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In Search of a Sum-Rule for the Parton Distributions of the Photon

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Abstract

The possibility of formulating a useful 'energy-momentum' sum-rule for the parton distributions of the photon is investigated in detail. The limitations of such an attempt in practical applications are pointed out.

The hadronic parton distributions, $f^{h=p,n,\pi}(x, Q^2)$, are known to satisfy the energy-momentum sum-rule

$$M_2^h(Q^2) \equiv \sum_{f=q,\bar{q},g} \int_0^1 x f^h(x, Q^2) dx = 1. \quad (1)$$

This very useful sum-rule follows from the definition of f^h within the framework of the operator product expansion (OPE) formalism [1]. Unfortunately, a similarly useful sum-rule for the photonic parton distributions, $f^\gamma(x, Q^2)$, does not follow directly from their definition within the framework of the OPE formalism [2]. Recently some attempts were undertaken [3, 4] to relate $M_2^\gamma(Q^2)$ to the 'running' finite hadronic part of the photon field renormalization coefficient, $Z_{3,h}^{-1/2}(Q^2)$, which is given via the dispersion relation [5]

$$\Pi_h(Q^2) \equiv Z_{3,h}^{-1}(Q^2) - 1 = \frac{Q^2}{4\pi^2\alpha} \int_{4m_\pi^2}^{\infty} \frac{\sigma_h(s)}{s + Q^2} ds \quad (2)$$

where $\sigma_h \equiv \sigma(e^+e^- \rightarrow \text{hadrons})$. Such a relation could be as useful as the corresponding sum-rule for $M_2^h(Q^2)$ due to the available detailed data [6] on $\sigma_h(s)$.

Since a relation between $M_2^\gamma(Q^2)$ and $\Pi_h(Q^2)$ does not strictly follow from their respective field theoretical definitions, it is interesting to see to what extent such a relation could hold in the perturbative regime where $\alpha_s(Q^2)/2\pi \ll 1$. From the lowest order expressions for the contributions of the subprocess $\gamma^*(Q^2)\gamma \rightarrow q\bar{q}$, as calculated via the 'box-diagram', one obtains [2, 7, 8] in the limit $Q^2/m_q^2 \gg 1$ and $\alpha_s(Q^2)/2\pi \ll 1$

$$M_{2,\text{box}}^\gamma(Q^2) = N_c \frac{\alpha}{3\pi} \sum_q e_q^2 \left(\ln \frac{Q^2}{m_q^2} - \frac{3}{4} \right), \quad (3)$$

while the corresponding 1-loop vacuum polarization diagram yields the well known result

$$\Pi_h^{1\text{-loop}}(Q^2) = N_c \frac{\alpha}{3\pi} \sum_q e_q^2 \left(\ln \frac{Q^2}{m_q^2} - \frac{5}{3} \right). \quad (4)$$

Combining these results with a conjectured [3, 4] relation between $M_2^\gamma(Q^2)$ and $Z_{3,h}^{-1}(Q^2) - 1$ leads, in general, unambiguously to the following proposal

$$M_2^\gamma(Q^2) = \Pi_h(Q^2) + C \quad (5)$$

where in the leading logarithmic order (LO) one should obviously drop the (non-logarithmic) constants in eqs. (3) and (4), i.e. take $C = 0$. In the next-to-leading logarithmic order (NLO) it is, however, unclear what the relevant value of C really is since the naive choice $C = N_c \frac{\alpha}{3\pi} \left(\sum_q e_q^2 \right) \left(\frac{3}{4} - \frac{5}{3} \right)$, implied by eqs. (3) and (4), is unjustified in view of the applied renormalizations needed when considering the standard massless partons.

A direct test of eq. (5) which does not depend on the constant term consists in comparing the logarithmic derivatives $d/d \ln Q^2$ (denoted by a dot), i.e. the relation

$$\dot{M}_2^\gamma(Q^2) = \dot{\Pi}_h(Q^2), \quad (6)$$

an equality which should follow from eq. (5). In LO it follows from the renormalization group evolution equations [2, 8] for $M_{2,\text{LO}}^\gamma(Q^2)$ that

$$\dot{M}_{2,\text{LO}}^\gamma(Q^2) = N_c \frac{\alpha}{3\pi} \sum_q e_q^2 \quad (7)$$

where the sum extends over all quarks satisfying $2m_q < \sqrt{Q^2}$. A useful parametrization [9] of $\Pi_h(Q^2)$ in the space-like region $Q^2 \equiv -q^2 \geq 0$ provides the opportunity to test the validity of eq. (6). In Table 1 we present the results for $r_{\text{LO}}(Q^2) \equiv \dot{\Pi}_h(Q^2)/\dot{M}_{2,\text{LO}}^\gamma(Q^2)$

$Q^2(\text{GeV}^2)$	0.36	1	4	50
$r_{\text{LO}}(Q^2)$	0.900	1.225	1.445	1.139

Table 1. The slope ratio $r_{\text{LO}} \equiv \dot{\Pi}_h/\dot{M}_{2,\text{LO}}^\gamma$ with $\dot{\Pi}_h$ calculated using the parametrization of [9] and $\dot{M}_{2,\text{LO}}^\gamma$ according to eq. (7). The relative error of r_{LO} is about $\pm 5\%$ [9] in the Q^2 range considered.

and note that $r_{\text{LO}}(Q^2)$ deviates significantly from the expected $r_{\text{LO}}(Q^2) = 1$, due to eq. (6), even in the asymptotic range $Q^2 \gg 1 \text{ GeV}^2$ where it is supposed to hold true.

The violation of eq. (6) in the aforementioned range of Q^2 can be traced back to the fact [6] that $\sigma_h(s)$ is not well approximated by its LO (or NLO) perturbative QCD value at $\sqrt{s} < 10$ GeV, i.e., that precocious scaling does not occur in the time-like region in contrast to the situation for deep inelastic scattering in the space-like region.

It is also interesting to test the validity of eq. (5) itself, for example in the case of the SaS 1D parametrization [4] of $f_{\text{LO}}^\gamma(x, Q^2 \geq Q_0^2)$ starting at $Q_0^2 = 0.36$ GeV² with the reasonable input $\alpha^{-1} M_2^\gamma(Q_0^2) = 0.551$. It should however be mentioned that any other parametrization of photonic densities (see, e.g. [10, 11]) will do as well. The evaluation of the right hand side of eq. (5) with $C = 0$, as appropriate in LO, yields however a significantly smaller value $\alpha^{-1} \Pi_h(Q_0^2) = 0.287 \pm 0.013$ using again the parametrization [9] for $\Pi_h(Q^2)$. Similarly, at $Q^2 \simeq 1$ GeV² where many modern photonic parton distributions are already compared to the data on $F_2^\gamma(x, Q^2)$ [4, 10, 12], one gets $\alpha^{-1} M_2^\gamma(1 \text{ GeV}^2) = 0.768$ whereas $\alpha^{-1} \Pi_h(1 \text{ GeV}^2) = 0.520 \pm 0.023$. At $Q^2 \gg Q_0^2$, $M_2^\gamma(Q^2)$ becomes obviously almost independent of its initial value, $M_2^\gamma(Q_0^2)$, and thus this region of Q^2 is unsuitable for confirming the validity of any conceived 'sum-rule' or photonic parton distribution.

The proposed relation in (5) can therefore not be used for selecting [3, 10, 11] the relevant input distributions $f_{\text{LO}}^\gamma(x, Q_0^2)$. This is of course in contrast to the situation with the well established energy-momentum sum-rule for $M_2^h(Q^2)$ in (1).

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