GUT COSMIC MAGNETIC FIELDS IN A WARM INFLATIONARY UNIVERSE

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August 1998

Abstract

Sources of magnetic fields from grand unified theories are studied in the warm inflation regime. A ferromagnetic Savvidy vacuum scenario is presented that yields observationally interesting large scale magnetic fields. As an intermediate step, a general analysis is made of defect production at the onset of warm inflation and monopole constraints are obtained. Many features of this Savvidy vacuum scenario are applicable within a supercooled inflation regime and these points are discussed.

PACS number: 98.80 Cq
hep-ph/9809404


1. INTRODUCTION

Although magnetic fields make up only a very small fraction of the energy density of the Universe, they make a profound impact on what we observe. They influence star and galaxy formation and evolution, they are responsible for diverse effects, from altering atomic spectra to the general lack of hydrostatic equilibrium in a plasma.

There is sufficient ionization in the galaxy that the galactic B-fields are locked into and flow as a fluid with the plasma. Given any contour $C$ embedded in the plasma at time $t$, then at a later time $t'$ the contour will have moved along with the plasma to $C'$. To a very good approximation the flux through $C$ is equal to the flux through $C'$. The relaxation time for the B-fields in our galaxy $\tau$ is approximately $L^2 \sigma / c^2$ where $L$ is the characteristic length scale of the magnetic fields (or coherence length of the plasma) and $\sigma$ is the conductivity. For $L$ a few hundred pc and typically one free electron per $cm^3$ contributing to the conductivity, $\tau$ is many orders of magnitude longer than the age of the Universe. Likewise the relaxation time for extra-galactic magnetic fields is also expected to be much longer than the age of
the Universe. Typical galactic B-fields are $\sim 10^{-6}$G while extragalactic magnetic fields are thought to be generally less than $\sim 10^{-11}$G, although in galactic sheets there is evidence for fields as strong as $\sim 10^{-7}$G [3].

The mechanism for the generation of cosmic magnetic seed fields is not known, although several possibilities have been suggested:

(1) Gauge theories may have ferromagnetic vacua, [4], often referred to as Savvidy vacua, which lead to nonvanishing B-fields imprinted on the pregalaxy formation plasma [6]. This method can work if the $\beta$-function for a gauge theory satisfies a plausible integration condition so that the effective Lagrangian has a minimum $g^2[TrF^2]_{\text{min}} \sim \Lambda^4$, away from the perturbative ground state $TrF^2 = 0$, where $\Lambda$ is the RG scale. In particular, the requirement on the $\beta$-function is

$$\left| \int_0^\infty \frac{dt}{\beta(t)} \right| < \infty. \quad (1)$$

One way to realize the nonperturbative minimum is with a constant B-field $B \sim F$ so that $g^2B^2 \sim \Lambda^4$. The one-loop (zero temperature) effective potential in an SU(N) background B-field is [4,5]:

$$V(B) = B^2/2 + \frac{11N}{96\pi^2}g^2B^2(\ln\frac{gB}{\mu^2} - \frac{1}{2}) \quad (2)$$

minimizing this expression gives

$$gB_{\text{min}} \equiv \Lambda^2_{\text{SU(N)}} = \mu^2 \exp\left(\frac{-48\pi^2}{11Ng^2}\right) \quad (3)$$

and

$$V(B_{\text{min}}) \sim -(gB_{\text{min}})^2\left(\frac{11N}{192\pi^2}\right) \quad (4)$$

In the early Universe there are thermal corrections to $V(B)$, but it can be argued [6] that these corrections are small enough to allow the $B_{\text{min}} \neq 0$ vacuum to exist at all $T$. In the Enqvist-Olesen scenario [6], the transition to the $B_{\text{min}} \neq 0$ phase is caused by local fluctuations due to currents in the plasma in the very early Universe. At the GUT scale, they show the possibility to have horizon-size persistent quark currents that lead to magnetic fields $B \sim T^2$, which in turn precipitate the phase transition to the $B_{\text{min}} \neq 0$ vacuum. This leads to a B-field of approximately $10^{42}$G at the GUT scale which would evolve to a $\sim 10^{-14}$G field today in the absence of dynamo effects.

(2) Gradients in the VEVs of Higgs fields at cosmological phase transitions can lead to B-fields on completion of the phase transition [7]. To summarize this method we consider a gauge theory with gauge group $G$. If a Higgs field $\phi$ develops a vacuum expectation value (VEV) $\langle \phi \rangle \neq 0$, then the theory undergoes a phase transition and $G$ breaks to some subgroup $H$. $\langle \phi \rangle$ will be correlated up to some length scale $\xi$, that can be the horizon scale for a second order phase transition, but is in general smaller for a first order phase transition. The fact that $\langle \phi \rangle$ has a maximum correlation length implies $\langle \phi \rangle$ is non-uniform over length scales.
larger than $\xi_i$ and therefore $\partial_\mu \langle \phi \rangle \neq 0$. Furthermore, it can be argued that the variation of $\langle \phi \rangle$ can not be compensated with a gauge transformation over length scales larger than the horizon size. Therefore, $D_\mu \langle \phi \rangle = (\partial_\mu - ieA_\mu)\langle \phi \rangle \neq 0$. For the phase transition taking place at temperature $T_c$ one estimates $\xi_c = (gT_c)^{-1}$ where $g$ is the gauge coupling constant; hence $\partial_\mu \langle \phi \rangle \sim gA_\mu \langle \phi \rangle \sim \langle \phi \rangle \xi_c^{-1}$ and so $B \sim F \sim (g\xi_c^2)^{-1} \sim gT_c^2$. For the electroweak phase transition $T_c \sim 10^2 \text{GeV}$ one finds a magnetic field $B \sim 10^{23} \text{G}$ which then evolves with the expansion of the Universe to provide predynamo seed fields $B \sim 10^{-19} \text{G}$ [1,6,7].

Both scenarios (1) and (2) give only small seed fields that must subsequently be amplified by dynamo effects.

(3) The very early Universe may have been devoid of magnetic fields, but seed B-fields could have formed in stars. These fields could then be amplified by dynamo effects and expelled into interstellar space. Interstellar B-fields could then be amplified again by galactic dynamo effects to give the fields we see today [8].

Scenarios (1) and (2) are both top down in the sense that they generate seed B-fields that are trapped and amplified in galaxies and stars. As these fields are generated very early, typically between the GUT scale and the electroweak scale, they are cosmology dependent. (3) is bottom up as the fields are generated on small (stellar) scales and then expelled into interstellar and subsequently intergalactic space. Consequently, (3) is rather insensitive to the cosmological model being employed.

In this paper cosmic magnetic field generation will be examined during warm inflation, based on the ferromagnetic vacuum mechanism (1) and the Higgs gradient mechanism (2). A scenario will be presented for generation of cosmologically interesting magnetic fields during a GUT symmetry breaking warm inflation. As a review, section II discusses coherent magnetic fields during generic GUT and electroweak phase transitions and section III summarizes warm inflation. The main analysis of magnetic fields during warm inflation is in section IV. Subsection IVA examines the Higgs gradient mechanism. As an aside, from this analysis monopole constraints are derived for warm inflation. Then in subsection IVB the ferromagnetic Savvidy vacuum scenario is presented. Also discussed here are several features about this Savvidy vacuum scenario that are applicable within a supercooled inflation regime. Finally the results are summarized and scrutinized in the conclusion.

II. GUT MAGNETIC FIELDS AND SYMMETRY BREAKING

To have a specific model exemplifying the results which follow, this paper will consider minimal SU(5). For this model, $SU(5)$ gauge symmetry exists at high temperature above a critical temperature $T_c$. GUT symmetry breaks in phase transitions via a pattern of spontaneous symmetry breaking (SSB) to the low energy vacuum, $SU(3)_C \times U(1)_{EM}$. In minimal SU(5) the breaking pattern is $SU(5) \xrightarrow{<24\Omega>} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{<5\zeta>} SU(3)_C \times U(1)_{EM}$ with symmetry breaking scales $T_c^{GUT} \sim 10^{15} \text{GeV}$, $T_c^{QCD} \sim 10^{-1} \text{GeV}$. Cosmic magnetic fields generated at or near the GUT phase transition will need to survive until the era of galaxy formation with sufficient strength to be candidate dynamo seed fields. We review the $SU(5)$ Yang-Mills Lagrangian, identify the magnetic fields, and subsequently consider the effect of SSB upon those fields.

The Yang-Mills term in the $SU(5)$ symmetric Lagrangian density is
\[ \mathcal{L}_{YM} = -\frac{1}{2} Tr(F_{\mu\nu} F^{\mu\nu}) \]  

(5)

where

\[ F_{\mu\nu} = \tau^a F_{\mu\nu}^a, \]

(6)

\[ F_{\mu\nu}^a = \partial_{\mu} A_{\nu}^a - \partial_{\nu} A_{\mu}^a + ig f^{abc} A_{\mu}^b A_{\nu}^c, \]

(7)

\( a (= 1, \ldots, 24) \) is the group index, \( \tau^a \) are the 24 generators of SU(5), \( f^{abc} \) are the structure constants, and \( A_{\mu}^a \) are the gauge fields with coupling \( g \). The magnetic fields are given by \( B_{i} = \epsilon_{ijk} F_{jk}^a \). The fields \( B^a \) are associated with massless gauge fields, but during SSB some gauge fields acquire a mass screening their respective components of the magnetic field. The long range fields after the GUT phase transition correspond to the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) generators that satisfy \( \tau^b < \phi_{24} >= 0 \), where \( \phi_{24} \) is the Higgs field configuration responsible for GUT symmetry breaking.

Magnetic fields could be established prior to SSB, as in scenario (1) which we show applies to warm inflation. What effect will SSB, originating in the Higgs sector, have upon coherent B-fields of the Yang-Mills sector? To present a simple analysis of this effect suppose the \( SU(5) \) symmetric theory is endowed with a magnetic field

\[ \sqrt{B^a B^a} = B_{GUT} \quad (a = 1, \ldots, 24), \]

(8)

where \( B_{GUT} = B_{\text{min}} \), in scenario (1). Following the GUT phase transition half of the gauge fields, the \( X \) and \( Y \) bosons, become massive and screen magnetic fields. The field strength then becomes

\[ \sqrt{B^b B^b} \sim \frac{1}{2} B_{GUT} \quad (b = 1, \ldots, 12) \]

(9)

This result follows from the assumption that \( B_{GUT} \) is shared equally among all massless field components. By the same argument \( B^{\text{color}} \sim \frac{1}{2} B_{GUT}, B^{\text{weak}} \sim \frac{1}{8} B_{GUT}, \) and \( B^{Y} \sim \frac{1}{24} B_{GUT} \).

Similarly for the electroweak phase transition, we suppose there to be a magnetic field immediately prior to SSB of strength

\[ \sqrt{B^c B^c} \sim B_{EW}, \quad (c = 1, \ldots, 4). \]

(10)

These fields could originate from either scenario (1) or (2). During SSB the \( W \) and \( Z \) bosons become massive and screen the weak magnetic field. The field strength is then \( B^{\text{em}} \sim \frac{1}{4} B_{EW} \). \( B^{em} \) is obtained via the usual weak mixing angle

\[ B_i^{em} = \epsilon_{ijk} F_{jk}^{em} = \epsilon_{ijk} \left[ -\sin \theta_W \psi^a F_{jk}^a + \cos \theta_W F_{jk}^Y \right] \]

(11)

where

\[ \psi^a = \frac{\psi_5^a \sigma^a \psi_5}{\psi_5^5 \psi_5}. \]

(12)
σ^a are generators for the SU(2) subgroup of SU(5), and ψ_5 is the Higgs field configuration responsible for electroweak SSB.

B^m could be the primordial seed field for galactic dynamos and thereby ultimately responsible for the large scale magnetic fields observed today. Based upon a counting of broken group generators, SSB alone reduces a GUT primordial magnetic field strength by a factor \( \sim 24 \). Of course, evolution of the cosmic scale factor until the time of galaxy formation will be primarily responsible for magnetic field suppression.

### III. REVIEW OF WARM INFLATION

Inflation in the presence of nonnegligible radiation is characterized by non-isentropic expansion [9–12] and thermal seeds of density perturbations [13,14]. Such a region can be realized as an extension to supercooled inflation scenarios, in which reheating still is necessary, or it can be realized in warm inflation scenarios [15] in which there is no reheating. In the warm inflation regime, the vacuum energy density \( \rho_v \) dominates the energy density, thus driving inflationary growth of the scale factor. In addition, there is a substantial component of radiation energy density \( \rho_r \). An analysis of two fluid Friedmann cosmology [12] composed of \( \rho_v(t) \) and \( \rho_r(t) \) shows that inflationary expansion requires \( \rho_r < \rho_v \) and, moreover, observationally interesting expansion already occurs for \( \rho_r \leq \rho_v/10 \). This is a sizable radiation energy density component for most particle physics early universe phenomena.

In the context of scalar field dynamics, warm inflation can be realized by a Ginzburg-Landau kinetic equation of motion

\[
\eta \dot{\phi} = -\frac{dV}{d\phi} \tag{13}
\]

Since \( \dot{\rho}_v(t) = \dot{\phi}(dV/d\phi) \), the dissipative term \( \eta \dot{\phi}^2 \) expresses the loss of energy from the \( \phi \)-system into the radiation system. Such an equation of motion, for an SU(5) potential was shown to yield an observationally consistent warm inflation scenario with respect to both expansion e-folds, \( N_e > 60 \), and density perturbations \( \delta \rho/\rho \sim 10^{-5} \) [15].

The fundamental equation of motion for scalar fields is second order in time. To obtain eq. (13) from first principles, two steps are necessary. First, a second order damped equation of motion must be derived

\[
\ddot{\phi} + \eta(\phi)\dot{\phi} + V'(\phi) = \xi(\phi, t), \tag{14}
\]

where \( \xi(\phi, t) \) is a random force term with \( \langle \xi(\phi, t) \rangle = 0 \). Second, the validity of the first step must be demonstrated in the overdamped limit \( \eta(\phi)\dot{\phi} \gg \ddot{\phi} \) for the ensemble average of eq. (14). With these two steps, the resulting first principles equation of motion is of the form eq. (13). The general consistency of this procedure with the principles of quantum mechanics and warm inflation cosmology has been demonstrated in a quantum mechanical model in [15]. Based on methods developed in [16–18], eq. (13) also has been derived from first principles in a quantum field theory model in [19] for a symmetry restored \( \lambda \phi^4 \)-model coupled to scalar bosons.
Here we will assume the general phenomenological validity of eq. (13) and examine its consequences for generation of magnetic fields during warm inflation. A compact analysis of a variety of warm inflation expansion behavior can be obtained from the class of potentials

\[ V(\phi) = \lambda M^{4-n}(\phi - M)^n \quad \text{(symmetry broken)} \]  

(15)

\[ V(\phi) = \lambda M^{4-n}\phi^n \quad \text{(symmetry restored)} \]  

(16)

Exact warm inflation scale factor solutions for this class of potentials are given in [12]. Here the results are summarized. Warm inflation scale factor trajectories, by definition, are required to start within a radiation dominated regime \( R(t) \sim \sqrt{t} \), go through a positive acceleration behavior starting at \( t_{BI} \) (begin inflation) and then return to a radiation dominated regime at \( t_{EI} \) (end inflation). During the course of this, radiation energy is being produced from vacuum decay and redshifted due to expansion, with a net effect of gradual monotonic decrease in the density. As such, in the warm inflation regime there is no reheating. In [12] it is shown that warm inflation scale factor solutions exist for potentials in eqs. (15) and (16) when \( 2 \leq n < 4 \). Within this range, for larger \( n \), the duration of the warm inflation period \( t_{EI} - t_{BI} \) as well as the ratio of the initial to final radiation energy density \( \rho_r(t_{BI})/\rho_r(t_{EI}) \) increase for fixed e-folds \( N_e = \ln[R(t_{EI})/R(t_{BI})] \). The case \( n = 4 \) yields an inflationary power-law expansion and sustains a sizable radiation component. However, for this potential the inflationary period never terminates.

In a realistic setting, no single index \( n \) describes the whole potential. For example, given the scalar symmetry restored potential, \( m^2/2\phi^2 + \lambda/24\phi^4 \), \( n = 4 \) behavior at large \( \phi \) goes over to \( n = 2 \) behavior as \( \phi \to 0 \). Also in realistic models, the dissipative coefficient \( \eta(\phi) \) may be a function of \( \phi \) [19]. For a simple power-law form \( \eta(\phi) = \eta_0\phi^m \), the scale factor solutions for potentials with index \( n' \) will be the solutions for the potentials in eq. (16) for index \( n = n' - m \) and dissipative coefficient \( \eta_0 \) in eq. (13).

For symmetry breaking warm inflation scenarios, it is important that the inflaton’s roll down the potential commences soon after vacuum domination, so as to allow radiation production from vacuum decay. This is necessary so that depletion of radiation energy density due to expansion is compensated adequately by production. Thus the inflaton should not get trapped in a metastable state once vacuum domination occurs. Second-order phase transitions are ideally suited for this purpose. Fluctuation induced first-order phase transitions ("weakly first-order") also can suffice if the decay rate from the metastable minima at \( \langle \phi \rangle = 0 \) is fast. Aside from the order of the transition the kinetics of the quench during symmetry breaking is also important. A complete treatment of order parameter behavior near symmetry breaking is beyond the scope of this work. It should be noted that analyses of gauge-Higgs system transitions include the range from first-order, weakly first-order and second-order [20].

IV. MAGNETIC FIELDS DURING WARM INFLATION

Local magnetic fields can be produced by agitation of charge carriers. At high temperature thermal fluctuations can initiate such a disturbance when \( T \gg m \), where \( m \) is a
characteristic scale for the charge carrier. In the particle physics phase transition scenario of the early universe, the chronology of this condition with respect to the charge carriers is equivalent to the chronology of the early universe. As such, thermal fluctuations are a generic source for production of magnetic fields in the early universe.

Due to the nonnegligible thermal component before and during warm inflation, magnetic field formation is apt to occur. Production mechanisms based on thermal fluctuations are random. In general the treatment of a random system requires identifying the random variables and specifying the statistical distribution. For warm inflation, two mechanisms are suitable for a GUT-based scenario, the Higgs field fluctuation mechanism of Vachaspati [7] and the ferromagnetic Saviddy vacuum of Enqvist and Olesen [6]. These mechanisms originally were formulated for the post-inflationary radiation dominated regime. We observe that the warm inflation regime provides suitable conditions for them to be active. 

A. Higgs Gradients - Monopole Constraint

In the Vachaspati mechanism, the random sources for production of magnetic fields are associated with the charged Higgs fields. In the original proposal, the Higgs field itself was taken as the random variable with a coherence length \( \xi \sim 1/m_H \). Here \( m_H \) is the thermal Higgs mass, which is assumed to be approximately equal to the vector boson mass, thus \( \xi \sim 1/(gT) \) [7]. A subsequent analysis [22] indicates that the appropriate random variables are the gradients of the Higgs field. The implication is, the rms-magnetic field over \( N_d \) independent coherent domains decreases as

\[
B \sim \frac{gT^2}{\sqrt{N_d}}. \tag{17}
\]

The fluctuations of the Higgs gradients are not stable sources of magnetic fields, however, near the critical temperature these fluctuations can get locked or “freeze-out”. For the SU(5) transition these fluctuations will lock into topologically stable monopole and antimonopole configurations and these are the magnetic sources. As such, a conflict of interest arises since an overdensity of monopoles has inappropriate cosmological consequences, in particular the critical density constraint and the Parker bound limit. These constraints in turn limit the size of magnetic fields that are allowed from magnetic monopole sources.

To estimate these constraints, the number density of the Higgs field fluctuations is required at the time of freeze-out. The conventional estimate for the freeze-out temperature is the Ginzburg temperature \( T_F = T_G \lesssim T_c \). \( T_G \) is based on an equilibrium estimate above which thermal fluctuations can still disorient correlated domains. An alternative argument for freeze-out has been given by Zurek [23], based on the kinetics of the quench as \( T_c \) is approached from above. His analysis suggests that \( T_F = T_Z > T_c \), which is in qualitative distinction to \( T_G \). Moreover, the number density of the defects, \( n_\phi \), depends on the kinetics. In 1-D and 2-D Ginzburg-Landau models, similar in form to eq. (14) [24], \( n_\phi \) at \( T_F = T_Z \) is

\[\text{Some of these results were presented in [21].}\]
shown to depend on the viscosity coefficient $\eta(\phi)$ and the duration of the quench $\tau_Q$. In the overdamped regime $n_\phi \sim (\eta/(m^2\tau_Q))^{1/4}$ and in the underdamped regime $n_\phi \sim (1/(m\tau_Q))^{1/3}$, where $m^2 = V''(0)$ is the characteristic dynamical mass scale. Although similar results have not been confirmed for 3-D systems, the scaling is expected to hold but the premultiplying coefficient should be smaller [25].

For the warm inflation scenario, these findings lead to very different possibilities. Above $T_c$, when the scalar field is in the symmetry restored regime, the viscosity coefficient has been shown [19] in a $\lambda \phi^4$ quantum field theory model to vanish as $\eta \sim \phi^2/T_c$. This allows for the possibility of substantially small defect production before entering the warm inflation regime. For our present purposes, we will use the upper limits on defect density production, since it is important to know the maximum monopole density that warm inflation must dilute. By either the Ginzburg or Zurek criteria, the defect density should not exceed the intrinsic microscopic scale, which at high temperature is set by $T \sim T_c$. Thus, we will consider the number density of monopoles at the onset of warm inflation to be $n_M \sim T_c^3$. Note the associated magnetic field energy density from eq. (17), $\rho_B \approx g^2 T_c^4 \ll \rho_r \approx g^* T_c^4$, where $g^*$ is the number of relativistic particles.

The critical density condition on monopoles requires [26]

$$\Omega_M h^2 \approx 10^{24} \left(\frac{R_M}{s}\right)(\frac{M}{10^{16}\text{GeV}}) \leq 1$$

(18)

where $M \sim 4\pi M_{\text{GUT}}/g^2 \sim 20 M_{\text{GUT}}$ is the monopole mass. Ignoring monopole-antimonopole annihilation, the ratio $n_M/s$ is constant [26], so it can be estimated at the end of the warm inflation, which yields

$$\frac{n_M}{s} = \frac{45}{2\pi^2 g^* e^3 N_e} \left(\frac{T_c}{T_{EI}}\right)^3 \approx \frac{1}{15 e^3 N_e} \left(\frac{T_c}{T_{EI}}\right)^3,$$

(19)

where the number of relativistic particles for minimal SU(5) GUT is $g^* \sim 170$. The temperature at the end of warm inflation $T_{EI}$ is required. For the quadratic potential in eqs. (15) and (16) this is $T_{EI} \approx T_c/\sqrt{N_e}$ [12]. In general for the potentials in eqs. (15) and (16), $T_{EI} \sim T_c N_e^{-p}$ for some small positive exponent $p$. The quadratic case is a good estimate, since near the minimum of a generic potential, when warm inflation ends, the behavior is quadratic. Based on eqs. (18) and (19) we find $N_e \approx 20$. The analysis of the Parker bound does not change this lower bound. This $N_e$ is sufficiently large that an initial magnetic field as large as $\sim 10^{16}\text{G}$ is miniscule by the end of warm inflation. A similar conclusion was arrived by Davis and Dimopoulos [27], who examined the Vachaspati mechanism during a false vacuum inflation.

The magnetic monopole constraints can be removed by going to flipped-SU(5). In this model stable monopole solutions are not admitted, although unstable monopole-antimonopole configurations attached by strings are possible [7,28]. At present, the knowledge of such configurations is limited, thus we will not conjecture about them. In any case, for a cosmologically interesting warm inflation, $N_e > 60$, without further assumptions, the direct effect of defects at the onset will be negligible for post-inflation magnetic field generation.
In the Enqvist-Olesen ferromagnetic Savvidy vacuum scenario [6], the random variables are quark currents in the GUT-scaled radiation plasma at temperature $T$. If a given quark traverses a distance $1/T$ without collision, it will generate a magnetic field $\sim T^2$. This could help to locally induce a transition into the Savvidy vacuum at sufficiently high temperature $T^2 \gg B_{\text{min}}$, where $B_{\text{min}}$ is given in eq. (3). The Savvidy vacuum would act as a memory of the local currents. At very high-$T$ the quark currents generate very high magnetic fields, thus it would be easy to reorient regions of Savvidy vacuum. For a plasma of quarks at a high temperature $T$, quarks will criss-cross each other’s paths significantly, which implies that a characteristic coherence region for a local Savvidy vacuum bubble is $\sim 1/T^3$. If these local bubbles were randomly distributed, then an estimate of the rms-magnetic field at large scale would be possible with magnitudes similar to those in the previous subsection. This is the scenario of magnetic fields that Enqvist and Olesen estimated. Such a scenario would work effectively during warm inflation.

However, a charged plasma tends to screen long range electromagnetic fields. Thus charge and current distributions in a charged plasma are not random, especially local to any given charge. Although rigorous calculations of magnetic screening in a charged non-Abelian plasma have not been possible, evidence from analytic [29] and numerical calculations [30] suggest that a local magnetic field is screened at distances greater than $1/(g^2T)$. Recently this point has been examined in the context of the Enqvist-Olesen ferromagnetic Savvidy vacuum scenario [31]. The conclusion in [31] is that the charged plasma is not an adequate source of large scale magnetic fields. The largest scale magnetic field produced by the charged plasma $1/(g^2T)$ is very small compared to the characteristic scale of the Savvidy vacuum, which from eq. (3) is

$$\Lambda_{SU(5)} = 2 \times 10^{11}\text{GeV}. \quad (20)$$

Here, and throughout this paper following [31], for SU(5) we set $\mu = M_{\text{GUT}} = 10^{15}\text{GeV}$ and $g(\mu) = 0.7$. As such, the plasma fields do not have an adequate long range affect to induce a transition into the Savvidy vacuum.

We observe that there is another source that can produce large scale magnetic fields and at sufficient magnitude to induce the transition into the ferromagnetic Savvidy vacuum, the monopoles that are produced at the onset of symmetry breaking, which also is the onset of warm inflation. Monopoles behave very differently from charged plasma particles as magnetic field sources. They are very heavy and thus decorrelated from the motions in the plasma. Although monopoles and antimonopoles are created simultaneously, the Higg’s gradients that initiate this process are decorrelated. Thus at freeze-out there is no reason to expect the monopole-antimonopole distribution to be so configured as to screen magnetic fields at large scales. Thus the monopoles at freeze-out are a viable source of large scale magnetic fields that can induce a transition of the gauge field system into the Savvidy vacuum.

Based on this observation, the following scenario suggests itself. At $T_F \sim M_{\text{GUT}}$ a density of monopoles $n_M$ is created. The local magnetic field $B_{\text{local}} \approx n_M^{2/3}$, with corresponding coherence scale $n_M^{-1/3}$, will produce, at the scale of the Hubble radius at the beginning of warm inflation
\begin{equation}
H_{Bl}^{-1} = \left( \sqrt{\frac{8\pi^3 g^* T_c^4}{45m_p^2}} \right)^{-1},
\end{equation}

an rms-magnetic field from eq. (17) of size

\begin{equation}
B_{rms}^H \approx B_{local} \sqrt{\frac{H_{Bl}}{n_M^{1/3}}} = \sqrt{n_M H_{Bl}}.
\end{equation}

To induce a transition to the Savvidy vacuum the source field must be the order of or bigger than the magnetic field at the minimum of the ferromagnetic effective potential

\begin{equation}
B_{rms} \geq B_{min} \sim \frac{\Lambda^2}{g} \sim 4 \times 10^{22}\text{GeV}^2.
\end{equation}

From eq. (22) this implies a monopole number density

\begin{equation}
n_M \geq \left( \frac{\Lambda^2}{g \sqrt{H_{Bl}}} \right)^2 = 2 \times 10^{34}\text{GeV}^3.
\end{equation}

From the lower bound, as few as \( \sim 10 \) monopoles within the Hubble radius just before warm inflation can induce the transition into the Savvidy vacuum. However a rapid creation of the Savvidy vacuum may be necessary, before full symmetry breaking has occurred. In this case, a larger source magnetic field is preferable, which in turn implies a monopole density above this lower limit.

Although the monopoles solve the problem of a source field, a second problem arises. The non-Abelian effective potential eq. (2) was obtained from a one-loop calculation [4,5,32] with a specific order of limits. If the large distance limit is taken first and then the source magnetic field is removed, the effective potential has a new minimum at the \( B_{min} \) in eq. (3). An analogy to the thermodynamic limit has been given in [32], in which to obtain a nonvanishing magnetization, first the infinite volume limit must be taken and then the external magnetic field can be removed. This underlies the need for an initial source magnetic field at length scales larger than \( > \Lambda^{-1} \). The validity of the one-loop approximation has been questioned in [33] on the basis that such classically unstable field configurations do not dominate the functional integral and the problem is entirely nonperturbative. On the other hand, Nielsen and Olesen [32] have argued that the behavior of the \( \beta \)-function eq. (1) is sufficient for the qualitative validity of the effective potential eq. (2). They also have shown that the effective potential has an imaginary part

\begin{equation}
\text{ImV} = \frac{1}{8\pi} \mu^4 \exp \left( \frac{-96\pi^2}{11Ng^2} \right)
\end{equation}

and identify this as the decay probability per unit spacetime volume.

They have interpreted the ferromagnetic vacuum as a metastable state, below the perturbative vacuum, which decays to possible lower states and eventually to the nonperturbative true non-abelian vacuum. Based on this interpretation, the Savvidy vacuum induced by the
monopoles at the onset of warm inflation will decay. If it decays too soon before the inflationary period terminates, subsequent expansion will dilute the magnetic fields produced up to then and they will not be cosmologically interesting. In warm inflation, observationally consistent expansion e-folds requires an inflationary time period of order $100/H_{BI}$ or larger.

After this time interval, the decay probability per unit three-volume from eq. (25) is

$$P_{\text{decay}}^{\text{volume}} = \frac{100\mu^4}{8\pi H_{BI}} \exp\left(\frac{-96\pi^2}{11Ng^2}\right) = 2 \times 10^{34}\text{GeV}^3.$$  \hspace{1cm} (26)

From this it follows that for Savvidy vacuum domains of radius $\sim (3 \times 10^{11}\text{GeV})^{-1}$ or smaller after warm inflation, the decay probability still is less than one.

For regions of Savvidy vacuum that have not decayed just after warm inflation, their effect will be imprinted on the plasma. Since the plasma conserves flux [2,34], these magnetic fields will be frozen into the plasma. Right after warm inflation, the largest coherence scale for magnetic fields made by the Savvidy vacuum will be $\sim 10^{-12}\text{GeV}^{-1}$, with magnitude from eq. (23)

$$B_{EI} \approx B_{\text{min}} = \frac{\Lambda^2}{g} \approx 10^{41}\text{G}.$$  \hspace{1cm} (27)

For comparison, in the warm inflation scenario considered above, the temperature at the end of warm inflation is $T_{EI} \approx T_c/10 \approx M_{\text{GUT}}/50 = 2 \times 10^{13}\text{GeV}$. This implies the magnetic regions of Savvidy vacuum are much bigger that the interparticle distance $1/T_{EI}$. Note that the magnetic energy density of the Savvidy vacuum $\rho_B \sim B_{\text{min}}^2$ is much smaller than either the radiation or vacuum energy during all of warm inflation, since they are both $> T_{EI}^4$. Also, there is no magnetic domain formation, since the Savvidy vacuum decays, after which, the magnetic fields evolve with the rest of the plasma. If the magnetic fields eq. (27) subsequently follow the Hubble flow, the magnitude at time of nucleosynthesis is $\sim 10^8\text{G}$ at the beginning ($T = 1\text{MeV}$) and $\sim 10^4\text{G}$ at the end ($T = 0.01\text{MeV}$) for coherent domains at these respective times of radius smaller than $67\text{MeV}^{-1}$ and $6700\text{MeV}^{-1}$. These magnitudes are below the big-bang nucleosynthesis bounds on magnetic fields [37]. Similarly, the size of the magnetic field eq. (27) at the time of galaxy formation (today) is $10^{-11}\text{G}$ at scales smaller than $\sim 5\text{cm}$. This implies from eq. (22) that at $\sim 100\text{kpc}$, which is an interesting galactic scale, the rms-magnetic field is $4 \times 10^{-23}\text{G}$ [21], which is a few orders of magnitude too small to seed the galactic dynamo. These estimates ignore nonlinear effects of evolution after and even during warm inflation, such as inverse cascade [35,36], which would enhance the magnitude.

These estimates suggest favorable conditions for such a scenario. As such it is worthwhile to examine arguments that could negate the scenario. Firstly, scale considerations indicate that patches of Savvidy vacuum of radius $\sim \Lambda_{SU(5)}^{-1}$ will coherently decay from the metastable state, rather than say smaller patches in a longer time or visa-versa. Since the domain size of Savvidy vacuum regions that are found to persist after warm inflation (given above eq. (27)) are of order this characteristic length, it is a borderline situation. Secondly, although the Savvidy vacuum can be created in the symmetry restored regime, the metastable state in the scenario must exist just below the SU(5) symmetry breaking temperature, $T_c$. Below $T_c$, twelve of the gauge fields have become massive, the X and Y bosons, but they played an
important role in establishing the Savvidy vacuum. Since the metastable state had already been created before these bosons became massive, the question is how much does the length and time scale for decay of this state change from eq. (25)? The SU(5) Savvidy vacuum may make its way to the SU(2) and SU(3) Savvidy vacua below \( T_c \), but due to the disparity in scales between these phases, this fact does not help to quantify our question. If one naively changes \( N \) from 5 to 2 or 3 in eq. (25), but still evaluates it at scales of order \( M_{\text{GUT}} \), it would, in fact, increase the lifetime of the metastable state.

Finally, note that the basic features of this scenario are not specific to warm inflation and also can be applied to supercooled inflation scenarios. The essential requirement is that after the source magnetic field created by the monopoles is negligible, the inflationary epoch finishes before the metastable Savvidy vacuum decays. For a symmetry breaking warm inflation scenario, this requirement is natural, since warm inflation prefers small expansion e-folds, within a few orders of magnitude above the lower observational bound. Also, due to the presence of sizeable radiation during warm inflation, the effects of the Savvidy vacuum already are being imprinted on the plasma. For a supercooled scenario, a primary concern is that preheating/reheating may have adverse effects on an existing Savvidy vacuum. The characteristic coherence scale during reheating \( \sim (M_{\text{GUT}}/10)^{-1} - (M_{\text{GUT}}/100)^{-1} \) is much smaller than \( \Lambda_{\text{SU(5)}}^{-1} \). Since reheating is a highly non-equilibrium process, this coherence scale will be the governing scale, thus existing Savvidy vacua will likely be dismantled.

V. CONCLUSION

In this paper, we have examined GUT sources for magnetic fields and their implications before and during a warm inflation regime. As a preliminary step, this has required a general determination of defect density production during a symmetry breaking warm inflation scenario. The thermal nature of the scenario implies this is determined at the onset of the transition with a defect density no larger than the intrinsic scale which at high temperature is set by \( T \sim T_c \). For SU(5) monopoles at a number density \( T_c^3 \) at the onset of warm inflation, the Parker bound and overdensity constraint require the number of e-folds \( N_e \geq 20 \). It has been noted that the initial density may be substantially less based on the considerations by Zurek [23,24].

We have observed that the soon-to-be exiled SU(5) monopoles initially in the early universe may be good for something: sources of large scale magnetic fields that can induce the gauge-field transition into the ferromagnetic Savvidy vacuum. A GUT warm inflation scenario has been proposed for production of large scale magnetic fields. Although the monopoles solve the problem of an initial large scale source, a second problem is the metastability of the Savvidy vacuum. Due to this, the inflationary epoch must be short, in order that the Savvidy vacuum survive inflation and imprint its effect on the plasma. We have found that the duration of warm inflation at the observational lower bound \( N_e \sim 60 \) is of order the lifetime of the Savvidy vacuum. Some modifications to the scenario can improve this borderline situation. For example, we estimated the source magnetic field from the monopoles at the onset of warm inflation, but within the first 5-10 e-folds of warm inflation it is sufficiently large to affect the Savvidy vacuum. Likewise, if the Savvidy vacuum decays a few e-folds before warm inflation ends, there still would be a sizeable imprint on
the plasma leftover. Irrespective of such refinements, the strong exponential dependence in the Savvidy vacuum results and its perturbative origin are the primary concerns.

Accepting the qualitative correctness of the one-loop Savvidy vacuum calculation, this touchy quantitative issue can be avoided by increasing the disparity between the lifetime of the Savvidy vacuum and the duration of warm inflation. This is possible at temperatures well above the GUT scale $T \gg T_c$. If this is a preinflation hot-big-bang regime, the characteristic Savvidy vacuum region will be larger than the causal horizon $1/H = t \approx m_p/(\sqrt{g^*T^2}) \ll \Lambda_{SU(5)}^{-1}$. In fact, very close to $m_p$, the causal horizon is smaller than the magnetic screening length $1/H < 1/(g^2T)$.

If the Savvidy vacuum is an acceptable solution in this regime, the plasma particles could be the source to create it, followed by a warm inflation regime. The duration of an observationally consistent warm inflation in this very high-$T$ region is orders of magnitude smaller than the lifetime of the Savvidy vacuum, since the Hubble parameter is much larger for $T \gg T_c$. As such, since the Savvidy vacuum easily persists after warm inflation, it will imprint its effect on the plasma. Furthermore, now the magnetic screening effects work in favor of sustaining the large scale fields, since the charged plasma is incapable of reorienting the Savvidy vacuum, even if the temperature is very high. To complete this scenario, the next step is the GUT transition. To avoid stable monopole production, flipped-SU(5) is appropriate if further inflation is disallowed. For the magnetic field scenario outlined above, cosmologically interesting magnetic fields are created. Since they are produced at an earlier time, assuming only Hubble flow evolution, their magnitude today would be a few orders smaller than for the scenario in subsection IVB. As an aside, recall from the discussion in subsection IVA that unstable monopole-antimonopole configurations could be created at the flipped-SU(5) GUT transition. If so, such configurations could be an alternative and sole source for large scale magnetic fields that induce the gauge field transition into the Savvidy vacuum. This point will not be pursued further here.

Although the Savvidy vacuum scenario for magnetic field formation has appealing features, it strongly relies on the quantitative properties of the gauge field solution. Unfortunately, to understand this beyond the perturbative level probably will come in-hand with the understanding of QCD confinement. Nevertheless, the one-loop expressions are available, so it is worth asking if they can be tested for the QCD Savvidy vacuum [21]. Neutron stars and heavy-ion collisions are cases in which Savvidy vacuum bubbles may be created in the charged plasma, yet amongst a plethora of other effects. The conspicuous property about the Savvidy vacuum is metastability, which may be exhibiting itself in resonance states. The lifetime of the $SU(3)$ Savvidy vacuum $\sim \Lambda^{-1}_{QCD}$, is typical of several resonances. The requirement of a strong initial color-magnetic field source of magnitude $\Lambda_{QCD}^2 \approx (200\text{MeV})^2$ can easily be realized in local regions during a high energy hadronic collision. The cleanest process that contains these two features is diffractive dissociation. The pp-elastic scattering slope parameter, which reflects the area of the scattered object is $b \approx 12\text{GeV}^{-2} \sim 1/\Lambda_{QCD}^2$ [38]. When the Reggeon collides into such a region on the hadronic disk, in some cases it could alter the local vacuum and place it in a metastable state, such as the Savvidy vacuum. Such a region would have small overlap with the rest of the hadron, thus easily separate. This picture to the present elaboration is not very specific to the Savvidy vacuum, beyond its metastability property. Nevertheless, that is the only metastable gauge-field vacuum that
is known.

Excepting the reservations stated already, the Savvidy vacuum warm inflation scenario involves generic features of gauge fields and monopoles, which are commonplace in the present-day early universe picture. No additional assumptions are made about field theory such as non-conformal coupling [34,39,40]. However if this effect was considered, it would increase the size of the magnetic fields that are created, since their redshift could be made to decrease slower than $1/R(t)^2$.

ACKNOWLEDGMENTS

We thank Tanmay Vachaspati for helpful discussions. AB also thanks Per Elmfors, Kari Enqvist, Pablo Laguna, and Poul Olesen for helpful discussions. This work was supported by the U. S. Department of Energy under grant DE-FG05-85ER40226. S.D.W. also gratefully acknowledges support from the NASA/Tennessee Space Grant Consortium.
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