Colour octet contribution to exclusive P-wave charmonium decay into nucleon-antinucleon

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We show that although there is no infrared divergence in exclusive P-wave quarkonium decays, the colour octet contribution is no less important than in the inclusive decay. Results from more proper calculations with only colour singlet contribution are not sufficient to account for the measured partial decay widths and so the next higher Fock state must be included. Using the $\chi_J$ decay into $NN$ as an example, a scheme is devised to yield estimates of the decay widths using both contributions, the results are comparable with experimental measurements.

1. Introduction

In the late 60’s and early 70’s, Barbieri et al [1,2] considered the inclusive decay of P-wave heavy quark-antiquark system into hadrons. Such decays proceed via the annihilation of the heavy fermions into massless gluons. For P-wave quarkonium, annihilation into two gluons is allowed and this is the leading contribution to the inclusive hadronic decay width, at least for the total angular momentum $J = 0$ and $J = 2$ system. For $J = 1$ P-wave quarkonium, the decay into two massless spin-1 gluons is forbidden. The leading decay process in this case is therefore one gluon and a light quark-antiquark pair. Whereas the kinematics of the leading decay process of the even-$J$ systems into two gluons is very simple and completely well defined, that of the $J = 1$ system is not. In the rest frame of the quarkonium, the decay into two gluons would proceed with them moving apart from each other with equal and opposite momentum. The three-body leading decay of the $J = 1$ quarkonium is, on the other hand, kinematically less restrictive so the situation where the quark and antiquark move apart with nearly equal and opposite momentum with an accompanying very soft gluon is allowed. The corresponding probability on the tree graph level from such a process with a very soft gluon is easily shown to contain infrared divergence when all the participants are on their respective mass shells. In view of the infrared divergence and the fact that the heavy quark and antiquark are really in a bounded system, one should perhaps keep them off shell by an amount corresponding to the binding energy $\varepsilon$ of the system. Thus using $\varepsilon$ as an infrared cutoff, one gets for the $J = 1$ quarkonium decay width

$$\Gamma_{J-1} \sim \frac{\alpha_s^3}{\varepsilon} \int \frac{dq}{q} \simeq \alpha_s^3 \ln \varepsilon.$$  (1)

This is the solution proposed in [2]. In fact, this infrared divergence also appears in the decay widths of the even-$J$ P-wave quarkonia but at the next-to-leading order.

With the more recent advances in quarkonium physics [3,4], we now know that this infrared divergence should not be present. The physical picture of the quarkonium in the early 70’s was a heavy quark bounded with an equally heavy antiquark. The missing component in this picture is the higher Fock states of the quarkonium. The contribution to the decay into $gq\bar{q}$ from the so-
called colour octet, the next higher Fock state of the P-wave quarkonium, becomes degenerate with that from the lowest valence state when the gluon approaches the infrared. When both are included, there is no infrared divergence. This is the situation of the inclusive P-wave quarkonium decay. Colour octet is needed and is introduced to cancel infrared divergence.

In exclusive decay, however, the heavy quark system decays from a bounded system into other bounded systems, the same infrared divergence in the inclusive decay does not appear. So there is no need for colour octet and the partial widths of the various exclusive decay modes have been calculated from the valence colour singlet component alone, see [5,6] for example. We will show that, on the contrary, the colour octet is very important for P-wave quarkonium.

2. Angular momentum suppression of P-wave wavefunctions

The decay of a quarkonium of mass $M$ through annihilation is a short distance process with the annihilation length given by $L \sim 1/M$. For a heavy quarkonium, $1/M$ is very small so we only need the quarkonium wavefunction near the origin $\psi(L \sim 0)$. This is true only for a S-wave quarkonium, for a P-wave, the wavefunction at the origin is zero so one expands the wavefunction around the origin and uses instead the quantity $Le^i(\theta) (L \sim 0)$ for the infrared confinement physics.

In momentum space, this means

\begin{align}
\text{S-wave:} & \quad \psi_S(0) \to \tilde{\psi}_S(k) \\
\text{P-wave:} & \quad \psi_P(0) \sim Le^i(0) \to \frac{k}{M} \tilde{\psi}_P(k).
\end{align}

(2)

(3)

Assuming that there is no significant difference between the S- and P-wave Fourier components $\tilde{\psi}(k)$, which seems reasonable, it follows that the P-wave wavefunction is weighted down by $1/M$ in comparison with that of the S-wave. Therefore P-wave is relatively suppressed on the level of the wavefunction. We will see that this is very important in the next section.

3. Comparing colour singlet with octet

In order to find out whether the colour octet component in exclusive quarkonium decay is negligible or not when compared with the colour singlet, we now perform power counting in the only large scale of the process, namely $M$. We must choose an explicit process and a scheme for the calculation of the decay probability amplitude in order to be able to do this. We use $\chi_J$ decay into $NN$ as the example. For the purpose of power counting, it is sufficient to choose the simpler standard hard scattering scheme of Brodsky and Lepage [7]. In this scheme, the probability amplitude is given by a convolution of distribution amplitudes $\phi(x)$ and hard perturbative part $T_H$.

\[ M \sim f_{\chi_J} \phi_{\chi_J}(x) \otimes f_N \phi_N(x) \otimes f_R \phi_R(x) \otimes T_H(x) \]

and this is related to the partial decay width by $\Gamma \sim |M|^2/M$. So $M$ must have mass dimension one. On the right hand side of Eq. (4), the only dimensional quantities are the decay constants $f$'s and a certain power of $1/M$ hidden in $T_H$. They must make up the right dimension to match the left hand side. The decay constants have the following mass dimension. $f_N$, $f_R$ and $f_{\chi_J}$, the octet constant are from 3-particle wavefunctions and so must have mass dimension two. The singlet constant $f_0$ is from a 2-particle wavefunction which would mean dimension one but being a P-wave increased this to two. Now extracting sufficient power of $1/M$ from $T_H$ to make up the right dimension for $M$ gives

\[ M^0 \sim M f_0^3 \left( \frac{f_N}{M^2} \right)^2 \sim \frac{1}{M^5} \]

(5)

\[ M^8 \sim M f_0^3 \left( \frac{f_N}{M^2} \right)^2 \sim \frac{1}{M^8} \]

(6)

We see that both the colour singlet and octet are weighed by the inverse fifth power of the large scale $M$. Therefore the colour octet is not suppressed when compared to the singlet contribution.

Now we are ready to return to the $1/M$ suppression of P-wave wavefunction discussed in the previous section. Only because of this suppression is the singlet amplitude scale like $1/M^5$. If
this were not present as in the case of the S-wave $J/\psi$, the singlet amplitude would scale like $1/M^4$ instead. In that case, it would be legitimate to neglect the colour octet contribution.

4. Calculation — singlet contribution

To obtain the colour singlet contribution to the decay width, instead of the original hard scattering scheme, we employ the modified version of Botts, Li and Sterman [8,9]. In this modified scheme, the probability amplitude is now a convolution of wavefunctions, hard perturbative part $T_H$ and the exponential Sudakov suppression factor

$$
M \sim \psi_{\chi J}(x, k_{\perp}) \otimes \psi_{N}(x, k_{\perp}) \otimes \psi_{\pi}(x, k_{\perp})
\otimes T_H(x, k_{\perp}, \alpha_s(x)) \otimes \exp \{-S(x, k_{\perp})\} .
$$

The essential differences between this and the standard scheme are that internal transverse momenta are kept everywhere, the strong coupling $\alpha_s$ is part of the convolution instead of a constant and Sudakov suppression is included. Actually even in the standard scheme, $\alpha_s$ need not be a constant and can be made part of the convolution, but in that case, some ad hoc arrangement such as freezing the value of the coupling at small momentum transfer will have to be made. The modified scheme has no such problem as the more natural Sudakov factor takes care of potential divergence of the coupling. This is achieved by using the adjacent largest virtuality in the neighbourhood of each coupling for its argument [13,14]. It can be seen that the neighbouring quark propagator, quark propagator or the inverse internal transverse separation of quark pairs in the outgoing nucleon-antinucleon. In this way, the scale of the decay process is determined dynamically.

Our objection to some of the calculations based on using the standard scheme is that if a fixed coupling is not used then freezing of the coupling or one of the other methods to deal with this is necessary. These methods have not a sound physical basis. If a constant coupling is used, then taking our current example of $\chi J$ decay into nucleon-antinucleon the probability amplitude goes like $\mathcal{M} \sim \alpha_s^3$ and the decay width is therefore $\Gamma \sim \alpha_s^6$.

A choice of $\alpha_s = 0.3$ instead of $\alpha_s = 0.5$ will therefore make a factor of 20 difference in the width so provided the singlet contribution is of reasonable size, one can always argue for a value of the coupling so that the singlet contribution is sufficient to account for the measured partial width. This in our opinion is quite arbitrary and unsatisfactory.

Using the modified scheme, we found that the colour singlet component makes up of no more than 6 and 12 % of the measured decay widths for $\chi_1$ and $\chi_2$ respectively. So colour singlet alone is not sufficient in agreement with our theoretical expectation.

5. Calculation — including octet contribution

To complete the calculation, one must include the colour octet contribution. It turned out that to use the modified scheme to include also the octet contribution was rather daunting because the internal transverse momentum must be kept everywhere and with an extra constituent gluon, the dimension of the numerical integrations became quite high. One could go back to the standard scheme but then had to face the objections raised in the previous section. Since the problem with using the standard scheme is mainly in the coupling, one can circumvent this by using a model coupling that is free of the Landau pole and thus can be included in the convolution of probability distribution amplitudes. With such a model, the $\chi J$ decay probability amplitude into $NN$ becomes

$$
\mathcal{M} \sim f_{\chi J} \phi_{\chi J}(x) \otimes f_N \phi_N(x) \otimes f_{\pi} \phi_{\pi}(x)
\otimes T_H(x, \alpha_{s,\text{model}}(x)) .
$$

There exist various models that fit our requirement. We choose the one constructed by Shirko and Solovtsov [10] which is less complicated and the Landau pole is removed by simple subtraction. To one-loop, it is

$$
\alpha_{s,\text{model}}(\mu) = \frac{4\pi}{\beta_0} \left\{ \frac{1}{\ln \mu^2/\Lambda^2} + \frac{\Lambda^2}{\Lambda^2 - \mu^2} \right\} .
$$
Preliminary estimates of the colour singlet plus octet contributions to $\chi J$ decay into $N\bar{N}$ in our scheme.

<table>
<thead>
<tr>
<th>$J$</th>
<th>$\Gamma(\chi J \rightarrow N\bar{N})$ [eV]</th>
<th>PDG [eV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>89.0</td>
<td>75.68</td>
</tr>
<tr>
<td>2</td>
<td>180.0</td>
<td>200.00</td>
</tr>
</tbody>
</table>

Although this model is comparatively simple, it agrees with known estimates of the quantity

$$A(Q) = \frac{1}{Q} \int_0^Q \, \, dk \, \alpha_s(k)$$

from jet physics. These estimates are done at a value of $Q \sim 2$ GeV [11,12].

Without the $k_{\perp}$ dependence, the calculation becomes much simpler and with our so constructed semi-modified hard scattering scheme, the dynamical setting of scale by the process itself is preserved. Details of our calculation parallel those of [13,14]. Our preliminary results of the total colour singlet plus octet contribution together with the experimental measurements [15] are shown in Table 1 [16]. As can be seen, the agreement with the measured values are quite reasonable. This confirm our theoretical argument in Sec. 3. In fact, this argument can be extended to even higher wave quarkonia and in such cases, not only the next higher Fock state must be included but also the next-next higher states.

In view of the above considerations and results, the constituent gluon in P-wave quarkonia is an important part of the heavy quark hadrons. The description that they are bounded heavy quark-antiquark systems becomes inaccurate. If the “quark” in quarkonium means heavy quark-antiquark object, then the three $\chi J$’s would each be less of a quarkonium than a $J/\psi$ and a D-wave would be even less so. Therefore the nomenclature is misleading in this sense.

REFERENCES