The elastic QCD dipole amplitude at one-loop

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We derive the analytic expression of the two one-loop dipole contributions to the elastic 4-gluon amplitude in QCD. The first one corresponds to the double QCD pomeron exchange, the other to an order $\alpha^2$ correction to one-pomeron exchange. Both are expressed in terms of the square of the recently derived triple QCD pomeron vertex and involve a summation over all conformal Eigenvectors of the BFKL kernel.

1. The bare (BFKL) pomeron in Quantum chromodynamics has been derived long ago [1] by resummation of the leading $(\alpha \log(1/x))^{n}$ contributions to the QCD perturbative expansion at small $x$, $\alpha$ being an arbitrarily fixed but small QCD coupling. This derivation allows to compute the 4-gluon elastic amplitude $A_Q(k, k'; Y)$, where $k, k'$ are the two-dimensional transverse components of the initial momenta, $Q$, the momentum transferred in the elastic reaction, and the total available rapidity interval $Y (\equiv \log(1/x)$ in DIS amplitudes). It is well-known that the bare BFKL pomeron has an energy dependence violating the Froissart bound, which is a basic consequence of unitarity and the existence of a minimum pion mass. It has been suggested that the unitarity constraint could be satisfied within the weak coupling regime of QCD through the computation of multipomeron exchange [2]. A first approximate attempt of calculating the double pomeron exchange has been performed [3]. Numerical estimates of the multi-pomeron contributions have also been presented [4]. However, the corresponding exact analytic expressions were not yet available.

The aim of our paper is to give the analytical expression of the one-loop dipole contribution to the elastic amplitude in the QCD-dipole picture of BFKL dynamics [5]. The QCD dipole formulation is known [6] to be equivalent at tree level to the $SL(2,C)$-invariant BFKL amplitude in terms of Feynman graphs. The idea is to use the dipole formalism as an effective theory defining the propagation and interaction vertices of two QCD Pomeron, which are compound states of reggeized gluons in the BFKL [1] representation.

A first attempt [5] has been made to compute the double-Pomeron contribution to the forward elastic amplitude as a first-order unitarity correction to the onium-onium total cross-section at large $Y$. However, this approach was using approximate expressions valid at large impact-parameter only, and thus the whole content of $SL(2,C)$ symmetry was lost. In the present paper we derive the exact expressions, without approximation, and valid for any $Q$ (thus for both forward and non-forward amplitudes).

Let us first introduce the $SL(2,C)$-invariant formalism for the 4-gluon elastic amplitude $A_Q(k, k'; Y)$ in the BFKL derivation. The solution of the BFKL equation is more easily expressed in terms of the Fourier transformed amplitude $f_Q(\rho, \rho'; Y)$ given by the relation

$$A_Q(k, k'; Y) = \frac{1}{(2\pi)^2} \int d^2 \rho d^2 \rho' \; \epsilon^{\rho(k - \hat{\rho}) - \rho'(k' - \hat{\rho}')} f_Q(\rho, \rho'; Y). \tag{1}$$

Using the $SL(2,C)$-invariant formalism, the solution of the BFKL equation reads [7]

$$f_Q(\rho, \rho'; Y) = \frac{1}{\alpha^2} \frac{1}{16} \int dh \; E_Q^h(\rho') E_Q^h(\rho) \; d(h) \; e^{\omega(h) Y}, \tag{2}$$

where the factor $\alpha^2$ comes from the coupling to incident dipoles. In equation (2), the symbolic notation $\int dh \equiv \sum_{n=-\infty}^{\infty} \int dv$ corresponds to the integration over the $SL(2,C)$ quantum numbers with $h = iv + \frac{1-n}{2}$. $E_Q^h(\rho)$ and $\omega(h)$ are, respectively, the $SL(2,C)$ Eigenvectors and Eigenvalues of the BFKL kernel [7]. The Eigenvalues read

$$\omega(h) = \frac{\alpha N_c}{\pi} \chi(h) \equiv \frac{\alpha N_c}{\pi} 2 \left\{ \Psi(1) - Re \left( \Psi \left( \left| \frac{1}{2} \right| \frac{1}{2} + iv \right) \right) \right\}, \tag{3}$$

where $\Psi \equiv (\log \Gamma)'$. The $SL(2,C)$ Eigenvectors are defined by

$$E_Q^h(\rho) = \frac{2\pi^2}{|\rho| b(h)} \int d^2 b \; e^{iQ \cdot b} \; E^h \left( b + \frac{b}{2} \right), \tag{4}$$

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with
\[ E^h \left( b - \frac{\rho}{2}, b + \frac{\rho}{2} \right) = (-)^{h-h^\prime} \left( \frac{\rho}{b^2 - \xi^2} \right)^h \left( \frac{\rho^\prime}{b^2 - \xi^2} \right)^{h^\prime}, \]
(5)
where \( h = 1 - h^\prime \), \( b \) is the 2-d impact-parameter, and
\[ d(h) = \left\{ \left[ \nu^2 + \frac{(n-1)^2}{4} \right] \left[ \nu^2 + \frac{(n+1)^2}{4} \right] \right\}^{-1}, \quad b(h) = \frac{\pi^n \nu^{h-h^\prime} 1 \gamma(1-h)}{2 - h} \]
(6)
with \( \gamma(z) = \frac{\Gamma(z)}{\Gamma(1-z)} \). Note that an analytic expression of the Eigenvectors \( E^h_Q(\rho) \) in the mixed representation has been provided [6] in terms of a combination of products of two Bessel functions. For simplicity, we did not include the impact factors corresponding to the external coupling (see [7,8]). Note also that the leading contribution of the amplitude (2) is the \( n=0 \) component which corresponds to the BFKL QCD Pomeron.

The final expressions to be obtained in this letter read:
\[ f_Q^{(P)}(\rho, \rho'; Y) = \alpha^4 \int dh E^h_Q(\rho')E^h_Q(\rho) \int dh dh_b \frac{g_{3P}(h, h_a, h_b)}{b(h_a)b(h_b)b(h)} \left( \chi(h) - \chi(h_a) - \chi(h_b)^2 \right), \]
(7)
\[ f_Q^{(P\oplus P)}(\rho, \rho'; Y) = \alpha^4 \int dh E^h_Q(\rho')E^h_Q(\rho) \int dh dh_b \frac{g_{3P}(h, h_a, h_b)}{b(h_a)b(h_b)b(h)} \left( \chi(h) - \chi(h_a) - \chi(h_b)^2 \right)^2, \]
(8)
where \( g_{3P} \) is the triple-QCD-Pomeron vertex (e.g., \( 1 \rightarrow 2 \) dipole vertex in the \( 1/N_C \) limit) which has been recently derived [9].

The two contributions correspond respectively to the one-dipole-loop correction to the BFKL Pomeron \( f_Q^{(P)} \) and to the two-Pomeron exchange \( f_Q^{(P\oplus P)} \). Indeed, the energy dependence of these contributions is fixed by the asymptotic behaviour in \( Y \) of formul\( a \)e (7,8), which we will show later to be, respectively, in the vicinity of the one-Pomeron and two-Pomeron intercepts. Notice that the forward amplitudes \( Q = 0 \) are obtained by replacing \( E^h_Q(\rho) \rightarrow (\rho)^{h-h}. \)

Formul\( a \)e (7,8) are the main results of this paper. We will comment these results and compare with the existing approximate evaluations in the further discussion.

**2. Let us come to the derivation of formul\( a \)e (7,8).** The formul\( a \)e of the general one-loop amplitude in the QCD dipole model can be written [3]:
\[ f_{\text{(one-loop)}}(\rho_0, \rho_1; \rho_0', \rho_1'; Y + y') = \frac{1}{2(2\pi)^8} \int_0^y dy' \int_0^{y'} dy'' \int_0^{\rho_0} d\rho_0 \int_0^{\rho_0'} d\rho_0' \int_0^{\rho_1} d\rho_1 \int_0^{\rho_1'} d\rho_1' \frac{n_2(\rho_0, \rho_0, \rho_0, \rho_1, \rho_0, \rho_1; \rho_0', \rho_0', \rho_0', \rho_1', y, y', y)}{T(\rho_0, \rho_0, \rho_0, \rho_1, \rho_0, \rho_1)}, \]
(9)
where \( \rho_0, \rho_1 \) are the transverse coordinates of one of the initially colliding dipoles (resp. \( \rho_0', \rho_1' \) for the second one), \( \rho_0, \rho_1, \rho_0, \rho_1, \rho_0', \rho_1', \rho_0', \rho_0', \rho_0', \rho_1 \) are the two interacting dipoles emerging from the dipole \( \rho_0, \rho_1 \) after evolution in rapidity (resp. \( \rho_0', \rho_1' \) for the second one), see figure 1a,b. It is important to notice that one has to introduce the probability distributions \( n_2(\rho_0, \rho_0, \rho_0, \rho_1, \rho_0, \rho_1; y, y', y) \) of producing two dipoles after a mixed rapidity evolution, namely with a rapidity \( y - y' \) with one-Pomeron type of evolution and a rapidity \( y \) with two-Pomeron type of evolution and integrate over \( y \). The interaction amplitudes \( T(\rho_0, \rho_0, \rho_0, \rho_1, \rho_0, \rho_1) \) and \( T(\rho_0, \rho_0, \rho_0, \rho_1, \rho_0, \rho_1) \) are the elementary two-gluon exchange amplitudes between two colorless dipoles, namely
\[ T(p_0, p_1, p_2; \rho_0) = \int d^2q \ e^{i \frac{\pi}{4} \left( \gamma_{\rho_0} + \gamma_{\rho_1} - \gamma_{\rho_2} \right)} f_\rho(p_0 - p_1, p_2, \rho_0' - \rho_1'; Y = 0). \]  

\[ n_2 \text{ obeys a mixed evolution equation [3,10] the solution of which is a mere extension of the one formulated in ref. [11]. It reads} \]

\[ n_2(\rho_0, \rho_1; \rho_0, \rho_1, \rho_0, \rho_1 | y - \bar{y}, \bar{y}) = \frac{\alpha N_C}{\pi} \int \frac{d\omega d\omega}{|\rho_0 \rho_1|^2} \int \frac{d\omega \omega}{|\rho_0 \rho_1|^2} \int \frac{d\omega}{\omega} a(h_a) e^{i \omega (h_a - h_b)} \]

\[ \times \int \frac{d^2p_\rho d^2p_\beta d^2p_\mu E^{h_\alpha}(\rho_0; \rho_1, \rho_0, \rho_1) E^{h_\gamma}(\rho_0; \rho_1, \rho_0, \rho_1) G_{a, \beta, \gamma}(h_a, h_\mu, h_\beta, h_\gamma), \]  

with

\[ \mathcal{R}_{a, \beta, \gamma}(h_a, h_\beta, h_\gamma) \equiv \int \frac{d^2r_0 d^2r_1 d^2r_2}{|r_0, r_1, r_2|^2} E^{h}(r_\gamma, r_1, r_2, r_0; r_\alpha, r_2, r_0) E^{h}(r_\gamma, r_1, r_2, r_0; r_\beta, r_0), \]  

where \( \rho = \rho_0 - \rho_1 \), \( \rho = \rho_0 - \rho_1 \), \( \rho = \rho_0 - \rho_1 \).

Conformal SL(2, C) invariance implies

\[ \mathcal{R}_{a, \beta, \gamma}(h_a, h_\beta, h_\gamma) \equiv [\rho_{a, \beta}]^{h_a - h_\beta} [\rho_{a, \gamma}]^{h_a - h_\gamma} [\rho_{a, \gamma}]^{h_a - h_\beta} g_{\beta, \gamma}(h_a, h_\beta, h_\gamma), \]

where \( g_{\beta, \gamma}(h_a, h_\beta, h_\gamma) \) is the triple Pomeron coupling in the 1/\( N_C \) limit obtained in the QCD dipole model, namely:

\[ g_{\beta, \gamma}(h_a, h_\beta, h_\gamma) = \int \frac{d^2r_0 d^2r_1 d^2r_2}{|r_0, r_1, r_2|^2} \frac{1}{|r_0, r_1, r_2|^2} \frac{1}{|r_0, r_1, r_2|^2} \frac{1}{|r_0, r_1, r_2|^2} \]

\[ \times \left[ \frac{1}{r_1} \right]^{h_a} \left[ \frac{1}{r_0} \right]^{h_\beta} \left[ \frac{1}{r_2} \right]^{h_\gamma} \left[ \frac{1}{r_1} \right]^{h_a} \left[ \frac{1}{r_0} \right]^{h_\beta} \left[ \frac{1}{r_2} \right]^{h_\gamma} \cdot \cdot \cdot \]  

From general arguments of conformal invariance, it is enough to calculate the forward elastic amplitude (\( Q = 0 \)), since the \( Q \)-dependence is given exclusively by the expansion over the conformal eigenvectors \( \int dh E_Q^h(\rho) \). This property can be directly checked by a tedious but explicit calculation, not reproduced here (see [12]).

It is convenient to introduce the double Fourier transform of the \( 1 \to 2 \) dipole distribution. One defines

\[ n_2(\rho_0, \rho_1, \rho_2; q_0, q_0, q_0 | y - \bar{y}, \bar{y}) \equiv \int e^{i \frac{\pi}{4} \left( \gamma_{\rho_0} + \gamma_{\rho_1} - \gamma_{\rho_2} \right)} \int \frac{d\omega d\omega}{|\rho_0 \rho_1|^2} \int \frac{d\omega}{\omega} a(h_a) e^{i \omega (h_a - h_b)} \]

\[ \times \int \frac{d^2p_\rho d^2p_\beta d^2p_\mu E^{h_\alpha}(\rho_0; \rho_1, \rho_0, \rho_1) E^{h_\gamma}(\rho_0; \rho_1, \rho_0, \rho_1) G(h, h_a, h_b), \]  

where \( q_0 \) (resp. \( q_0 \)) is the transverse momentum of the dipole \( \rho_0, \rho_1 \) (resp. \( \rho_0, \rho_1 \)) with respect the forward direction and \( 2b = \rho_0 + \rho_1 \) (resp. \( 2b = \rho_0 + \rho_1 \)), is the impact parameter.

Indeed, the computation of the forward amplitude (\( Q = q_0 + \bar{q}_0 = 0 \)) requires only the simpler solution for \( n_2(\rho_0, \rho_1, \rho_2; q_0, -q_0, \bar{q}_0, \bar{q}_0) \). Inserting formula (11) in definition (15) the integration over \( d^2p_\rho d^2p_\beta d^2p_\mu \) can be performed using the Eigenvectors (4) in the mixed representation. After removing the \( \delta \)-function of transverse momentum conservation, we get:

\[ n_2(\rho_0, \rho_1; q_0, q_0, \bar{q}_0, \bar{q}_0) = \frac{\alpha N_C}{\pi} \int \frac{d\omega d\omega}{|\rho_0 \rho_1|^2} \int \frac{d\omega}{\omega} a(h_a) e^{i \omega (h_a - h_b)} \]

\[ \times \int \frac{d\omega}{\omega} \omega (h_a + h_b + h - 1) (h_a - h_b + h) (h_a + h_b + h - 1) \gamma(2h) \]

\[ G(h, h_a, h_b) = \frac{g_{\beta, \gamma}(h_a, h_b, h_b)}{b(h_a) b(h_b)} \gamma(h_a + h_b + h - 1) \gamma(h_a - h_b + h) \gamma(h_a + h_b + h), \]

\[ \]
and, by definition, $\gamma(z) \equiv \frac{\Gamma(z)}{\Gamma(1-z)}$. To perform the eight integrals over transverse coordinates in the formula (9) for the one-loop amplitude we use the expression (11) for $n_2$ and introduce the double Fourier transform (cf. $\tilde{n}_2(q_1, q_2)$, see (15)) and the one-dimensional Fourier transform of the interaction amplitudes $T$, see (10). The integral over the intermediate impact parameters leads to $\delta$-functions of the various transverse momenta ($q_1, q_2$, etc...) while the remaining integrations over the differences $p_i - p_j$ give rise to $\delta$-functions over the conformal weights, according to the known orthogonality properties [7,6] of the $SL(2,\mathbb{C})$ Eigenvectors (4). This boils down to the equivalence theorem [6] between the BFKL and QCD dipole expression for the 4-gluon amplitude at tree-level.

All in all, the forward one-loop dipole amplitude reads:

$$f_{Q^0}(\rho, \rho'; Y) \equiv 4 \left( \frac{\alpha N_C}{\pi} \right)^2 \int dh d\bar{h} d\eta \, \frac{g_3 \rho \eta (h, \eta, h_0) \bar{g}_2 (h, \bar{h}, h)}{\bar{b} (h, \bar{h}) b (h)} \right)^2$$

\begin{equation}
\times \left[ \left( \frac{\rho}{\eta} \right)^{\lambda - \frac{1}{2}} \left( \frac{\bar{\eta}}{\bar{h}} \right)^{\lambda - \frac{1}{2}} \right] \int_0^y d\bar{y} \int_0^{\bar{y}} d\bar{y}' \int d\omega d\omega' d\omega_1 d\omega'_1 \left( \frac{\omega (h_a) + \omega (h_b) - \omega (\omega_1) - \omega (\omega_1)}{\omega (h_a) + \omega (h_b) - \omega (\omega_1) - \omega (\omega_1)} \right) \frac{e^{\omega \varphi + \omega_1 (\omega - \omega_1)}}{e^{\omega \varphi + \omega_1 (\omega - \omega_1)}}.
\end{equation}

(18)

The integration over $d^2 q$ yields a $\delta(h - h') \equiv \delta_{n_1, n_2} \delta(\nu - \nu')$. Integrating over $h'$ and noting that $\left| \frac{g_3 \rho (h, \eta, h)}{\bar{b} (h, \bar{h}) b (h)} \right|^2 = \left| \frac{g_3 \rho (h, \eta, h)}{\bar{b} (h, \bar{h}) b (h)} \right|^2$, we finally get

$$f_{Q^0}(\rho, \rho'; Y) \equiv 4 \left( \frac{\alpha N_C}{\pi} \right)^2 \int dh d\bar{h} \frac{g_3 \rho \eta (h, \eta, h_0) \bar{g}_2 (h, \bar{h}, h)}{\bar{b} (h, \bar{h}) b (h)} \right)^2$$

\begin{equation}
\times \left[ \left( \frac{\rho}{\eta} \right)^{\lambda - \frac{1}{2}} \left( \frac{\bar{\eta}}{\bar{h}} \right)^{\lambda - \frac{1}{2}} \right] \int_0^y d\bar{y} \int_0^{\bar{y}} d\bar{y}' \int d\omega d\omega' d\omega_1 d\omega'_1 \left( \frac{\omega (h_a) + \omega (h_b) - \omega (\omega_1) - \omega (\omega_1)}{\omega (h_a) + \omega (h_b) - \omega (\omega_1) - \omega (\omega_1)} \right) \frac{e^{\omega \varphi + \omega_1 (\omega - \omega_1)}}{e^{\omega \varphi + \omega_1 (\omega - \omega_1)}}.
\end{equation}

(19)

It is worthwhile to notice that the quantity between brackets in formula (19) is nothing but $E_{Q^0}(\rho') - E_{Q^0}(\rho)$ at $Q \equiv 0$, corresponding to the forward $SL(2,\mathbb{C})$ Eigenvectors in the mixed representation (cf. (4)). The generalization of (19) to the non-forward amplitude amounts to replace the quantity between brackets by the expression for arbitrary $Q$. This is the global conformal invariance of the 4-gluon amplitude. We have explicitly checked [12] this property which is thus an hint of the global conformal invariance of the 4-gluon amplitude maintained at one-loop level in the dipole formalism.

The integration over rapidity variables yields two different contributions depending on the sign of the quantity $\omega (h_a) + \omega (h_b) - \omega (h)$. Indeed for $\omega (h_a) + \omega (h_b) < \omega (h)$, the relevant poles are situated at $\omega = \omega_1 = \omega_1 = \omega_1 = \omega (h)$, leading to expression (7) which is associated with the single Pomeron dependence $e^{\omega \varphi} Y$. In the opposite case, namely $\omega (h_a) + \omega (h_b) > \omega (h)$, the relevant poles are situated at $\omega = \omega_1 = \omega_1 = \omega_1 = \omega (h) + \omega (h_b)$. The resulting amplitude is given by (8), which corresponds to the double-Pomeron energy behaviour $e^{\omega (h_a) + \omega (h_b)} Y$. Notice that either expression depends only on the sum $Y = y + y'$, as it should from longitudinal boost invariance.

3. Let us discuss the physical interpretation of the two components (7) and (8) of the one-loop dipole amplitude. At high $Y$, the behaviour of these components is driven by the dominant $n = 0$ Eigenvalue of the function $\omega$, see Eqn. (3), for the appropriate argument ($h$ or $h_a, h_b$, depending on the component). For the first component (7), the leading behaviour is fixed by the integration over $h$ in the vicinity of the BFKL Pomeron, namely $n \equiv 0, \nu = 0, \omega (h) \approx \omega (1/2) = \frac{4 \pi N_c \ln 2}{3}$. The integration over $h_a, h_b$ remains free provided $\omega (h_a, h_b) < \omega (1/2)$. A natural interpretation is that it corresponds to the one-loop dipole contribution $f_{Q^0}(\rho, \rho'; Y)$ to the BFKL Pomeron.

The dominant $Y$-behaviour of the second component (8) is fixed by integration over $h_a, h_b$, namely $n_a = n_b = 0$, and $\nu_a, \nu_b \approx 0, \omega (h_a) + \omega (h_b) \approx 2 \omega (1/2) = \frac{8 \pi N_c \ln 2}{3}$. This behaviour is typical of a double Pomeron contribution. The interpretation is the Pomeron-Pomeron cut unitarity correction $f_{Q^0, P^0}(\rho, \rho'; Y)$ following from the one-loop dipole...
amplitude. This second component was already estimated [3] in the forward direction. The form of the resulting amplitude was the same, with an unknown factor which has to be identified now with the triple Pomeron vertex squared. Another result of the exact calculation is that the integral over the conformal quantum numbers $h$ has to be performed on all values of the conformal spin $n$, and has no reason to be restricted to $n = 0$, as found in [3].

The resulting one-loop dipole contributions to the elastic 4-gluon amplitude are related to the triple Pomeron vertex function $|g_{3P}(h_h,h_h)|^2$ which also appears in the perturbative QCD derivation of hard diffraction [13]. On a QCD theoretical ground, the strength of hard diffraction is indeed connected to the saddle-point value $g_{3P} \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)$ which is known to be a large coefficient ($\approx 7700$, see [9]).

Comparing the one-loop results (7,8) with the tree-level amplitude (2), the dipole loop is of order $\hat{a}^2$. This is expected for a Pomeron-Pomeron cut contribution to the elastic amplitude (9). On the other hand, the dipole loop correction (7) to the BFKL Pomeron is thus only of next-next leading order. However it is useful to notice that the integrand may be quite large and a summation over the conformal quantum numbers has to be performed. A more quantitative estimate certainly deserves further study. The presence of the denominator $(\chi(h) - \chi(h_b))$ leads to series of double poles. For the component $f_{Q}^{(P)}$, the double poles are solutions of $\chi(h) \approx 8 \log 2$, value which corresponds to the saddle points at $h_0 = h_b \approx \frac{1}{2}$. The component $f_{Q}^{(P \otimes P)}$ corresponds to summation over both $h_0$ and $h_b$ satisfying the relation $\chi(h_0) + \chi(h_b) \approx 41 \log 2$. Note that this double summation may be related to the toroidal geometry of a one-loop string amplitude which often appears in string theory calculations as double summations and thus may give some new insight on the stringy nature of an effective theory [11].

In conclusion, we hope that the analytical calculation of the double BFKL pomeron exchange in the dipole formulation may open the way to a derivation of the unitarity corrections to the bare pomeron at weak coupling. Indeed, it has been remarked [2] that the full unitary elastic amplitude cannot be reconstructed from approximate evaluations of the multi pomeron exchanges. Analytical solutions of the multi pomeron amplitudes are thus likely to be evaluated following the same line as the present paper. Concerning the higher order corrections to the bare pomeron trajectories the situation is more intricate. Knowing that the next-leading order correction is large and of order $\hat{a}^{2}$ [14], the physical meaning and relevance of the dipole contribution $f_{Q}^{(P)}$ has to be clarified. These matters certainly deserve work to be done in the future.

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FIGURES

Figure 1-a
One-loop dipole amplitude in transverse coordinate space
The transverse coordinates of the incident dipoles ($\rho_1, \rho_1', \rho_2, \rho_2'$) are denoted by their indices together with the two interacting dipoles. The coordinates $\rho_2, \rho_2'$ refer to the points where two independent dipole evolutions start.

Figure 1-b
One-loop dipole amplitude in rapidity space The one-loop dipole amplitude is sketched in rapidity space. The initial dipoles $\rho_0, \rho_1, \rho_1', \rho_2'$ evolve with rapidity $y - \tilde{y}', y' - \tilde{y}$ with one-Pomeron type of evolution and then with two-Pomeron type of evolution during $\tilde{y}, \tilde{y}'$. Note that $y + y' \equiv Y$, the total available rapidity. After integration over $\tilde{y}, \tilde{y}'$ (see text) the resulting amplitude depends only on $Y$. Note that the amplitude $f^{(P)}$, formula (7) corresponds to $\tilde{y} = \tilde{y}' \approx 0$, while the two-Pomeron component $f^{(P\otimes P)}$, formula (8) corresponds to $\tilde{y} + \tilde{y}' \approx Y$. 

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