) do show some variation from channel to channel, especially for the weak decays of charmed mesons [14–16]. An eminent feature emerged from the data analysis is that \( a_2^{\text{eff}} \) is negative in charm decay, whereas it becomes positive in the two-body decays of the \( B \) meson [14,17,7]:

\[ a_2^{\text{eff}}(D \rightarrow K\pi) \sim -0.50, \quad a_2^{\text{eff}}(B \rightarrow D\pi) \sim 0.20 - 0.28. \]  
(2.6)

It should be stressed that the magnitude of \( a_{1,2} \) depends on the model results for form factors. It follows that

\[ \chi_2(D \rightarrow K\pi) \sim -0.36, \quad \chi_2(B \rightarrow D\pi) \sim 0.12 - 0.19. \]  
(2.7)

The observation \( |\chi_2(B)| \ll |\chi_2(D)| \) is consistent with the intuitive picture that soft gluon effects become stronger when the final-state particles move slower, allowing more time for
significant final-state interactions after hadronization [14]. Phenomenologically, it is often to
treat the number of colors $N_c$ as a free parameter to model the nonfactorizable contribution
to hadronic matrix elements and its value can be extracted from the data of two-body
nonleptonic decays. Theoretically, this amounts to defining an effective number of colors
$N_c^{\text{eff}}$, called $1/\xi$ in [18], by

$$1/N_c^{\text{eff}} \equiv (1/N_c) + \chi. \quad (2.8)$$

It is clear from (2.7) that

$$N_c^{\text{eff}}(D \to K\pi) \gg 3, \quad N_c^{\text{eff}}(B \to D\pi) \sim 1.8 - 2.2. \quad (2.9)$$

The effective Wilson coefficients appear in the factorizable decay amplitudes in the combinations
$a_{2i} = c_{2i}^{\text{eff}} + \frac{1}{N_c} c_{2i-1}^{\text{eff}}$ and $a_{2i-1} = c_{2i-1}^{\text{eff}} + \frac{1}{N_c} c_{2i}^{\text{eff}} \ (i = 1, \ldots, 5)$. As discussed in
the Introduction, nonfactorizable effects in the decay amplitudes of $B \to PP, VP$ can be
absorbed into the parameters $a_i^{\text{eff}}$. This amounts to replacing $N_c$ in $a_i^{\text{eff}}$ by $(N_c^{\text{eff}})_i$. Explicitly,

$$a_{2i}^{\text{eff}} = c_{2i}^{\text{eff}} + \frac{1}{(N_c^{\text{eff}})_{2i}} c_{2i-1}^{\text{eff}}, \quad a_{2i-1}^{\text{eff}} = c_{2i-1}^{\text{eff}} + \frac{1}{(N_c^{\text{eff}})_{2i-1}} c_{2i}^{\text{eff}}, \quad (i = 1, \ldots, 5). \quad (2.10)$$

It is customary to assume in the literature that $(N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \cdots \approx (N_c^{\text{eff}})_{10}$ so that the
subscript $i$ can be dropped; that is, the nonfactorizable term is usually assumed to behave in
the same way in penguin and tree decay amplitudes. A closer investigation shows that this
is not the case. We have argued in [12] that nonfactorizable effects in the matrix elements
of $(V - A)(V + A)$ operators are a priori different from that of $(V - A)(V - A)$ operators.
One reason is that the Fierz transformation of the $(V - A)(V + A)$ operators $O_{5,6,7,8}$ is quite
different from that of $(V - A)(V - A)$ operators $O_{1,2,3,4}$ and $O_{9,10}$. As a result, contrary to the
common assumption, $N_c^{\text{eff}}(LR)$ induced by the $(V - A)(V + A)$ operators are theoretically
different from $N_c^{\text{eff}}(LL)$ generated by the $(V - A)(V - A)$ operators [12]. From Eq. (2.10) it is
expected that

$$N_c^{\text{eff}}(LL) \equiv (N_c^{\text{eff}})_1 \approx (N_c^{\text{eff}})_2 \approx (N_c^{\text{eff}})_3 \approx (N_c^{\text{eff}})_4 \approx (N_c^{\text{eff}})_9 \approx (N_c^{\text{eff}})_{10},$$

$$N_c^{\text{eff}}(LR) \equiv (N_c^{\text{eff}})_5 \approx (N_c^{\text{eff}})_6 \approx (N_c^{\text{eff}})_7 \approx (N_c^{\text{eff}})_8, \quad (2.11)$$

and $N_c^{\text{eff}}(LR) \neq N_c^{\text{eff}}(LL)$ in general. In principle, $N_c^{\text{eff}}$ can vary from channel to channel, as
in the case of charm decay. However, in the energetic two-body $B$ decays, $N_c^{\text{eff}}$ is expected to
be process insensitive as supported by data [7].

The $N_c^{\text{eff}}$-dependence of the effective parameters $a_i^{\text{eff}}$’s are shown in Table I for several
representative values of $N_c^{\text{eff}}$. From Table I we see that (i) the dominant coefficients are
$a_1, a_2$ for current-current amplitudes, $a_3$ and $a_6$ for QCD penguin-induced amplitudes, and
$a_9$ for electroweak penguin-induced amplitudes, and (ii) $a_1, a_4, a_6$ and $a_9$ are $N_c^{\text{eff}}$-stable, while
others depend strongly on $N_c^{\text{eff}}$. Therefore, for charmless $B$ decays whose decay amplitudes
depend dominantly on $N_c^{\text{eff}}$-stable coefficients, their decay rates can be reliably predicted
within the factorization approach even in the absence of information on nonfactorizable
effects.
The CLEO data of $B^\pm \to \omega \pi^\pm$ available last year clearly indicate that $N_{\text{eff}}^{\text{LL}}(LL)$ is favored to be small, $N_{\text{eff}}^{\text{LL}}(LL) < 2.9$ [12]. If the value of $N_{\text{eff}}^{\text{LL}}(LL)$ is fixed to be 2, the branching ratio of $B^\pm \to \omega \pi^\pm$ for positive $\rho$ ($\rho$ being a Wolfenstein parameter; see Sec. II.D), which is preferred by the current analysis [19], will be of order $(0.9 - 1.0) \times 10^{-5}$, which is very close to the central value of the measured one. Unfortunately, the significance of $B^\pm \to \omega \pi^\pm$ is reduced in the recent CLEO analysis and only an upper limit is quoted [20]. Nevertheless, the central value of $B(B^\pm \to \pi^\pm \omega)$ remains about the same. Therefore, a measurement of its branching ratio is urgently needed. A very recent CLEO analysis of $B^0 \to \pi^+ \pi^-$ [21] presents an improved upper limit, $B(B^0 \to \pi^+ \pi^-) < 0.84 \times 10^{-5}$. If the form factor $F_0^{B\pi}(0)$ is known, this tree-dominated decay could offer a useful constraint on $N_{\text{eff}}^{\text{LL}}(LL)$ as its branching ratio increases slightly with $N_{\text{eff}}^{\text{LL}}$. For $F_0^{B\pi}(0) = 0.30$, we find $N_{\text{eff}}^{\text{LL}}(LL) \lesssim 2.0$. The fact that $N_{\text{eff}}^{\text{LL}}(LL)$ is favored to be at the value of 2 in hadronic charmless two-body decays of the $B$ meson is consistent with the nonfactorizable term extracted from $B \to (D, D^*)\pi$, $D\rho$ decays, namely $N_{\text{eff}}^{\text{LL}}(B \to D\pi) \approx 2$. Since the energy release in the energetic two-body decays $B \to \omega \pi$, $B \to D\pi$ is of the same order of magnitude, it is thus expected that $N_{\text{eff}}^{\text{LL}}(LL)|_{B \to \omega \pi} \approx 2$. In analogue to the class-III $B \to D\pi$ decays, the interference effect of spectator amplitudes in charged $B$ decays $B^- \to \pi^- \pi^0, \rho^- \pi^0, \pi^- \rho^0$ is sensitive to $N_{\text{eff}}^{\text{LL}}(LL)$; measurements of them will be very useful to pin down the value of $N_{\text{eff}}^{\text{LL}}(LL)$.

As for $N_{\text{eff}}^{\text{LR}}(LR)$, it is found in [12] that the constraints on $N_{\text{eff}}^{\text{LR}}(LR)$ derived from $B^\pm \to \phi K^\pm$ and $B \to \phi K^*$ are not consistent. Under the factorization hypothesis, the decays $B \to \phi K$ and $B \to \phi K^*$ should have almost the same branching ratios, a prediction not borne out by current data. Therefore, it is crucial to measure the charged and neutral decay modes of $B \to \phi(K, K^*)$ in order to see if the generalized factorization approach is applicable to $B \to \phi K^*$ decay. Nevertheless, the analysis of $B \to \eta'K$ in [12] indicates that $N_{\text{eff}}^{\text{LL}}(LL) \approx 2$ is favored and $N_{\text{eff}}^{\text{LR}}(LR)$ is preferred to be larger. Since the energy release in the energetic two-body charmless $B$ decays is not less than that in $B \to D\pi$ decays, it is thus expected that

$$|\chi(2 \text{- body rare B decay})| \lesssim |\chi(B \to D\pi)|. \quad (2.12)$$

It follows from Eqs. (2.7) and (2.8) that $N_{\text{eff}}^{\text{LL}}(LL) \approx N_{\text{eff}}^{\text{LL}}(B \to D\pi) \sim 2$ and $N_{\text{eff}}^{\text{LR}}(LR) \sim 2-5$, depending on the sign of $\chi$. Since $N_{\text{eff}}^{\text{LR}}(LR) > N_{\text{eff}}^{\text{LL}}(LL)$ implied by the data, therefore, we conjecture that

$$N_{\text{eff}}^{\text{LL}}(LL) \approx 2, \quad N_{\text{eff}}^{\text{LR}}(LR) \lesssim 5. \quad (2.13)$$

C. Factorizable amplitudes and their classification

Applying the effective Hamiltonian (2.1), the factorizable decay amplitudes of $B_s \to PP, VP, VV$ obtained within the generalized factorization approach are summarized in the Appendices A,B,C, where, for simplicity, we have neglected $W$-annihilation, space-like penguins and final-state interactions. All the penguin contributions to the decay amplitudes can
Table I. Numerical values for the effective coefficients $a_i^\text{eff}$ at $N_c^{\text{eff}} = 2, 3, 5, \infty$ (in units of $10^{-4}$ for $a_3, \ldots, a_{10}$). For simplicity we will drop the superscript “eff” henceforth.

<table>
<thead>
<tr>
<th></th>
<th>$N_c^{\text{eff}} = 2$</th>
<th>$N_c^{\text{eff}} = 3$</th>
<th>$N_c^{\text{eff}} = 5$</th>
<th>$N_c^{\text{eff}} = \infty$</th>
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<tbody>
<tr>
<td>$a_1$</td>
<td>0.986</td>
<td>1.04</td>
<td>1.08</td>
<td>1.15</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.25</td>
<td>0.058</td>
<td>-0.095</td>
<td>-0.325</td>
</tr>
<tr>
<td>$a_3$</td>
<td>-13.9 – 22.6i</td>
<td>61</td>
<td>121 + 18.1i</td>
<td>211 + 45.3i</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-344 – 113i</td>
<td>-380 – 121i</td>
<td>-408 – 127i</td>
<td>-450 – 136i</td>
</tr>
<tr>
<td>$a_5$</td>
<td>-146 – 22.6i</td>
<td>-52.7</td>
<td>22.0 + 18.1i</td>
<td>134 + 45.3i</td>
</tr>
<tr>
<td>$a_6$</td>
<td>-493 – 113i</td>
<td>-515 – 121i</td>
<td>-533 – 127i</td>
<td>-560 – 136i</td>
</tr>
<tr>
<td>$a_7$</td>
<td>0.04 – 2.73i</td>
<td>-0.71 – 2.73i</td>
<td>-1.24 – 2.73i</td>
<td>-2.04 – 2.73i</td>
</tr>
<tr>
<td>$a_8$</td>
<td>2.98 – 1.37i</td>
<td>3.32 – 0.91i</td>
<td>3.59 – 0.55i</td>
<td>4</td>
</tr>
<tr>
<td>$a_9$</td>
<td>-87.9 – 2.73i</td>
<td>-91.1 – 2.73i</td>
<td>-93.7 – 2.73i</td>
<td>-97.6 – 2.73i</td>
</tr>
<tr>
<td>$a_{10}$</td>
<td>-29.3 – 1.37i</td>
<td>-13.1 – 0.91i</td>
<td>-0.04 – 0.55i</td>
<td>19.48</td>
</tr>
</tbody>
</table>

be derived from Table II by studying the underlying $b$ quark weak transitions. To illustrate this, let $X^{(BM_1,M_2)}$ denote the factorizable amplitude with the meson $M_2$ being factored out:

$$X^{(BM_1,M_2)} = \langle M_2 | (\bar{q}_2 q_3)_{V-A} | 0 \rangle \langle M_1 | (\bar{q}_1 b)_{V-A} | B \rangle.$$  \hspace{1cm} (2.14)

In general, when $M_2$ is a charged state, only $a_{\text{even}}$ penguin terms contribute. For example, from Table II we obtain

$$A(\bar{B}_s \to K^+ \pi^-)_{\text{peng}} \propto [a_4 + a_{10} + (a_6 + a_8) R] X^{(B_s K^+, \pi^-)},$$

$$A(\bar{B}_s \to K^{*+} \pi^-)_{\text{peng}} \propto [a_4 + a_{10} - (a_6 + a_8) R'] X^{(B_s K^{*+}, \pi^-)},$$

$$A(\bar{B}_s \to K^+ \rho^-)_{\text{peng}} \propto [a_4 + a_{10}] X^{(B_s K^+, \rho^-)},$$  \hspace{1cm} (2.15)

with $R' \approx R \approx m_\pi^2/(m_b m_d)$. When $M_2$ is a neutral meson with $I_3 = 0$, namely, $M_2 = \pi^0, \rho^0, \omega$ and $\eta^{(')}$, $a_{\text{odd}}$ penguin terms start to contribute. From Table II we see that the decay amplitudes of $\bar{B}_s \to M \pi^0$, $\bar{B}_s \to M \rho^0$, $\bar{B}_s \to M \omega$, $\bar{B}_s \to M \eta^{(')}$ contain the following respective factorizable terms:

$$\frac{3}{2}(-a_7 + a_9) X_u^{(B_s M, \pi^0)},$$

$$\frac{3}{2}(a_7 + a_9) X_u^{(B_s M, \rho^0)},$$

$$(2a_5 + 2a_7 + \frac{1}{2}a_7 + \frac{1}{2}a_9) X_u^{(B_s M, \omega)},$$

$$(2a_5 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9) X_u^{(B_s M, \eta^{(')})},$$  \hspace{1cm} (2.16)

where the subscript $u$ indicates the $u\bar{u}$ quark content of the neutral meson:

$$X_u^{(B_s M, \pi^0)} = \langle \pi^0 | (\bar{u} u)_{V-A} | 0 \rangle \langle M_1 | (\bar{q}_1 b)_{V-A} | B \rangle.$$  \hspace{1cm} (2.17)
For example, the penguin amplitudes of $B \to \eta \omega$ and $K^0 \pi^0$ are given by

\[
A(B_s \to \eta \omega)_{\text{peng}} \propto \left[2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9)\right] X_u^{(B_s \eta \omega)},
\]

\[
A(B_s \to K^0 \pi^0)_{\text{peng}} \propto \frac{3}{2}(-a_7 + a_9) X_u^{(B_s K^0 \pi^0)} + \left[a_4 - \frac{1}{2}a_{10} + (a_6 - \frac{1}{2}a_8)R\right] X_d^{(B_s K^0 \pi^0)},
\]

\[
\propto \left[-a_4 + \frac{3}{2}(-a_7 + a_9) + \frac{1}{2}a_{10} - (a_6 - \frac{1}{2}a_8)R\right] X_u^{(B_s K^0 \pi^0)},
\]

(2.18)

respectively. It is interesting to note that the decays $B_s \to (\eta', \phi)(\pi^0, \rho^0)$ do not receive any contributions from QCD penguin diagrams and they are dominated by electroweak penguins. We will come back to this interesting observation later.

Table II. Penguin contributions to the factorizable $B \to PP, VP,VV$ decay amplitudes multiplied by $-(G_F/\sqrt{2})V_{tb}V_{ts}^*$, where $q = d,s$. The notation $B \to M_1, M_2$ means that the meson $M_2$ can be factored out under the factorizable approximation. In addition to the $a_{\text{even}}$ terms, the decay also receives contributions from $a_{\text{odd}}$ penguin effects when $M_2$ is a neutral meson with $I_3 = 0$. Except for $\eta$ or $\eta'$ production, the coefficients $R$ and $R'$ are given by $R = 2m_{B_s}^2/[(m_1 + m_2)(m_b - m_3)]$ and $R' = -2m_{B_s}^2/[(m_1 + m_2)(m_b + m_3)]$, respectively.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$b \to qu\bar{u}$, $b \to qc\bar{c}$</th>
<th>$b \to qdd$, $b \to qss$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to P, P$</td>
<td>$a_4 + a_{10} + (a_6 + a_8)R$</td>
<td>$a_4 - \frac{1}{2}a_{10} + (a_6 - \frac{1}{2}a_8)R$</td>
</tr>
<tr>
<td>$B \to V, P$</td>
<td>$a_4 + a_{10} + (a_6 + a_8)R'$</td>
<td>$a_4 - \frac{1}{2}a_{10} + (a_6 - \frac{1}{2}a_8)R'$</td>
</tr>
<tr>
<td>$B \to P, V$</td>
<td>$a_4 + a_{10}$</td>
<td>$a_4 - \frac{1}{2}a_{10}$</td>
</tr>
<tr>
<td>$B \to V, V$</td>
<td>$a_4 + a_{10}$</td>
<td>$a_4 - \frac{1}{2}a_{10}$</td>
</tr>
<tr>
<td>$B \to P, P^0$</td>
<td>$a_3 - a_5 - a_7 + a_9$</td>
<td>$a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9$</td>
</tr>
<tr>
<td>$B \to V, P^0$</td>
<td>$a_3 - a_5 - a_7 + a_9$</td>
<td>$a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9$</td>
</tr>
<tr>
<td>$B \to P, V^0$</td>
<td>$a_3 + a_5 + a_7 + a_9$</td>
<td>$a_3 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9$</td>
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<td>$a_3 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9$</td>
</tr>
</tbody>
</table>

Just as the charm decays or $B$ decays into the charmed meson, the tree-dominated amplitudes for hadronic charmless $B$ decays are customarily classified into three classes [18]:

- **Class-I** for the decay modes dominated by the external $W$-emission characterized by the parameter $a_1$. Examples are $B_s \to K^+ \pi^-$, $K^{*+} \pi^-$, $\cdots$.

- **Class-II** for the decay modes dominated by the color-suppressed internal $W$-emission characterized by the parameter $a_2$. Examples are $B_s \to K^0 \pi^0$, $K^0 \rho^0$, $\cdots$.

- **Class-III** decays involving both external and internal $W$ emissions. Hence the class-III amplitude is of the form $a_1 + ra_2$. This class does not exist for the $B_s$. 

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Likewise, penguin-dominated charmless $B_s$ decays can be classified into three categories: 

- **Class-IV** for those decays whose amplitudes are governed by the QCD penguin parameters $a_4$ and $a_6$ in the combination $a_4 + Ra_6$, where the coefficient $R$ arises from the $(S - P)(S + P)$ part of the operator $O_6$. In general, $R = 2m_{P_b}^2/[(m_1 + m_2)(m_b - m_3)]$ for $B \to P_aP_b$ with the meson $P_b$ being factored out under the factorizable approximation, $R = -2m_{P_b}^2/[(m_1 + m_2)(m_b + m_3)]$ for $B \to V_aP_b$, and $R = 0$ for $B \to P_aV_b$ and $B \to V_aV_b$. Note that $a_4$ is always accompanied by $a_{10}$, and $a_6$ by $a_8$. In short, class-IV modes are governed by $a_{\text{even}}$ penguin terms. Examples are $\B_s \to K^0\K^0$, $\phi\eta^{(')}$, $\eta\eta^{(')}$, $\cdot\cdot\cdot$.

- **Class-V** modes for those decays whose amplitudes are governed by the effective coefficients $a_3, a_5, a_7$ and $a_9$ (i.e. $a_{\text{odd}}$ penguin terms) in the combinations $a_3 \pm a_5$ and/or $a_7 \pm a_9$ (see Table II). Examples are $\B_s \to \pi\eta^{(')}$, $\omega\eta^{(')}$, $\pi\rho$, $\cdot\cdot\cdot$.

- **Class-VI** involving the interference of class-IV and class-V decays, e.g. $\B_s \to \eta^{(')}\eta^{(')}$, $\phi\eta^{(')}$, $K^0\phi$, $\cdot\cdot\cdot$.

Sometimes the tree and penguin contributions are comparable. In this case, the interference between penguin and spectator amplitudes is at work. There are three such decays: $\B_s \to K^0\omega$, $K^0\eta^{(')}$, $K^*0\omega$; they involve class-II and -VI amplitudes (see Tables IV and V).

**D. Input parameters**

In this subsection we specify the values for various parameters employed in the present paper. For current quark masses, we employ the running masses at the scale $\mu = m_b$:

$$
\begin{align*}
m_u(m_b) &= 3.2 \text{ MeV}, & m_d(m_b) &= 6.4 \text{ MeV}, & m_s(m_b) &= 105 \text{ MeV}, \\
m_c(m_b) &= 0.95 \text{ GeV}, & m_b(m_b) &= 4.34 \text{ GeV}.
\end{align*}
$$

(2.19)

As for the Wolfenstein parameters $A, \lambda, \rho$ and $\eta$, which are utilized to parametrize the quark mixing matrix, we use $A = 0.804$, $\lambda = 0.22$, $\rho = 0.16$ and $\eta = 0.34$. The values for $\rho$ and $\eta$ follow from a recent analysis of all available experimental constraints imposed on the Wolfenstein parameters [19]:

$$
\bar{\rho} = 0.156 \pm 0.090, \quad \bar{\eta} = 0.328 \pm 0.054,
$$

(2.20)

where $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$. For the values of decay constants, we use $f_s = 132$ MeV, $f_K = 160$ MeV, $f_{K^*} = 210$ MeV, $f_{K^*} = 221$ MeV, $f_{\omega} = 195$ MeV and $f_{\phi} = 237$ MeV.

To determine the decay constant $f^{(q)}_{\eta^{(')}}$, defined by $\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | \eta^{(')} \rangle = i f^{(q)}_{\eta^{(')}} p_\mu$, it has been emphasized [24,25] that the decay constants do not simply follow the $\eta - \eta^{(')}$ state mixing given by

---

\[\text{Our classification of factorizable penguin amplitudes is not the same as that in [22]; we introduce three new classes in the same spirit as the classification of tree-dominated decays.}\]
\[ \eta' = \eta_8 \sin \theta + \eta_0 \cos \theta, \quad \eta = \eta_8 \cos \theta - \eta_0 \sin \theta. \]  \tag{2.21}

Introduce the decay constants \( f_8 \) and \( f_0 \) by

\[ \langle 0 | A_{\mu}^0 | \eta_0 \rangle = i f_0 p_{\mu}, \quad \langle 0 | A_{\mu}^8 | \eta_8 \rangle = i f_8 p_{\mu}. \]  \tag{2.22}

Because of SU(3) breaking, the matrix elements \( \langle 0 | A_{\mu}^0(8) | \eta_8(0) \rangle \) do not vanish in general and they will induce a two-angle mixing among the decay constants, that is, \( f_\eta \) and \( f_\eta' \) are related to \( f_8 \) and \( f_0 \) by

\[ f_\eta' = \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0, \quad f_\eta = -2 \frac{f_8}{\sqrt{6}} \sin \theta_8 + \frac{f_0}{\sqrt{3}} \cos \theta_0. \]  \tag{2.23}

Likewise,

\[ f_\eta'^* = \frac{f_8}{\sqrt{6}} \cos \theta_8 - \frac{f_0}{\sqrt{3}} \sin \theta_0, \quad f_\eta'^* = -2 \frac{f_8}{\sqrt{6}} \cos \theta_8 - \frac{f_0}{\sqrt{3}} \sin \theta_0. \]  \tag{2.24}

Based on the ansatz that the decay constants in the quark flavor basis follow the pattern of particle state mixing, relations between \( \theta_8, \theta_0 \) and \( \theta \) are derived in [25], where \( \theta \) is the \( \eta - \eta' \) mixing angle introduced in (2.21). It is found in [25] that phenomenologically

\[ \theta_8 = -21.2^\circ, \quad \theta_0 = -9.2^\circ, \quad \theta = -15.4^\circ, \]  \tag{2.25}

and

\[ f_8/f_\pi = 1.26, \quad f_0/f_\pi = 1.17. \]  \tag{2.26}

The decay constant \( f_\eta'^* \), defined by \( \langle 0 | \bar{c} \gamma_{\mu} \gamma_5 c | \eta' \rangle = i f_\eta'^* q_{\mu} \), has been determined from theoretical calculations [27–29] and from the phenomenological analysis of the data of \( J/\psi \to \eta c \gamma, J/\psi \to \eta' \gamma \) and of the \( \eta \gamma \) and \( \eta' \gamma \) transition form factors [11,25,30–32]; it lies in the range \(-2.3 \text{ MeV} \leq f_\eta'^* \leq -18.4 \text{ MeV} \). In this paper we use the values

\[ f_\eta'^* = -(6.3 \pm 0.6) \text{ MeV}, \quad f_\eta'^* = -(2.4 \pm 0.2) \text{ MeV}, \]  \tag{2.27}

as obtained in [25].

For form factors, the Bauer-Stech-Wirbel (BSW) model [26] gives [5]

\[ F_0^{B,K}(0) = 0.274, \quad F_0^{B,\eta \gamma}(0) = 0.335, \quad F_0^{B,\eta' \gamma}(0) = 0.282, \]

\[ A_0^{B,\phi}(0) = 0.272, \quad A_1^{B,\phi}(0) = 0.273, \quad A_2^{B,\phi}(0) = 0.273, \]

\[ A_0^{B,K^*}(0) = 0.236, \quad A_1^{B,K^*}(0) = 0.232, \quad A_2^{B,K^*}(0) = 0.231, \]

\[ V^{B,\phi}(0) = 0.319, \quad V^{B,K^*}(0) = 0.281. \]  \tag{2.28}

\[ \dagger \text{The form factors adopted in [6] are calculated using the light-front quark model and in general they are larger than the BSW model's results.} \]
It should be stressed that the $\eta - \eta'$ wave function normalization has not been included in the form factors $F_0^{B_s\eta}$ and $F_0^{B_s\eta'}$; they are calculated in a relativistic quark model by putting the $s\bar{s}$ constituent quark mass only. To compute the physical form factors, one has to take into account the wave function normalizations of the $\eta$ and $\eta'$:

$$F_0^{B_s\eta} = - \left( \frac{2}{\sqrt{6}} \cos \theta + \frac{1}{\sqrt{3}} \sin \theta \right) F_0^{B_s\eta s\bar{s}}, \quad F_0^{B_s\eta'} = \left( -\frac{2}{\sqrt{6}} \sin \theta + \frac{1}{\sqrt{3}} \cos \theta \right) F_0^{B_s\eta s\bar{s}}. \quad (2.29)$$

It is clear that the form factors $F_0^{B_s\eta}$ and $F_0^{B_s\eta'}$ have opposite signs.

For the $q^2$ dependence of form factors in the region where $q^2$ is not too large, we shall use the pole dominance ansatz, namely,

$$f(q^2) = \frac{f(0)}{(1 - q^2/m_s^2)^n}, \quad (2.30)$$

where $m_s$ is the pole mass given in [18]. A direct calculation of $B \to P$ and $B \to V$ form factors at time-like momentum transfers is available in the relativistic light-front quark model [33] with the results that the $q^2$ dependence of the form factors $A_0, A_2, V, F_1$ is a dipole behavior (i.e. $n = 2$), while $F_0, A_1$ exhibit a monopole dependence ($n = 1$).

Recently, the $B_s \to K^*$ and $B_s \to \phi$ form factors have also been calculated in the light-cone sum rule approach [34] with the parametrization

$$f(q^2) = \frac{f(0)}{1 - a(q^2/m_{B_s}^2) + b(q^2/m_{B_s}^2)^2} \quad (2.31)$$

for the form-factor $q^2$ dependence. The results are [34]

$$A_0^{B_s\phi}(0) = 0.382, \quad a = 1.77, \quad b = 0.856,$$
$$A_1^{B_s\phi}(0) = 0.296, \quad a = 0.87, \quad b = -0.061,$$
$$A_2^{B_s\phi}(0) = 0.255, \quad a = 1.55, \quad b = 0.513,$$
$$V^{B_s\phi}(0) = 0.433, \quad a = 1.75, \quad b = 0.736,$$
$$A_0^{B_sK^*}(0) = 0.254, \quad a = 1.87, \quad b = 0.887,$$
$$A_1^{B_sK^*}(0) = 0.190, \quad a = 1.02, \quad b = -0.037,$$
$$A_2^{B_sK^*}(0) = 0.164, \quad a = 1.77, \quad b = 0.729,$$
$$V^{B_sK^*}(0) = 0.262, \quad a = 1.89, \quad b = 0.846. \quad (2.32)$$

It is obvious that the $q^2$ dependence for the form factors $A_0, A_2$ and $V$ is dominated by the dipole terms, while $A_1$ by the monopole term in the region where $q^2$ is not too large. In Tables IV and V we will present results using these two different parametrizations for $B_s \to V$ form factors.

We will encounter matrix elements of pseudoscalar densities when evaluating the penguin amplitudes. Care must be taken to consider the pseudoscalar matrix element for $\eta^{(*)} \to$ vacuum transition: The anomaly effects must be included in order to ensure a correct chiral behavior for the pseudoscalar matrix element [12]. The results are [35,11]
$\langle \eta^{(\prime)} | \bar{s} \gamma_5 s | 0 \rangle = -i \frac{m_s^2}{2 m_s} \left( f_{\eta^{(\prime)}}^* - f_{\eta^{(\prime)}} \right),$

$\langle \eta^{(\prime)} | \bar{u} \gamma_5 u | 0 \rangle = \langle \eta^{(\prime)} | d \gamma_5 d | 0 \rangle = r_{\eta^{(\prime)}} \langle \eta^{(\prime)} | \bar{s} \gamma_5 s | 0 \rangle,$

(2.33)

with [12]

$$r_{\eta^{(\prime)}} = \frac{\sqrt{2} f_0 - f_8^\prime}{\sqrt{2} f_8 - f_0^\prime} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta,$$

$$r_\eta = -\frac{1}{2} \frac{\sqrt{2} f_0 - f_8}{\sqrt{2} f_8 - f_0} \frac{\cos \theta - \sqrt{2} \sin \theta}{\cos \theta + \frac{1}{\sqrt{2}} \sin \theta}.$$  (2.34)

### III. NUMERICAL RESULTS AND DISCUSSIONS

With the factorizable decay amplitudes summarized in Appendices and the input parameters shown in Sec. II, we are ready to compute the branching ratios for the two-body charmless nonleptonic decays of the $B_s$ meson. The decay rates for $B_s \rightarrow PP, VP$ are given by

$$\Gamma(B_s \rightarrow P_1 P_2) = \frac{p_c}{8 \pi m_{B_s}^2} |A(B_s \rightarrow P_1 P_2)|^2,$$

$$\Gamma(B_s \rightarrow VP) = \frac{p_c^2}{8 \pi m_V^2} |A(B_s \rightarrow VP)/(\varepsilon \cdot p_{B_s})|^2.$$  (3.1)

The decay $B_s \rightarrow VV$ is more complicated as its amplitude involves three form factors. In general, the factorizable amplitude of $B_s \rightarrow V_1 V_2$ is of the form:

$$A(B_s \rightarrow V_1 V_2) = \alpha X^{(B_s, V_1, V_2)} + \beta X^{(B_s, V_2, V_1)}$$

$$= (\alpha_1 A_{B_s V_1} + \beta_1 A_{B_s V_2}) \varepsilon_1^* \cdot \varepsilon_2 + (\alpha_2 A_{B_s V_1} + \beta_2 A_{B_s V_2}) (\varepsilon_1^* \cdot p_{B_s}) (\varepsilon_2^* \cdot p_{B_s})$$

$$+ i \varepsilon_{\mu \nu \rho \sigma} \varepsilon_2^\mu \varepsilon_1^\nu \rho_{B_s}^\rho \eta_{B_s}^\sigma (\alpha_3 V_{B_s V_1} + \beta_3 V_{B_s V_2}),$$  (3.2)

where use of Eq. (C1) has been made. Then

$$\Gamma(B_s \rightarrow V_1 V_2) = \frac{p_c}{8 \pi m_{B_s}^2} |\alpha_1 (m_{B_s} + m_1) m_2 f_{V_2} A_{B_s V_1}^* (m_2^2) |^2 (H + 2 \zeta H_1 + 2 \xi^2 H_2),$$  (3.3)

where

$$H = (a - bx)^2 + 2(1 + c^2 y^2),$$

$$H_1 = (a - bx)(a - b' x') + 2(1 + cc' y y'),$$

$$H_2 = (a - b' x')^2 + 2(1 + c^2 y^2),$$  (3.4)

with

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\[ a = \frac{m_B^2 - m_1^2 - m_2^2}{2m_1m_2}, \quad b = \frac{2m_B^2p_c^2}{m_1m_2(m_B + m_1)^2}, \quad c = \frac{2m_Bp_c}{(m_B + m_1)^2}, \]

\[ \zeta = \frac{\beta_1A_1^{B,V}(m_1^2)}{\alpha_1A_1^{B,V}(m_2^2)}, \quad x = \frac{A_2^{B,V}(m_1^2)}{A_1^{B,V}(m_2^2)}, \quad y = \frac{V_B^{V}(m_1^2)}{A_1^{B,V}(m_2^2)}, \]

(3.5)

where \( p_c \) is the c.m. momentum, \( m_1 \) (\( m_2 \)) is the mass of the vector meson \( V_1 \) (\( V_2 \)), and \( b', c', x', y' \) can be obtained from \( b, c, x, y \), respectively, with the replacement \( V_1 \leftrightarrow V_2 \).

The calculated branching ratios for \( B_s \to PP, VP, VV \) decays averaged over CP-conjugate modes are shown in Tables III-V, respectively; where the nonfactorizable effects are treated in two different cases: (i) \( \text{LL} \) and (ii) \( \text{LR} \). Obviously, if the tree, QCD penguin and electroweak penguin amplitudes are of the same sign, then \( (1 - R_{W}) \) measures the fraction of non-electroweak penguin contributions to \( B(B_s \to h_1h_2) \). It is evident from Table VI that the decays listed in (3.7) all have the same \( N_{c}^{\text{eff}} \)-dependence: For \( N_{c}^{\text{eff}}(LL) = 2 \), the electroweak penguin contributions account for 85% of the branching ratios for \( B_s \to H \text{\pi}, \text{\eta\pi}, \text{\eta\rho}, \text{\eta'\rho}, \text{\phi\rho}, \) and the ratio \( R_{W} \) is very sensitive to \( N_{c}^{\text{eff}} \) when \( N_{c}^{\text{eff}}(LL) = N_{c}^{\text{eff}}(LR) \). We also see that electroweak penguin contributions to \( B_s \to \omega \eta, \text{\omega}\eta', \text{\phi}\eta, \text{\phi}\eta', \text{\omega}\phi, \text{\phi}\phi, \text{K}\phi, \text{K}\phi' \) do not receive any QCD penguin contributions \([2]\). Therefore, these six decay modes are in general small, ranging from \( 4 \times 10^{-8} \) to \( 0.4 \times 10^{-6} \), but they could be accessible at the future hadron colliders with large \( b \) production.

In order to see the relative importance of electroweak penguin effects in penguin-dominated \( B_s \) decays, we follow \([22]\) to compute the ratio

\[ R_{W} = \frac{\mathcal{B}(B_s \to h_1h_2)(\text{with } a_7, \cdots , a_{10} = 0)}{\mathcal{B}(B_s \to h_1h_2)}. \]

(3.8)

Obviously, if the tree, QCD penguin and electroweak penguin amplitudes are of the same sign, then \( (1 - R_{W}) \) measures the fraction of non-electroweak penguin contributions to \( B(B_s \to h_1h_2) \). It is evident from Table VI that the decays listed in (3.7) all have the same \( N_{c}^{\text{eff}} \)-dependence: For \( N_{c}^{\text{eff}}(LL) = 2 \), the electroweak penguin contributions account for 85% of the branching ratios for \( B_s \to H \text{\pi}, \text{\eta\pi}, \text{\eta\rho}, \text{\eta'\rho}, \text{\phi\rho}, \) and the ratio \( R_{W} \) is very sensitive to \( N_{c}^{\text{eff}} \) when \( N_{c}^{\text{eff}}(LL) = N_{c}^{\text{eff}}(LR) \). We also see that electroweak penguin contributions to

\[ B_s \to \omega \eta, \omega \eta', \phi \eta, \phi \eta', \omega \phi, \phi \phi, \text{K}\phi, \text{K}\phi' \]

(3.9)

depending very sensitively on \( N_{c}^{\text{eff}} \), are in general as important as QCD penguin effects and even play a dominant role. For example, about 50% of \( \mathcal{B}(B_s \to K^0\phi) \) comes from the electroweak penguin contributions at \( N_{c}^{\text{eff}}(LL) = 2 \) and \( N_{c}^{\text{eff}}(LR) = 5 \).
Strictly speaking, because of variously possible interference of the electroweak penguin amplitude with the tree and QCD penguin contributions, $R_W$ is not the most suitable quantity for measuring the relative importance of electroweak penguin effects. For example, it appears at the first sight that only 21% of $\mathcal{B}(B_s \to \omega\eta')$ and $\mathcal{B}(B_s \to \omega\phi)$ arises from the electroweak penguins at $N_c^{\text{eff}}(LL) = N_c^{\text{eff}}(LR) = 3$. However, the decay amplitudes are proportional to (see Appendix B)

$$V_{ub}V_{ts}^*a_2 - V_{tb}V_{ts}^*[2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9)].$$

(3.10)

Since $a_2$ and $(a_2 + a_5)$ are minimum at $N_c^{\text{eff}} \sim 3$ (see Table I), the decay is obviously dominated by the electroweak penguin transition when $N_c^{\text{eff}}(LL) = N_c^{\text{eff}}(LR) = 3$. Numerically, we find at the amplitude level

$$\text{tree} : \text{QCD penguin} : \text{electroweak penguin} = 0.28 : 1 : -2.72.$$  

(3.11)

It is clear that although $R_W = 0.79$ for $N_c^{\text{eff}} = 3$, the decays $B_s \to \omega\eta'$ and $B_s \to \omega\phi$ are actually dominated by the electroweak penguin.

The branching ratios for the class-V and -VI modes shown in (3.9) depend strongly on the value of $N_c^{\text{eff}}$. As pointed out in Sec. II, the preferred values for the effective number of colors are $N_c^{\text{eff}}(LL) \approx 2$ and $N_c^{\text{eff}}(LR) \sim 5$. We believe that the former will be confirmed soon by the forthcoming measurements of $B \to \pi\pi$, $\pi\rho$, $\cdots$. However, the branching ratios for some of the decay modes, e.g., $B_s \to \omega\eta, \omega\eta', \phi\eta$, become very small at the values of $N_c^{\text{eff}}$ given by Eq. (2.13). As suggested in [22], these decays involve large cancellation among competing amplitudes and they may receive significant contributions from annihilation and/or final-state interactions.

As noted in passing, class-IV modes involve the QCD penguin parameters $a_4$ and $a_6$ in the combination $a_4 + Ra_6$, where $R > 0$ for $B_s \to P_aP_b$, $R = 0$ for $P_aV_b$ and $V_aV_b$ final states, and $R < 0$ for $B_s \to V_aP_b$, where $P_b$ or $V_b$ is factorizable under the factorization assumption. Therefore, the decay rates of class-IV decays are expected to follow the pattern:

$$\Gamma(B_s \to P_aP_b) > \Gamma(B_s \to P_aV_b) \sim \Gamma(B_s \to V_aV_b) > \Gamma(B_s \to V_aP_b),$$

(3.12)

as a consequence of various possibilities of interference between the penguin terms characterized by the effective coefficients $a_4$ and $a_6$. From Tables III-V, we see that

$$\Gamma(B_s \to K^+K^-) > \Gamma(B_s \to K^+K^0) \gg \Gamma(B_s \to K^{*+}K^-) > \Gamma(B_s \to K^{*+}K^0),$$

$$\Gamma(B_s \to K^0\bar{K}^0) > \Gamma(B_s \to K^0\bar{K}^{*0}) \gg \Gamma(B_s \to K^{*0}\bar{K}^0) > \Gamma(B_s \to K^{*0}\bar{K}^{*0}).$$

(3.13)

Note that the pattern $\Gamma(B \to P_aV_b) > \Gamma(B \to P_aP_b)$, which is often seen in tree-dominated decays, for example $\Gamma(B_s \to K^+\rho^-) > \Gamma(B_s \to K^+\pi^-)$, occurs because of the larger spin phase space available to the former due to the existence of three different polarization states for the vector meson. On the contrary, the hierarchy (3.13) implies that the spin phase-space suppression of the penguin-dominated decay $B_s \to P_aP_b$ over $B_s \to P_aV_b$ or $B_s \to V_aP_b$ is overcome by the constructive interference between penguin amplitudes in the former. Recall that the coefficient $R$ is obtained by applying equations of motion to the hadronic
matrix elements of pseudoscalar densities induced by penguin operators. Hence, a test of
the hierarchy shown in (3.13) is important for understanding the calculation of the penguin
matrix element. $^5$

Among the 39 charmless two-body decay modes of the $B_s$ meson, we find that only seven
of them have branching ratios at the level of $10^{-5}$:

$$
\mathcal{B}_{s} \rightarrow K^+K^-, K^0\bar{K}^0, \eta \eta', \eta' \eta', K^+\rho^-, K^{++}\rho^-, \phi\phi. \tag{3.14}
$$

It is interesting to note that among the two-body rare decays of $B^-$ and $B_d$, the class-VI
decays $B^- \rightarrow \eta'K^-$ and $B_d \rightarrow \eta'K^0$ have the largest branching ratios [36]:

$$
\mathcal{B}(B^\pm \rightarrow \eta'K^{\pm}) = \left(6.5^{+1.5}_{-1.4} \pm 0.9\right) \times 10^{-5},
$$

$$
\mathcal{B}(B_d \rightarrow \eta'K^0) = \left(4.7^{+2.7}_{-2.0} \pm 0.9\right) \times 10^{-5}. \tag{3.15}
$$

The decay rate of $B^- \rightarrow \eta'K^-$ and $B_d \rightarrow \eta'K^0$ is large because they receive two different sets
of penguin contributions proportional to $a_4 + Ra_6$ with $R > 0$. By contrast, $VP, VV$ modes
in charm decays or bottom decays involving charmed mesons usually have larger branching
ratios than the $PP$ mode. Because of the strange quark content of the $B_s$, one will expect
that the decay $B_s \rightarrow \eta'\eta'$ or $B_s \rightarrow \eta'\eta'$, the $B_s$ counterpart of $B_d \rightarrow \eta'K^0$, is the dominant
two-body $B_s$ decay. Our calculation indicates that while the branching ratio of $B_s \rightarrow \eta\eta'$ is
large,

$$
\mathcal{B}(B_s \rightarrow \eta\eta') \approx 2 \times 10^{-5} \quad \text{for } N_c^{\text{eff}}(LL) = 2, \ N_c^{\text{eff}}(LR) = 5, \tag{3.16}
$$

it is only slightly larger than that of other decay modes listed in (3.14), see Tables III-V.

What is the role played by the intrinsic charm content of the $\eta'$ to the hadronic charmless
$B_s$ decay? Just as the case of $B \rightarrow \eta'K$, $B_s \rightarrow \eta'\eta'$ receives an internal $W$-emission
contribution coming from the Cabibbo-allowed process $b \rightarrow c\bar{c}s$ followed by a conversion
of the $c\bar{c}$ pair into the $\eta'$ via gluon exchanges. Although the charm content of the $\eta'$ is
$a \ priori$ expected to be small, its contribution is potentially important because the CKM
mixing angle $\sqrt{V_{cb}V_{cs}^*}$ is of the same order of magnitude as that of the penguin amplitude [cf.
Eqs. (A10,A11)] and yet its effective coefficient $a_2$ is larger than the penguin coefficients by
an order of magnitude. Since $a_2$ depends strongly on $N_c^{\text{eff}}(LL)$ (see Table I), the contribution
of $c\bar{c} \rightarrow \eta'$ is sensitive to the variation of $N_c^{\text{eff}}(LL)$. It is easy to check that the $\eta'$ charm
content contributes in the same direction as the penguin terms at $1/N_c^{\text{eff}}(LL) > 0.28$ where
$a_2 > 0$, while it contributes destructively at $1/N_c^{\text{eff}}(LL) < 0.28$ where $a_2$ becomes negative.
In order to explain the abnormally large branching ratio of $B \rightarrow \eta'K$, an enhancement from
the $c\bar{c} \rightarrow \eta'$ mechanism is certainly welcome in order to improve the discrepancy between
theory and experiment. This provides another strong support for $N_c^{\text{eff}}(LL) \approx 2$. Note that a
similar mechanism explains the recent measurement of $B^- \rightarrow \eta_cK^-$ [37].

$^5$For a direct estimate of $R$ using the perturbative QCD method rather than the equation of
motion, see [23].
It turns out that the effect of the \( c\bar{c} \) admixture in the \( \eta' \) is more important for \( B_s \rightarrow \eta'\eta' \) than for \( B_s \rightarrow \eta\eta' \). It is clear from Eq. (A1) that the destructive interference between \( X_{c}(B_s,\eta\eta') \propto f_{\eta'}^{c}F_{0}^{B_s,\eta} \) and \( X_{c}(B_s,\eta'\eta) \propto f_{\eta}^{c}F_{0}^{B_s,\eta'} \) in the decay amplitude of \( B_s \rightarrow \eta\eta' \), recalling that the form factors \( F_{0}^{B_s,\eta} \) and \( F_{0}^{B_s,\eta'} \) have opposite signs, renders the contribution of \( c\bar{c} \rightarrow \eta' \) smaller for \( B_s \rightarrow \eta\eta' \).

A very recent CLEO reanalysis of \( B \rightarrow \eta'K \) using a data sample 80% larger than in previous studies yields the preliminary results [38]:

\[
\begin{align*}
B(B_s \rightarrow \eta'K^\pm) & = \left( 7.4^{+0.8}_{-1.3} \pm 1.0 \right) \times 10^{-5}, \\
B(B_d \rightarrow \eta'K^0) & = \left( 5.9^{+1.8}_{-1.6} \pm 0.9 \right) \times 10^{-5},
\end{align*}
\]

suggesting that the original measurements (3.15) were not an upward statistical fluctuation. This result certainly favors a slightly larger \( f_{\eta'}^{c} \) in magnitude than that used in (2.27). In fact, a more sophisticated theoretical calculation gives \( f_{\eta}^{c} \sim -(12.3 \sim 18.4) \) MeV [29], which is consistent with all the known phenomenological constraints. This value of \( f_{\eta}^{c} \) will lead to an enhanced decay rate for \( B \rightarrow \eta'K \). Numerically, we find that for \( N_{c}^{\text{eff}}(LL) = 2 \), \( N_{c}^{\text{eff}}(LR) = 5 \) and \( f_{\eta}^{c} = -15 \) MeV,

\[
\begin{align*}
B(B_s \rightarrow \eta\eta') & = 2.2 \times 10^{-5}, \\
B(B_s \rightarrow \eta'\eta') & = 1.8 \times 10^{-5},
\end{align*}
\]

(3.18)
to be compared with

\[
\begin{align*}
B(B_s \rightarrow \eta\eta') & = 1.8 \times 10^{-5}, \\
B(B_s \rightarrow \eta'\eta') & = 1.2 \times 10^{-5},
\end{align*}
\]

(3.19)
in the absence of the intrinsic charm content of the \( \eta' \).

Finally, we should point out the uncertainties associated with our predictions. Thus far, we have neglected \( W \)-annihilation, space-like penguin diagrams, and final-state interactions; all of them are difficult to estimate. It is argued in [22] that these effects may play an essential role for our class-V and -VI decay modes. Other major sources of uncertainties come from the form factors and their \( q^2 \) dependence, the running quark masses at the scale \( m_b \), the virtual gluon's momentum in the penguin diagram, and the values for the Wolfenstein parameters \( \rho \) and \( \eta \).

**IV. CONCLUSIONS**

Using the next-to-leading order QCD-corrected effective Hamiltonian, we have systematically studied hadronic charmless two-body decays of \( B_s \) mesons within the framework of generalized factorization. Nonfactorizable effects are parametrized in terms of \( N_{c}^{\text{eff}}(LL) \) and \( N_{c}^{\text{eff}}(LR) \), the effective numbers of colors arising from \( (V - A)(V - A) \) and \( (V - A)(V + A) \) 4-quark operators, respectively. The branching ratios are calculated as a function of \( N_{c}^{\text{eff}}(LR) \) with two different considerations for \( N_{c}^{\text{eff}}(LL) \): (i) \( N_{c}^{\text{eff}}(LL) \) being fixed at the value of 2, and (ii) \( N_{c}^{\text{eff}}(LL) = N_{c}^{\text{eff}}(LR) \). Depending on the sensitivity of the effective coefficients \( a_{i}^{\text{eff}} \) on \( N_{c}^{\text{eff}} \), we have classified the tree and penguin transitions into six different classes. Our results are:
1. The decays $\bar{B}_s \to \eta\pi, \eta'\pi, \eta\rho, \eta'\rho, \phi\pi, \phi\rho$ receive contributions only from the tree and electroweak penguin diagrams and are completely dominated by the latter. A measurement of them can be utilized to fix the effective electroweak penguin parameter $a_9$. For $N_{\text{eff}}(LL) = 2$, we found that electroweak penguin contributions account for 85% of their decay rates. Their branching ratios, though small [in the range of $(0.4 - 4.0) \times 10^{-7}$], could be accessible at hadron colliders with large $b$ production.

2. For class-V and -VI penguin-dominated modes: $\bar{B}_s \to \omega\eta, \omega\eta', \phi\eta, \omega\eta, K\phi, K^*\phi, \phi\phi$, electroweak penguin corrections, depending strongly on $N_{\text{eff}}$, are as significant as QCD penguin effects and can even play a dominant role.

3. Current experimental information on $B^- \to \omega\pi^-$ and $B^0 \to \pi^+\pi^-$ favors a small $N_{\text{eff}}^{(LL)}$, that is, $N_{\text{eff}}^{(LL)} \approx 2$, which is also consistent with the nonfactorizable term extracted from $B \to (D, D^*)\pi, \rho$ decays, $N_{\text{eff}}(B \to D\pi) \approx 2$. We have argued that the preferred value for the effective number of colors $N_{\text{eff}}^{(LR)}$ is $N_{\text{eff}}^{(LR)} \approx 5$.

4. Because of various possibilities of interference between the penguin amplitudes governed by the QCD penguin parameters $a_4$ and $a_6$, the decay rates of class-IV decays follow the pattern: $\Gamma(\bar{B}_s \to P_a P_b) > \Gamma(\bar{B}_s \to P_a V_b) \approx \Gamma(\bar{B}_s \to V_a V_b) > \Gamma(\bar{B}_s \to V_a P_b)$, where $P_a = K^+, P_b = K^-$ or $P_a = K^0, P_b = \overline{K}^0$. A test of this hierarchy is important to probe the penguin mechanism.

5. The decay $B \to \eta'K$ is known to have the largest branching ratios in the two-body hadronic charmless $B^-$ and $B_d$ decays. Its analogue in the $B_s$ system, namely $B_s \to \eta\eta'$, has a branching ratio of order $2 \times 10^{-5}$, but it is only slightly larger than that of $\eta'\eta', K^{*+}\rho^-, K^+K^-, K^0\overline{K}^0$ decay modes, which have the branching ratios of order $10^{-5}$.

6. The recent CLEO reanalysis of $B \to \eta K$ favors a slightly large decay constant $f_{\eta}$. Using $f_{\eta} = -15$ MeV, which is consistent with all the known theoretical and phenomenological constraints, we found that the intrinsic charm content of the $\eta'$ is important for $B_s \to \eta\eta'$, but less significant for $B_s \to \eta\eta'$.

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APPENDIX

A. The $\bar{B}_s \to PP$ decay amplitudes

For $\bar{B}_s \to PP$ decays, we use $X(B_s P_1, P_2)$ to denote the factorizable amplitude with the meson $P_2$ being factored out. Explicitly,
\[ X^{(B,P_1,P_2)} \equiv \langle P_2 | \langle \bar{q} q \rangle_{v-A} | 0 \rangle \langle P_1 | \langle \bar{q} b \rangle_{v-A} | \bar{B}_s \rangle = i f_{P_1} (m_{B_s}^2 - m_{P_1}^2) F_{0}^{B_s,P_1} (m_{P_2}^2). \] (A1)

For a neutral \( P_1 \) with the quark content \( N(\bar{q} q + \cdots) \), where \( N \) is a normalization constant,

\[ X^{(B,P_1,P_2)}_q \equiv \langle P_2 | \langle \bar{q} q \rangle_{v-A} | 0 \rangle \langle P_1 | \langle \bar{q} b \rangle_{v-A} | \bar{B}_s \rangle = i f_{P_1} (m_{B_s}^2 - m_{P_1}^2) F_{0}^{B_s,P_1} (m_{P_2}^2). \] (A2)

As an example, the factorizable amplitudes \( X^{(B,q',K)} \) and \( X^{(B,K,q')}_q \) of the decay \( \bar{B}_s \to K^0 \eta' \) read

\[ X^{(B,q',K)} = \langle K^0 | (\bar{s}d)_{v-A} | 0 \rangle \langle \eta' | (\bar{d}b)_{v-A} | \bar{B}_s \rangle = i f_{K} (m_{B_s}^2 - m_{\eta'}^2) F_{0}^{B_s,q'} (m_{K}^2), \]

\[ X^{(B,K,q')}_q = \langle \eta' | (\bar{q} q)_{v-A} | 0 \rangle \langle K^0 | (\bar{s}b)_{v-A} | \bar{B}_s \rangle = i f_{q'}^3 (m_{B_s}^2 - m_{K}^2) F_{0}^{B_s,K} (m_{\eta'}^2). \] (A3)

For simplicity, \( W \)-annihilation, space-like penguins and final-state interactions are not included in the decay amplitudes given below.

1. \( b \to d \) processes:

\[ A(\bar{B}_s \to K^+ \pi^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left[ a_4 + a_{10} + 2(a_6 + a_8) \frac{m_{\pi}^2}{(m_u + m_d)(m_b - m_u)} \right] \right\} X^{(B_s K^+, \pi^-)}, \] (A4)

\[ A(\bar{B}_s \to K^0 \pi^0) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[ -a_4 + \frac{3}{2} (-a_7 + a_9) + \frac{1}{2} a_{10} - 2(a_6 - \frac{1}{2} a_8) \frac{m_{\pi}^2}{(m_d + m_d)(m_b - m_d)} \right] \right\} X^{(B_s K^0, \pi^0)}, \] (A5)

\[ A(\bar{B}_s \to K^0 \eta^{(*)}) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 X^{(B_s K, \eta^{(*)})}_u + V_{tb} V_{td}^* a_2 X^{(B_s K, \eta^{(*)})}_c \right. \]

\[ - V_{tb} V_{td}^* \left[ (a_4 - \frac{1}{2} a_{10} + 2(a_6 - \frac{1}{2} a_8) \frac{m_{K}^2}{(m_u + m_d)(m_b - m_d)}) X^{(B_s \eta^{(*)}, K)}_s \right. \]

\[ + (a_3 - a_5 - a_7 + a_9) X^{(B_s K, \eta^{(*)})}_u + (a_3 - a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9) X^{(B_s K, \eta^{(*)})}_s \]

\[ + (a_3 - a_5 - a_7 + a_9) X^{(B_s K, \eta^{(*)})}_c \]

\[ + \left( a_3 + a_4 - a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right) \frac{m_{\eta'}^2}{m_s (m_b - m_s)} \left[ \left( \frac{f_{\eta'}^{(*)}}{f_{\eta'}^{(*)}} - 1 \right) r_{\eta'} \right] X^{(B_s K, \eta^{(*)})}_d \right\}. \] (A6)

2. \( b \to s \) processes:
\[
A(B_s \to K^+K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \left[ a_4 + a_{10} + 
+ 2(a_6 + a_8) \frac{m_K^2}{m_u + m_s (m_b - m_u)} \right] \right\} X^{(B_s K^+,K^-)}, \tag{A7}
\]

\[
A(B_s \to \pi^0 \eta'') = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ \frac{3}{2} (-a_7 + a_9) \right] \right\} X_u^{(B_s \eta'',\pi^0)}, \tag{A8}
\]

where

\[
X_u^{(B_s \eta'',\pi^0)} \equiv \langle \pi^0 | (\bar{u}u)_{V-A} | 0 \rangle \langle \eta'' | (s\bar{b})_{V-A} | B_s \rangle = i \frac{f_\pi}{\sqrt{2}} (m_{B_s}^2 - m_{\eta''}^2) F_0^{B_s \eta''} (m_{\pi}^2), \tag{A9}
\]

\[
A(B_s \to \eta''') = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 \left( X_u^{(B_s \eta''',\eta')} + X_u^{(B_s \eta'',\eta')} \right) + V_{cb}V_{cs}^* a_2 \left( X_c^{(B_s \eta''',\eta')} + X_c^{(B_s \eta'',\eta')} \right) 
- V_{tb}V_{ts}^* \left[ a_4 + a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} 
+ \left( a_6 - \frac{1}{2} a_8 \right) \frac{m_{\eta''}^2}{m_s (m_b - m_s)} \left( 1 - \frac{f_{\eta'}^2}{f_{\eta}^2} \right) X_s^{(B_s \eta''',\eta')} 
+ \left( 2a_3 - 2a_5 - \frac{1}{2} a_7 + \frac{1}{2} a_9 \right) X_u^{(B_s \eta''',\eta')} + (a_3 - a_5 - a_7 + a_9) X_c^{(B_s \eta''',\eta')} \right] \right\}, \tag{A10}
\]

\[
A(B_s \to \eta'') = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 X_u^{(B_s \eta'',\eta')} + V_{cb}V_{cs}^* a_2 X_c^{(B_s \eta'',\eta')} 
- V_{tb}V_{ts}^* \left[ a_4 + a_5 + \frac{1}{2} a_7 - \frac{1}{2} a_9 - \frac{1}{2} a_{10} 
+ \left( a_6 - \frac{1}{2} a_8 \right) \frac{m_{\eta''}^2}{m_s (m_b + m_s)} \left( 1 - \frac{f_{\eta'}^2}{f_{\eta}^2} \right) X_s^{(B_s \eta'',\eta')} 
+ \left( 2a_3 - 2a_5 - \frac{1}{2} a_7 + \frac{1}{2} a_9 \right) X_u^{(B_s \eta'',\eta')} + (a_3 - a_5 - a_7 + a_9) X_c^{(B_s \eta'',\eta')} \right] \right\}. \tag{A11}
\]

The amplitude of $B_s \to \eta \eta$ is obtained from $A(B_s \to \eta''\eta')$ by the replacement $\eta' \to \eta$.

3. pure penguin process:
\[
A(B_s \to K^0\bar{K}^0) = \frac{G_F}{\sqrt{2}} \left\{ -V_{tb}V_{ts}^* \left[ a_4 - \frac{1}{2}a_{10} \\
+ 2(a_6 - \frac{1}{2}a_8) \frac{m_K^2}{(m_s + m_d)(m_b - m_d)} \right] \right\} X^{(B_s K^0\bar{K}^0)},
\] (A12)

B. The \( \bar{B}_s \to VP \) decay amplitudes

The factorizable amplitudes of \( \bar{B}_s \to VP \) decays have the form:

\[
\begin{align*}
X^{(B_s P,V)} &\equiv \langle V| (\bar{q}_2 q_3)_{\nu - \lambda} |0\rangle \langle P| (\bar{q}_1 b)_{\nu - \lambda} |\bar{B}_s\rangle = 2f_V m_V F_{1,0}^{B_s P}(m_{\pi}^2)(\varepsilon \cdot p_{B_s}), \\
X^{(B_s V,P)} &\equiv \langle P| (\bar{q}_2 q_3)_{\nu - \lambda} |0\rangle \langle V| (\bar{q}_1 b)_{\nu - \lambda} |\bar{B}_s\rangle = 2f_P m_V A_{1,0}^{B_s V}(m_{\pi}^2)(\varepsilon \cdot p_{B_s}).
\end{align*}
\] (B1)

For example, the factorizable terms \( X^{(B_s \eta',K^*)} \) and \( X^{(B_s K^*,\eta')} \) of \( \bar{B}_s \to K^+\eta' \) decay are given by

\[
\begin{align*}
X^{(B_s \eta',K^*)} &\equiv \langle K^0| (\bar{s}u)_{\nu - \lambda} |0\rangle \langle \eta'| (\bar{u}b)_{\nu - \lambda} |\bar{B}_s\rangle = 2f_{\eta'} m_{K^*} F_{1,0}^{B_s,0}(m_{\eta'}^2)(\varepsilon \cdot p_{B_s}), \\
X^{(B_s K^*,\eta')} &\equiv \langle \eta'| (\bar{q}q)_{\nu - \lambda} |0\rangle \langle K^0| (\bar{s}u)_{\nu - \lambda} |\bar{B}_s\rangle = 2f_{\eta'} m_{K^*} A_{1,0}^{B_s,0}(m_{\eta'}^2)(\varepsilon \cdot p_{B_s}).
\end{align*}
\] (B2)

1. \( b \to d \) processes:

\[
A(\bar{B}_s \to K^{+}\pi^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^* \left[ a_4 + a_{10} \\
- 2(a_6 + a_8) \frac{m_{\pi}^2}{(m_u + m_d)(m_b + m_u)} \right] \right\} X^{(B_s K^+\pi^-)},
\] (B3)

\[
A(\bar{B}_s \to K^{+}\rho^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* a_1 - V_{tb}V_{td}^* \left( a_4 + a_{10} \right) \right\} X^{(B_s K^+\rho^-)},
\] (B4)

\[
A(\bar{B}_s \to K^{0}\pi^0) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^* \left[ -a_4 - \frac{3}{2}a_7 + \frac{3}{2}a_9 + \frac{1}{2}a_{10} \\
+ 2(a_6 - \frac{1}{2}a_8) \frac{m_{\pi}^2}{2m_d(m_b + m_d)} \right] \right\} X^{(B_s K^0\pi^0)},
\] (B5)

\[
A(\bar{B}_s \to K^{0}\rho^0) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^* \left( -a_4 + \frac{3}{2}a_7 + \frac{3}{2}a_9 + \frac{1}{2}a_{10} \right) \right\} X^{(B_s K^0\rho^0)},
\] (B6)

\[
A(\bar{B}_s \to K^{0}\omega) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{ud}^* a_2 - V_{tb}V_{td}^* \left( 2a_3 + a_4 + 2a_{10} \right) \\
+ \frac{1}{2}a_7 + \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right\} X^{(B_s K^0\omega)},
\] (B7)
\[ A(\mathcal{B}_s \rightarrow K^{*0}\eta^{(*)}) = \frac{G_F}{\sqrt{2}} \left\{ \left[ V_{ub}V_{us}^* a_2 X_u^{(B_sK^{*}\eta^{(*)})} + V_{cb}V_{cs}^* a_2 X_c^{(B_sK^{*}\eta^{(*)})} \right] \\
- V_{tb}V_{ts}^* \left[ (a_4 - \frac{1}{2}a_{10}) X^{(B_s\eta^{(*)},K^{*})} + \left( a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) \right] \right\} \]
\[ - \left( a_6 - \frac{1}{2}a_8 \right) \frac{m^2_{\eta^{(*)}}}{m_s(m_b + m_s)} \left( \frac{f_{\eta^{(*)}}^s}{f_{\eta^{(*)}}^u} - 1 \right) r_{\eta^{(*)}} X_d^{(B_sK^{*}\eta^{(*)})} \]
\[ + \left( a_3 - a_5 - a_7 + a_9 \right) X_u^{(B_sK^{*}\eta^{(*)})} + (a_3 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9) X_s^{(B_sK^{*}\eta^{(*)})} \]
\[ + (a_3 - a_5 - a_7 + a_9) X_c^{(B_sK^{*}\eta^{(*)})} \} \]. \quad (B8) \]

2. \( b \rightarrow s \) processes:

\[ A(\mathcal{B}_s \rightarrow K^+K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \left[ (a_4 + a_{10}) \right] \right\} X^{(B_sK^+,K^-)}, \quad (B9) \]

\[ A(\mathcal{B}_s \rightarrow K^{*+}K^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* \left[ a_4 + a_{10} \right] \\
- 2(a_6 + a_8) \frac{m^2_K}{m_s(m_b + m_s)} \right\} X^{(B_sK^{*+},K^-)}, \quad (B10) \]

\[ A(\mathcal{B}_s \rightarrow \rho^0\eta^{(*)}) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ \frac{3}{2}(a_7 + a_9) \right] \right\} X_u^{(B_s\eta^{(*)},\rho^0)}, \quad (B11) \]

\[ A(\mathcal{B}_s \rightarrow \omega\eta^{(*)}) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ 2(a_3 + a_5) + \frac{1}{2}(a_7 + a_9) \right] \right\} X_u^{(B_s\eta^{(*)},\omega)}, \quad (B12) \]

where

\[ X_u^{(B_s\eta^{(*)},\omega)} \equiv \langle \omega| (\bar{u}u)_{V-A} | 0 \rangle \langle \eta^{(*)} | (s\bar{b})_{V-A} | \mathcal{B}_s \rangle = \sqrt{2} f_{\omega} m_{\omega} F^{B_s\eta^{(*)}}(m^2_{\omega}) (\varepsilon \cdot p_{B_s}), \quad (B13) \]

\[ A(\mathcal{B}_s \rightarrow \pi^0\phi) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ \frac{3}{2}(-a_7 + a_9) \right] \right\} X_u^{(B_s\phi,\pi^0)}, \quad (B14) \]

\[ A(\mathcal{B}_s \rightarrow \phi\eta^{(*)}) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 X_u^{(B_s\phi,\eta^{(*)})} + V_{cb}V_{cs}^* a_2 X_c^{(B_s\phi,\eta^{(*)})} \right\} \\
- V_{tb}V_{ts}^* \left[ (a_3 + a_4 - a_5 + \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right] \right\} \]
\[ - \left( a_6 - \frac{1}{2}a_8 \right) \frac{m^2_{\eta^{(*)}}}{m_s(m_b + m_s)} \left( 1 - \frac{f_{\eta^{(*)}}^s}{f_{\eta^{(*)}}^u} \right) r_{\eta^{(*)}} X_s^{(B_s\phi,\eta^{(*)})} \]
\[ + \left( 2a_3 - 2a_5 - \frac{1}{2}a_7 + \frac{1}{2}a_9 \right) X_u^{(B_s\phi,\eta^{(*)})} + (a_3 - a_5 - a_7 + a_9) X_c^{(B_s\phi,\eta^{(*)})} \]
\[ + \left( a_3 + a_4 + a_5 - \frac{1}{2}a_7 - \frac{1}{2}a_9 - \frac{1}{2}a_{10} \right) X_c^{(B_s\eta^{(*)},\phi)} \} \]. \quad (B15) \]
3. pure penguin processes:

\[
A(\overline{B}_s \to K^0\overline{K}^0) = -\frac{G_F}{\sqrt{2}} V_{tb} \overline{V}_{ts} \left( a_4 - \frac{1}{2} a_{10} \right) X^{(B_sK^0K^0)},
\]

\[
(\mathrm{B16})
\]

\[
A(\overline{B}_s \to K^{0*}\overline{K}^0) = -\frac{G_F}{\sqrt{2}} V_{td} \overline{V}_{ts} \left[ a_4 - \frac{1}{2} a_{10} - 2 \left( a_6 - \frac{1}{2} a_8 \right) \frac{m_K^2}{(m_s + m_d)(m_b + m_d)} \right] X^{(B_sK^{0*}\overline{K}^0)},
\]

\[
(\mathrm{B17})
\]

\[
A(\overline{B}_s \to K^0\phi) = -\frac{G_F}{\sqrt{2}} \left\{ a_3 + a_5 - \frac{1}{2} \left( a_7 + a_9 \right) \right\} X^{(B_sK^0\phi)}
\]

\[
+ \left[ a_4 - \frac{1}{2} a_{10} - 2 \left( a_6 - \frac{1}{2} a_8 \right) \frac{m_K^2}{(m_s + m_d)(m_b + m_d)} \right] X^{(B_s\phi,\overline{K}^0)}.
\]

\[
(\mathrm{B18})
\]

C. The $\overline{B}_s \to VV$ decay amplitudes

The factorizable amplitude of $B_s \to VV$ decays has the form:

\[
X^{(B_sV_1V_2)} = if_{V_2} m_{V_2} \left[ (\varepsilon_1^* \cdot \varepsilon_2^*)(m_{B_s} + m_{V_1}) A_1^{B_sV_1}(m_{V_2}^2)
\right.
\]

\[
- (\varepsilon_1^* \cdot p_{B_s})(\varepsilon_2^* \cdot p_{B_s}) \frac{2 A_2^{B_sV_1}(m_{V_2}^2)}{(m_{B_s} + m_{V_1})}
\]

\[
+ i \epsilon_{\mu\nu\alpha\beta} \varepsilon_2^{*\mu} \varepsilon_1^{*\nu} p_{B_s}^{\alpha} p_1^{\beta} \frac{2 V_{B_sV_1}(m_{V_2}^2)}{(m_{B_s} + m_{V_1})} \right].
\]

\[
(\mathrm{C1})
\]

1. $b \to d$ processes:

\[
A(\overline{B}_s \to K^{++}\rho^-) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* \left( a_4 + a_{10} \right) \right\} X^{(B_sK^{++}\rho^-)},
\]

\[
(\mathrm{C2})
\]

\[
A(\overline{B}_s \to K^{0*}\rho^0) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left( -a_4 + \frac{3}{2} a_7 + \frac{3}{2} a_9 + \frac{1}{2} a_{10} \right) \right\} X^{(B_sK^{0*}\rho^0)},
\]

\[
(\mathrm{C3})
\]

\[
A(\overline{B}_s \to K^{0*}\omega) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left( 2a_3 + a_4 + 2a_5 + \frac{1}{2} a_7 + \frac{1}{2} a_9 - \frac{1}{2} a_{10} \right) \right\} X^{(B_sK^{0*}\omega)}.
\]

\[
(\mathrm{C4})
\]

22
2. $b \to s$ processes:

$$A(B_s \to K^{*+}K^{-*}) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_1 - V_{tb}V_{ts}^* (a_4 + a_{10}) \right\} X^{(B_s K^{*+}, K^{-*})},$$  \hfill (C5)

$$A(B_s \to \rho^0 \phi) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ \frac{3}{2} (a_7 + a_9) \right] \right\} X^{(B_s \phi, \rho^0)},$$  \hfill (C6)

$$A(B_s \to \omega \phi) = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{us}^* a_2 - V_{tb}V_{ts}^* \left[ 2(a_3 + a_5) + \frac{1}{2} (a_7 + a_9) \right] \right\} X^{(B_s \phi, \omega)}.$$  \hfill (C7)

3. pure penguin processes:

$$A(B_s \to K^{0*}K^{0*}) = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left( a_4 - \frac{1}{2} a_{10} \right) X^{(B_s K^{0*}, \bar{K}^{0*})},$$  \hfill (C8)

$$A(B_s \to K^{0*} \phi) = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* \left\{ a_3 + a_5 - \frac{1}{2} (a_7 + a_9) \right\} X^{(B_s K^{0*}, \phi)}$$
\hspace{1cm} + \left( a_4 - \frac{1}{2} a_{10} \right) X^{(B_s \phi, K^{0*})},$$  \hfill (C9)

$$A(B_s \to \phi \phi) = -\frac{G_F}{\sqrt{2}} V_{tb}V_{ts}^* 2 \left[ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right] X^{(B_s \phi, \phi)}.$$  \hfill (C10)
REFERENCES


[21] CLEO Collaboration, J. Roy, invited talk presented at the XXIX International Confer-


Table III. Branching ratios (in units of $10^{-6}$) averaged over CP-conjugate modes for charmless $\bar{B}_s \rightarrow PP$ decays. Predictions are for $k^2 = m_b^2/2$, $\eta = 0.34$, $\rho = 0.16$, and $N_{\text{eff}}(LR) = 2, 3, 5, \infty$ with $N_{\text{eff}}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_{\text{eff}}(LR)$ in the second case. We use the BSW model for form factors [see (2.28)].

<table>
<thead>
<tr>
<th>Decay</th>
<th>Class</th>
<th>$N_{\text{eff}}(LL) = 2$</th>
<th></th>
<th>$N_{\text{eff}}(LL) = N_{\text{eff}}(LR)$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B}_s \rightarrow K^+\pi^-$</td>
<td>I</td>
<td>6.64 6.66 6.67 6.70</td>
<td></td>
<td>6.64 7.38 8.01 8.99</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow K^0\pi^0$</td>
<td>II</td>
<td>0.24 0.24 0.25 0.25</td>
<td></td>
<td>0.24 0.08 0.12 0.46</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow K^+K^-$</td>
<td>IV</td>
<td>9.88 10.9 10.9 11.6</td>
<td></td>
<td>9.88 10.9 11.7 12.9</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow K^0K^0$</td>
<td>IV</td>
<td>10.3 10.9 11.4 12.1</td>
<td></td>
<td>10.3 12.0 13.5 15.8</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \pi^0\eta'$</td>
<td>V</td>
<td>0.04 0.04 0.04 0.04</td>
<td></td>
<td>0.04 0.05 0.06 0.09</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \pi^0\eta$</td>
<td>V</td>
<td>0.04 0.04 0.04 0.04</td>
<td></td>
<td>0.04 0.05 0.06 0.09</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow K^0\eta'$</td>
<td>VI</td>
<td>0.63 0.86 1.06 1.42</td>
<td></td>
<td>0.63 0.54 0.57 0.76</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow K^0\eta$</td>
<td>VI</td>
<td>0.81 0.84 0.87 0.91</td>
<td></td>
<td>0.81 0.82 0.96 1.39</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \eta\eta'$</td>
<td>VI</td>
<td>12.5 16.3 19.6 25.3</td>
<td></td>
<td>12.5 14.4 15.9 18.5</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \eta'\eta'$</td>
<td>VI</td>
<td>6.28 10.3 14.3 21.4</td>
<td></td>
<td>6.28 6.80 7.23 7.91</td>
<td></td>
</tr>
<tr>
<td>$\bar{B}_s \rightarrow \eta\eta$</td>
<td>VI</td>
<td>5.30 4.80 4.41 3.89</td>
<td></td>
<td>5.30 6.23 7.05 8.37</td>
<td></td>
</tr>
</tbody>
</table>
Table IV. Branching ratios (in units of $10^{-6}$) averaged over CP-conjugate modes for charmless $B_s \to VP$ decays. Predictions are for $k^2 = m_b^2/2$, $\eta = 0.34$, $\rho = 0.16$, and $N_{c}^{\text{eff}}(LR) = 2, 3, 5, \infty$ with $N_{c}^{\text{eff}}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_{c}^{\text{eff}}(LR)$ in the second case. For decay modes involving the $B_s \to K^*$ or $B_s \to \phi$ transition, we use two different models for form factors: the BSW model [18] (the upper entry) and the light-cone sum rule approach [34] (the lower entry).

<table>
<thead>
<tr>
<th>Decay Class</th>
<th>Class</th>
<th>$N_{c}^{\text{eff}}(LL) = 2$</th>
<th>$N_{c}^{\text{eff}}(LL) = N_{c}^{\text{eff}}(LR)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$B_s \to K^{+}\pi^-$</td>
<td>I</td>
<td>4.30</td>
<td>4.30</td>
</tr>
<tr>
<td>$B_s \to K^{+}\rho$</td>
<td>I</td>
<td>17.2</td>
<td>17.2</td>
</tr>
<tr>
<td>$B_s \to K^{0*}\pi^0$</td>
<td>II</td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td>$B_s \to K^0\rho^0$</td>
<td>II</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>$B_s \to K^{0}\eta'$</td>
<td>II,VI</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>$B_s \to K^0\eta$</td>
<td>II,VI</td>
<td>0.11</td>
<td>0.15</td>
</tr>
<tr>
<td>$B_s \to K^{0}\omega$</td>
<td>II,VI</td>
<td>0.71</td>
<td>0.60</td>
</tr>
<tr>
<td>$B_s \to K^{+*}K^-$</td>
<td>IV</td>
<td>0.68</td>
<td>0.78</td>
</tr>
<tr>
<td>$B_s \to K^{0}\kappa^0$</td>
<td>IV</td>
<td>0.26</td>
<td>0.34</td>
</tr>
<tr>
<td>$B_s \to K^{+}K^{-*}$</td>
<td>IV</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>$B_s \to K^{0}\kappa^{0*}$</td>
<td>IV</td>
<td>3.28</td>
<td>3.28</td>
</tr>
<tr>
<td>$B_s \to \pi^0\phi$</td>
<td>V</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>$B_s \to \rho\eta'$</td>
<td>V</td>
<td>0.35</td>
<td>0.34</td>
</tr>
<tr>
<td>$B_s \to \rho\eta$</td>
<td>V</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$B_s \to \omega\eta'$</td>
<td>V</td>
<td>0.79</td>
<td>0.18</td>
</tr>
<tr>
<td>$B_s \to \omega\eta$</td>
<td>V</td>
<td>0.80</td>
<td>0.18</td>
</tr>
<tr>
<td>$B_s \to \phi\eta'$</td>
<td>VI</td>
<td>1.06</td>
<td>1.18</td>
</tr>
<tr>
<td>$B_s \to \phi\eta$</td>
<td>VI</td>
<td>0.55</td>
<td>0.86</td>
</tr>
<tr>
<td>$B_s \to \phi\eta$</td>
<td>VI</td>
<td>2.03</td>
<td>0.79</td>
</tr>
<tr>
<td>$B_s \to K^{0}\phi$</td>
<td>VI</td>
<td>0.002</td>
<td>0.01</td>
</tr>
<tr>
<td>$B_s \to K^{0}\phi$</td>
<td>VI</td>
<td>0.004</td>
<td>0.03</td>
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</tbody>
</table>
Table V. Same as Table IV except for $\bar{B}_s \to VV$ decays.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Class</th>
<th>$N_{\text{eff}}(LL) = 2$</th>
<th>$N_{\text{eff}}(LL) = N_{\text{eff}}(LR)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B}_s \to K^{+}\rho^{-}$</td>
<td>I</td>
<td>12.5 12.5 12.5 12.5</td>
<td>12.5 13.9 15.0 16.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.4 14.4 14.4 14.4</td>
<td>14.4 16.0 17.4 19.5</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{0*}\rho^0$</td>
<td>II</td>
<td>0.40 0.40 0.40 0.40</td>
<td>0.40 0.044 0.094 0.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.46 0.46 0.46 0.46</td>
<td>0.46 0.051 0.11 0.84</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{0*}\omega$</td>
<td>II,VI</td>
<td>0.26 0.21 0.19 0.17</td>
<td>0.26 0.04 0.02 0.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.30 0.25 0.22 0.19</td>
<td>0.30 0.044 0.031 0.32</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{+<em>}K^{-</em>}$</td>
<td>IV</td>
<td>2.53 2.53 2.53 2.53</td>
<td>2.53 2.80 3.03 3.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.91 2.91 2.91 2.91</td>
<td>2.91 3.22 3.48 3.88</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{0*}K^{0*}$</td>
<td>IV</td>
<td>2.44 2.44 2.44 2.44</td>
<td>2.44 3.09 3.66 4.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.80 2.80 2.80 2.80</td>
<td>2.80 3.55 4.21 5.30</td>
</tr>
<tr>
<td>$\bar{B}_s \to \rho^0\phi$</td>
<td>V</td>
<td>0.17 0.18 0.18 0.18</td>
<td>0.17 0.22 0.28 0.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.33 0.34 0.34 0.35</td>
<td>0.33 0.42 0.53 0.79</td>
</tr>
<tr>
<td>$\bar{B}_s \to \omega\phi$</td>
<td>V</td>
<td>0.65 0.15 0.01 0.25</td>
<td>0.65 0.004 0.30 2.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.22 0.27 0.02 0.48</td>
<td>1.22 0.007 0.56 3.92</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{0*}\phi$</td>
<td>VI</td>
<td>0.007 0.049 0.10 0.19</td>
<td>0.007 0.13 0.28 0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.014 0.098 0.17 0.30</td>
<td>0.014 0.22 0.43 0.86</td>
</tr>
<tr>
<td>$\bar{B}_s \to \phi\phi$</td>
<td>VI</td>
<td>13.8 8.77 5.57 2.15</td>
<td>13.8 7.15 3.40 0.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>25.1 15.9 10.1 3.91</td>
<td>25.1 13.0 6.18 0.68</td>
</tr>
</tbody>
</table>
Table VI. Fractions of non-electroweak penguin contributions to the branching ratios of penguin-dominated two-body $B_s$ decays, as defined by Eq. (3.8). Predictions are for $k^2 = m_b^2/2$, $\eta = 0.34$, $\rho = 0.16$, and $N_{c\text{eff}}(LR) = 2, 3, 5, \infty$ with $N_{c\text{eff}}(LL)$ being fixed to be 2 in the first case and treated to be the same as $N_{c\text{eff}}(LR)$ in the second case. We use the BSW model for form factors.

<table>
<thead>
<tr>
<th>Decay</th>
<th>$N_{c\text{eff}}(LL) = 2$</th>
<th>$N_{c\text{eff}}(LL) = N_{c\text{eff}}(LR)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$\bar{B}_s \to \pi^0 \eta'$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\bar{B}_s \to \pi^0 \eta$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\bar{B}_s \to \pi^0 \phi$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\bar{B}_s \to \rho^0 \eta'$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\bar{B}_s \to \rho^0 \eta$</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\bar{B}_s \to \omega \eta'$</td>
<td>0.78</td>
<td>0.57</td>
</tr>
<tr>
<td>$\bar{B}_s \to \omega \eta$</td>
<td>0.78</td>
<td>0.57</td>
</tr>
<tr>
<td>$\bar{B}_s \to \phi \eta'$</td>
<td>1.73</td>
<td>1.70</td>
</tr>
<tr>
<td>$\bar{B}_s \to \phi \eta$</td>
<td>1.71</td>
<td>2.16</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^0 \phi$</td>
<td>3.25</td>
<td>0.23</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{<em>+} K^{</em>-}$</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{*0} \bar{K}^{*0}$</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>$\bar{B}_s \to \rho^0 \phi$</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>$\bar{B}_s \to \omega \phi$</td>
<td>0.78</td>
<td>0.57</td>
</tr>
<tr>
<td>$\bar{B}_s \to K^{*0} \phi$</td>
<td>3.82</td>
<td>0.55</td>
</tr>
<tr>
<td>$\bar{B}_s \to \phi \phi$</td>
<td>1.25</td>
<td>1.32</td>
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</table>