Towards the Solution of the Solar Neutrino Problem *

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We discuss various aspects of the solar neutrino spectrum distortion and time variations of fluxes. (i) Oscillations of neutrinos which cross the mantle and the core of the Earth can be parametrically enhanced. The parametric effect gives correct physical interpretation of the calculated day-night asymmetry. (ii) Solution of the $\nu_\odot$-problem in schemes with three and more neutrinos which accommodate explanations of other neutrino anomalies, in particular, the atmospheric neutrino anomaly, can lead to complicated distortion of the boron neutrino spectrum. (iii) The study of correlations between time (seasonal or day-night) variations and spectrum distortion will help to identify the solution of the $\nu_\odot$-problem.

1. Introduction

Specific time variations of signals and distortion of the energy spectrum (along with the charged to neutral current events ratio) are the key signatures of the neutrino physics solutions of the solar neutrino problem. Preliminary SuperKamiokande (SK) data [1] indicate that the effects (if exist) are not strong: $(1 - 2)\sigma$, i.e. at the level of present sensitivity. Study of correlations between time variations and distortion of the spectrum strengthens a possibility of identification of the solution. In this connection, I will discuss some aspects of the time variations of signals (sect. II), distortion of the energy spectrum (sect. III) and correlation between time variations and spectrum distortion (sect. IV).

2. What Happens With Neutrinos Inside the Earth?

The matter of the Earth can modify properties of solar, atmospheric and supernova neutrinos. Numerical calculations have been performed in a number papers previously [2], however, physics of the effects has been understood only recently.

The density profile of the Earth has two main structures: the core and the mantle. Density changes slowly within the mantle and the core but it jumps sharply by a factor of two at their border. It is known for a long while that in the first approximation one can consider the mantle and the core as layers with constant density. Neutrinos arriving at the detector at zenith angle $\cos \Theta > -0.84$ cross the mantle only. For $\cos \Theta < -0.84$, neutrinos cross three layers: mantle, core and again mantle.

Let us introduce $\Phi_m$ and $\Phi_c$ — the oscillation phases acquired by neutrino in the mantle (one layer) and in the core of the Earth:

$$
\Phi_i = 2\pi \int_{L_i}^{L_i^c} \frac{dL}{\Delta H_i}, \quad i = m, c,
$$

where $L_i = 2\pi/\Delta H_i$ is the oscillation length in matter, and $\Delta H_i$ is the level splitting (difference of the eigenvalues of two neutrino states). In the layer with constant density: $\Phi_i = \Delta H_i L_i$.

In [3] it was realized that for neutrinos which cross both the mantle and the core of the Earth the equalities

$$
\Phi_m \approx \Phi_c \approx \pi
$$

can be approximately satisfied, and this leads to significant enhancement of oscillations. (The phases in both layers of mantle are obviously equal.) The transition probability can reach

$$
P_{\text{max}} = \sin^2(4\theta_m - 2\theta_c),
$$

where $\theta_m$ and $\theta_c$ are the mixing angles in the mantle and the core respectively. $P_{\text{max}}$ can be

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much larger than \( \sin^2 2\theta_m \) and \( \sin^2 2\theta_c \) which correspond to maximal oscillation effect in one density layer.

This is a kind of enhancement of oscillations which has been introduced by Ermilova et al., [4] and Akhmedov [5] (see also [6]) and called the parametric enhancement of neutrino oscillations. The parametric enhancement occurs when the parameter of system (the density in our case) changes periodically and the period, \( r_f \), coincides with period of system.

The parametric enhancement of oscillations is due to certain synchronization of oscillation effects in the mantle and in the core. The frequencies of oscillations are different in the core and in the mantle. The enhancement occurs when the frequency change is synchronized with the frequency itself.

The condition (2) means that the size of the layer, \( L \), (in mantle or core) coincides with half of the oscillation length: \( L = l_M/2 \).

In the approximation of constant densities in the mantle and the core the resonance condition for phases (2) can be written as

\[
\Delta H_m L_m = \pi, \quad \Delta H_c L_c = \pi.
\]

(In general, the phase should be equal \( \pi(2k + 1) \), where \( k = 0, 1, 2, \ldots \) fixes the order of resonance.)

In 1987 E. Akhmedov [5] has considered the case of the “castle wall” density profile when the period of perturbation consists of two layers with constant but different densities. The Earth realizes, in a sense, the case of “1.5 period”.

The enhancement depends on number of periods (perturbations) and on the amplitude of perturbations which can be characterized by “swing” angle \( \Delta \theta \equiv 2\theta_m - 2\theta_c \). For small perturbations, large transition probability can be achieved after many periods. In the Earth the perturbation is large \( \Delta \theta \sim 2\theta_c \), and strong effect is realized even for “1.5 periods”.

Physics of the effect can be well understood from the graphical representation [6] based on analogy of the neutrino evolution with behaviour of spin of the electron in the magnetic field. Indeed, a neutrino state can be described by vector

\[
\vec{\nu} = (\Re \psi_\mu^1 \psi_s, \ \Im \psi_\mu^1 \psi_s, \ \psi_\mu^1 \psi_\mu - 1/2),
\]

where \( \psi_i \), \( i = \mu, s \) are the neutrino wave functions. (The elements of this vector are nothing but components of the density matrix.) Introducing vector:

\[
\vec{B} \equiv \frac{2\pi}{l_M} (\cos 2\theta_m, \ 0, \ \sin 2\theta_M)
\]

\((\theta_M \text{ is the mixing angle in medium})\) which corresponds to the magnetic field, one gets from the Schrödinger-like equation for \( \psi \) the evolution equation

\[
\frac{d\vec{\nu}}{dt} = (\vec{B} \times \vec{\nu})
\]

In medium with constant density \( (\theta_M = \text{const}) \), the evolution consists of \( \vec{\nu} \)-precession around \( \vec{B} \): \( \vec{\nu} \) is moves according to increase of the oscillation phase, \( \Phi \), on the surface of the cone with axis \( \vec{B} \). The direction of the axis, \( \vec{B} \), is determined uniquely by \( 2\theta_M \) (6). We will denote by \( \vec{B}_m \) and \( \vec{B}_c \) the axis in the mantle and in the core respectively. In fig. 1 we show a projection of the 3-dimensional picture on \( (\Re \psi_\mu^1 \psi_s, \ \psi_\mu^1 \psi_\mu - 1/2) \) plane [3].

The cone angle, \( \theta_{\text{cone}} \) (the angle between \( \vec{\nu} \) and \( \vec{B} \)) depends both on mixing angle and on the initial state. If an initial state coincides with \( \nu_\mu \), the angle equals \( \theta_{\text{cone}} = 2\theta_M \). The projection of \( \vec{\nu} \) on the axis \( z \), \( \nu_z \), gives the probability to find \( \nu_\mu \) in a state \( \vec{\nu} \):

\[
P \equiv \psi_\mu^1 \psi_\mu = \nu_z + \frac{1}{2} = \cos^2 \frac{\theta_z}{2}.
\]

Here \( \nu_z \equiv 0.5 \cos \theta_z \), and \( \theta_z \) is the angle between \( \vec{\nu} \) and the axis \( z \).

Let us consider an evolution of the neutrino which crosses the mantle, the core and then again the mantle and for which the resonance condition (2) is fulfilled. In the fig. 1, \( 2\theta_c < 2\theta_m < \pi/2 \), so that both axes \( \vec{B}_m \) and \( \vec{B}_c \) are in the first quadrant. (Actually, such a situation corresponds to mixing above the resonance \( 2\theta_c > 2\theta_m > \pi/2 \), when the axes are in the second quadrant. In fig. 1 for convenience of presentation we made redefinition \( 2\theta_c \rightarrow \pi - 2\theta_c, 2\theta_m \rightarrow \pi - 2\theta_m \) which does not change result.) The initial state, \( \vec{\nu}(1) \), coincides with flavor state, \( \nu_\mu \). (The picture corresponds to \( \nu_\mu - \nu_s \) mixing considered
Neutrino first propagates in the mantle and this corresponds to $\vec{v}$ precession around $\vec{B}_m = \vec{B}(2\theta_m)$. At the border between the mantle and the core the neutrino vector is in position $\vec{v}(2)$ (which corresponds to phase acquired in the mantle, $\Phi_m = \pi$). At the border the mixing angle changes suddenly: $\theta_m \to \theta_c$. In the core, $\vec{v}$ precesses around new position of axis, $\vec{B}_c \equiv \vec{B}(2\theta_c)$, with initial condition $\vec{v}(2)$. At the exit from the core, $\vec{v}$ will be in position $\vec{v}(3)$. When neutrino enters the mantle again, the value of mixing angle jumps back: $\theta_c \to \theta_m$. In the second layer of mantle, $\vec{v}$ precesses around $\vec{B}_m$ again. At the detector the neutrino vector will be in position $\vec{v}(4)$. After each jump of density the cone angle increases by the value of “swing” angle $\Delta \theta \equiv 2\theta_m - 2\theta_c$, thus enhancing the oscillations. According to fig. 1, a projection of $\vec{v}(4)$ on the axis $z$ equals

$$\theta_z = 2\theta_m + 2\theta_m + 2\Delta \theta = 2(4\theta_m - 2\theta_c).$$

Inserting this into (8) we get the survival probability $\cos^2(4\theta_m - 2\theta_c)$ which reproduces result in (3).
On the other hand, the MSW resonance can be considered as “the oscillation length resonance”: in the MSW resonance the oscillation length coincides for small vacuum mixing with refraction length.

Detailed interpretation of the effect in terms of the parametric resonance has been given in [8].

In the case of solar neutrinos the survival probability (due to the averaging and lost of coherence) depends on the transition probability $\nu_2 \rightarrow \nu_e$ inside the Earth, where $\nu_2$ is the heaviest mass eigenstate:

$$P \approx (1 - 2P_\odot)P_{2e}.$$  \hfill (9)

Here $P_\odot$ is the $\nu_e$ survival probability inside the Sun.

Graphical representation of the evolution of the solar neutrinos inside the Earth in the case of parametric resonance is shown in fig. 2. Now $2\theta_2 > \pi/2$ and $2\theta_m < \pi/2$, that is, the axis $\vec{B}_m$ is in the first and in the third quadrants, whereas $\vec{B}_c$ is in the second and in the fourth quadrants.

Such a situation corresponds to neutrino energies between the MSW resonance energies in the core and in the mantle. (It is easy to show that when $2\theta_m < 2\theta_c < \pi/2$ the oscillations are suppressed.) The initial state is $\vec{\nu}(1) = \nu_2$. Neutrino vector $\vec{\nu}$ first precesses around $\vec{B}_m$ and at the border between the mantle will be in position $\vec{\nu}(2)$. Then in the core, $\vec{\nu}$ precesses around $\vec{B}_c$, with initial condition $\vec{\nu}(2)$, and at the exit from the core $\vec{\nu}$ turns out to be in position $\vec{\nu}(3)$. In the second layer of mantle, the vector $\vec{\nu}$ precesses around $\vec{B}_m$ with initial condition: $\vec{\nu} = \vec{\nu}(3)$, and at the detector it will be in position $\vec{\nu}(4)$. According to fig. 2, a projection of $\vec{\nu}(4)$ on the axis $z$ equals

$$\theta_z = 2(4\theta_m - 2\theta_c) - 2\theta,$$  \hfill (10)

and consequently, $P_{2e} = \sin^2(2\theta_m - \theta_c + \theta)$ [7], where the difference from (3) is related to difference in the initial state.

One can see from figs. 1 and 2 that enhancement considered in [3] for $\nu_\mu - \nu_\tau$ oscillations and the one in [7] for $\nu_2 - \nu_e$ are of the same nature: the swing of axes leads to an enhancement of oscillations. The difference is in the initial state and in inclination of the swing angle.

Maximal transition probability (3) can be achieved when the parametric resonance condition is fulfilled exactly. The oscillation phases are functions of the neutrino energy and the zenith angle $\Theta$, and the two resonance conditions $\Phi_c(\Theta, E) = \pi$, $\Phi_m(\Theta, E) = \pi$ can be satisfied only for certain (resonance) values $\Theta_R$ and $E_R$. Deviations from $\Theta_R$ and $E_R$ weaken the enhancement. Thus the parametric resonance leads to appearance of the peak (parametric peak) in the energy or/and zenith angle dependence of the transition probability. The width of the parametric peak is inversely proportional to number of periods of density perturbation: $\propto 1/n$ [6]. (The bigger the number of periods the sharper the synchronisation condition.) In the case of the Earth the number of periods is small, $n \sim 1.5$, which means that the width of the peak is of the order one. Here the enhancement occurs even for significant detuning.

The probability $P_{2e}$ (as well as $P(\nu_\mu \rightarrow \nu_e)$) [2] has rather complicated structure with three large peaks: two of them correspond to the MSW resonance enhancement of oscillations in the core and in the mantle. The third peak is between the MSW peaks and its height is bigger than $\sin^2 2\theta_m$ and $\sin^2 2\theta_c$ at the peak energy fig. 3. The appearance of this third peak associated with resonance condition (2) is the consequence of parametric enhancement. Notice that certain interplay of the oscillation effects in the mantle and in the core leads not only to appearance of the parametric peak but it also modifies the MSW peaks in the mantle and in the core. The MSW peaks become suppressed in comparison with peaks from only one layer (core or mantle).

Although the parametric enhancement can be rather strong: $P_{2e} \sim 1$, the regeneration effect turns out to be suppressed by factor $(1 - 2P_\odot)$ (9). Recent changes in the solar model predictions [9,10] indicate that the suppression can be even stronger than it was supposed before. Indeed, the predicted flux of the boron neutrinos is now smaller (due to smaller cross section of $pB\nu_e$ reaction). This means that suppression of the boron neutrino flux due to oscillations should be weaker. We get $P_\odot \sim 0.5$ for the neutrino energy $E \sim 10$ MeV – in the center of the detectable
Figure 3. Transition probability for $\nu_e - \nu_\mu$ oscillations in the Earth (solid curve) as the function of $\delta \equiv \Delta m^2 / 4E$. Also shown are $\sin^2 2\theta_e$ (dashed curve) and $\sin^2 2\theta_m$ (dotted curve); vacuum angle: $\sin^2 2\theta = 0.01$, the zenith angle $\cos \Theta = -0.88$. (From [8]).

### 3. Beyond the Solar Neutrino Problem

The solar neutrino problem should be considered in general particle physics context which allows one also to accommodate solutions of other neutrino anomalies, and first of all, the atmospheric neutrino anomaly whose oscillation interpretation has received strong confirmation [11]. In fact, the results on atmospheric neutrinos make even more plausible the solution of the solar neutrino problem in terms of neutrino mass and mixing.

Clearly, the same oscillation channel can not explain both the solar neutrino and the atmospheric neutrino anomalies. One should consider mixing of three or even more neutrino species. This can have some impact on solutions of the $\nu_{\odot}$-problem. In particular, one may expect additional modifications of the neutrino energy spectrum. On the other hand, the solution of the $\nu_{\odot}$-problem may shed some light on the origin of other neutrino anomalies.

The distortion can be characterized by a sole slope parameter $s_e$ [12] defined as:

$$\frac{N_{\text{osc}}}{N_0} \approx R_0 + s_e T_e,$$

where $N_{\text{osc}}$ and $N_0$ are the numbers of events with and without oscillations correspondingly, $R_0$ is a constant, $T_e$ is the recoil electron energy in MeV, $s_e$ is in the units MeV$^{-1}$. In fig. 4 we show the slope parameter predicted by different two neutrino solutions of the $\nu_{\odot}$-problem [13]. The dots correspond to the best fit points of the total rates. The ellipses show the experimental result. Clearly, at the moment it is impossible to make discrimination among solutions.

Let us describe some possibilities beyond simple two neutrino case.

1. In the three neutrino schemes which solve both the solar and the atmospheric neutrino problems there is the hierarchy: $\Delta m^2_{12} \ll \Delta m^2_{13}$. In this case the heaviest state “decouples” from dynamics of the rest of system (leading to the averaged oscillation result) and the survival probability can be written as

$$P = \cos^4 \theta_{e3} P_2 + \sin^4 \theta_{e3},$$

where $\theta_{e3}$ describes the admixture of the $\nu_e$ in the heaviest state, and $P_2$ is the two neutrino survival probability which is characterized by $\Delta m^2_{12}$ and $\sin^2 2\theta_{12}$. For $\Delta m^2_{12} > 2 \times 10^{-3} \text{ eV}^2$ the CHOOZ [15] and BUGEY [14] experiments give strong bounds on $\theta_{e3}$, and therefore corrections due presence of the third neutrino are small. For $\Delta m^2_{13} < 10^{-3} \text{ eV}^2$, the mixing can be large thus leading to strong modification of the probability. Notice, however, that these changes do not improve the fit of the solar neutrino data. For small $\theta_{e3}$, the solutions of these two problems essentially decouple [16].

2. All three active neutrinos can be involved in the solar neutrino oscillations. This possibility
Figure 4. Deviation from an undistorted energy spectrum. The points with error bars show predictions from five possible $2\nu$ solutions: “SMA” stands for small mixing angle MSW conversion $\nu_e \rightarrow \nu_\mu$, “sterile” is the small mixing angle MSW conversion $\nu_e \rightarrow \nu_s$, VAC is the “just-so oscillations”, LMA is the large mixing angle MSW solution and LOW is the large mixing angle MSW solution with low $\Delta m^2$. The points correspond to the best fit points of the total rates in four experiments. The ellipses show $1\sigma$, $2\sigma$ and $3\sigma$ regions allowed by SK data. The errors in $R_0$ are large (not shown) so that all solution cross the ellipses in the horizontal scale. (From [13].)

Figure 5. The expected distortion of the recoil electron energy spectrum in the SuperKamiokande (solid lines) and SNO (long dashed lines) experiments for hybrid solution of the $\nu_\odot$-problem with parameters: $\sin^2 2\theta_{e\mu} = 0.5$, $\sin^2 2\theta_{e\tau} = 6 \times 10^{-4}$, $\Delta m^2_{31} = 8 \times 10^{-6}$ eV$^2$ and $\Delta m^2_{21} = 2 \times 10^{-10}$ eV$^2$. (From [17].)

can be naturally realized in the so called Grand Unification (GU) scenario [17]. Neutrino masses are generated by the see-saw mechanism; the neutrino Dirac mass matrix is similar to the mass matrix of the upper quarks at GU scale; the Majorana mass matrix of the RH neutrinos has weak mixing and linear mass hierarchy with the heaviest eigenvalue at the GU scale. This scenario predicts naturally $\Delta m^2_{23} \sim 10^{-5}$ eV$^2$ – in the range of the MSW solution of the solar neutrino problem and $\Delta m^2_{12} \sim 10^{-10}$ eV$^2$ in the “just-so” oscillation region. It also leads to relatively large $\nu_e - \nu_\mu$ mixing. The solar neutrinos undergo both the $\nu_e - \nu_\tau$ resonance conversion and the $\nu_e - \nu_\mu$ oscillations on the way from the Sun to the Earth. The interplay of both effects results in a peculiar (oscillatory) distortion of the boron neutrino energy spectrum [18]. The corresponding distortion of the recoil energy spectrum is shown in fig. 5. Notice that the curve has a kink whose position depends on $\Delta m^2$. This may be relevant for interpretation of the SK data.

3. The atmospheric neutrino problem can be solved by oscillations $\nu_\mu \leftrightarrow \nu_\tau$ which involve the sterile neutrino. This opens a possibility to rescue small flavor mixing in lepton sector in analogy with quark mixing. Now inside the Sun the electron neutrino is converted into the mix-
ture of the muon neutrino and sterile neutrino: \( \nu_2 = \cos \theta_{\text{atm}} \nu_\mu + \sin \theta_{\text{atm}} \nu_\tau \), where \( \theta_{\text{atm}} \) is the angle responsible for deficit of the atmospheric neutrinos. Correspondingly, properties of this solution of the \( \nu_\circ \rightarrow S \) problem are intermediate between properties of solutions based on conversion into pure active and pure sterile states. In particular, a distortion of the spectrum is stronger than in pure active case but weaker than in pure sterile case [3] (fig. 6).

\[
\nu_2 = \cos \theta_{\text{atm}} \nu_\mu + \sin \theta_{\text{atm}} \nu_\tau, \quad \theta_{\text{atm}} \text{ is the angle responsible for deficit of the atmospheric neutrinos.}
\]

Figure 6. The expected distortion of the recoil electron energy spectrum in the SuperKamiokande experiment. The solid line corresponds to pure \( \nu_\tau \rightarrow \nu_\mu \) conversion, dotted line is for \( \nu_\tau \rightarrow \nu_\mu \), the bold solid line is for the mixed case \( \nu_\tau \rightarrow \nu_\mu, \nu_\tau \) with \( \Delta m^2 = 5 \times 10^{-6} \text{ eV}^2 \), \( \sin^2 \theta_{12} = 8.8 \times 10^{-3} \) and \( \sin^2 2\theta_{\text{atm}} = 1 \).

4. In the supergravity, the hidden sector and the observable sector communicate via the Planck scale \( (1/M_P) \) suppressed interactions. In particular, a singlet field \( S \) from the hidden sector may have the coupling \( (m_{3/2}/M_P) \tilde{l} H S \), where \( m_{3/2} \approx 1 \text{ TeV} \) is the gravitino mass, \( H \) and \( l \) are the Higgs and the lepton doublets correspondingly. This interaction generates the \( \nu \rightarrow S \) mixing mass term

\[
m_{\nu_S} = \frac{m_{3/2}}{M_P} \langle H \rangle \sim 10^{-4} \text{ eV}, \tag{12}
\]

where \( \langle H \rangle \) is the VEV of \( H \) [19].

Consequences of this mixing depend on the mass of the scalar, \( m_S \). It turns out that for \( m_S \approx m_{3/2}/M_P \sim 3 \times 10^{-3} \text{eV} \) one gets \( \Delta m^2 \sim 10^{-5} \text{ eV}^2 \) and mixing angle \( \sin^2 2\theta \sim 10^{-2} \), so that the \( \nu_e \rightarrow S \) resonance conversion can solve the \( \nu_\circ \rightarrow S \) problem[19].

If \( m_S \) differs from the above value substantially, the other channel, e.g., \( \nu_e \rightarrow \nu_\mu \), can give a solution of the problem. In this case the \( \nu_e \rightarrow S \) mixing will modify the two neutrino effect. For \( m_S > m_2 \) \( (m_2 \sim 3 \times 10^{-3} \text{eV}) \), the \( \nu_e \rightarrow S \) mixing can lead to a dip in the non-adiabatic edge of the suppression pit at \( E \sim (m_S/m_2)^2 E_a \), where \( E_a \sim (0.5 - 0.7) \text{ MeV} \) is the energy of the adiabatic edge. This will manifest as a dip in the recoil electron spectrum and can be relevant for explanation of the spectrum observed by the SuperKamiokande. Also flavor composition of the neutrino flux will depend on energy. The flux of the beryllium \( \nu_e \) neutrinos is converted mainly to \( \nu_\mu \), whereas boron neutrinos are transferred both to \( \nu_\mu \) and \( S \). Correspondingly, an effect of the neutral currents is larger for low energies. Comparison of signals in BOREXINO and SNO experiments will check this effect.

4. Time Variations Versus Distortion

Existing solutions of the \( \nu_\circ \rightarrow S \) problem lead to specific correlations between time variations of signals and spectrum distortion. Therefore, using the data on spectrum distortion one can make predictions for time variations and vice versa. A study of these correlations strengthens the possibility to identify the solution.

1. For vacuum oscillation solution there is a strict correlation between a spectrum distortion and the amplitude of seasonal variations of neutrino flux [20]. The seasonal variations are due to ellipticity of the Earth orbit. The correlation originates from dependence of the oscillation probability \( P \) on the neutrino energy and distance to the Sun. Indeed, the phase of oscillations is
proportional to $\Phi \propto L/E$ which gives immediately
\[ \frac{dP}{dL} = -\frac{dP}{dE} \cdot \frac{E}{L}. \] (13)
Here $P^{-1}dP/dE$ is the slope of the neutrino spectrum distortion. According to (13), a positive slope, $dP/dE > 0$, is accompanied by decrease of probability with distance, so that the seasonal variations due to geometrical factor, $L^{-2}$, will be enhanced. In the case of negative slope, oscillations will suppress the seasonal variations due to geometrical factor.

The correlations can be expressed as correlations between the slope parameter for the energy spectrum of the recoil electrons (11) and the summer-winter asymmetry defined as
\[ A_e \equiv \frac{2N_W - N_S}{N_{SP} + N_A}. \] (14)
Here $N_W$, $N_S$, $N_{SP}$, $N_A$ are the numbers of events detected from November 20 to February 19, from May 22 to August 20, February 20 to May 21, from August 21 to November 19 respectively. It is convenient to describe the asymmetry due to oscillations by the parameter
\[ r_e \equiv \frac{A_e}{A_0} - 1, \] (15)
where $A_0$ is the asymmetry related to the geometrical factor. Obviously, $r_e = 0$ in the no-oscillation case; $r_e > 0$ ($r_e < 0$) corresponds to enhancement (damping) of the geometrical effect. Fig. 7 shows the $\mu_e - r_e$ correlation. For the best fit value of the slope (fig. 4) we get $r_e \sim 0.4$, so that one expects an enhancement of asymmetry. This can be checked after 4 - 5 years of the SK operation.

2. In the case of the MSW solution there is a correlation between the day-night asymmetry and spectrum distortion. This helps to disentangle the large and small mixing solutions of the problem [21]. For large mixing solution one expects strong day-night asymmetry and weak distortion of the spectrum. In contrast, for small mixing solution stronger spectrum distortion is accompanied by weak day-night effect. In fig. 8 the distortion of spectrum is characterized by deviation of the average electron kinetic energy $T_e$ from its standard value without oscillations. As follows from the figure the data favor a small mixing solution.

3. The correlation of the day-night effect and spectrum distortion allows one also to disentangle solutions based on conversion to active and to sterile neutrinos. Main difference comes from presence of the $\nu_\mu(\nu_\tau)$ contribution to $\nu_e$-scattering in the case of active neutrino conversion. This contribution, being proportional to $(1 - P(E))$, leads to smearing of the spectrum distortion. Therefore for the same values of parameters the distortion is stronger in the sterile case. In contrast, the regeneration effect is weaker in the sterile neutrino case. This is related to the fact, that in the $\nu_e - \nu_\mu$ case the effective potential (which describes matter effect) is approximately two times smaller than in the $\nu_e - \nu_\mu$ case. Thus for $\nu_e - \nu_\mu$ conversion one expects larger day-night asymmetry and smaller slope, whereas $\nu_e - \nu_\mu$ conversion leads to larger slope but weaker asymmetry. In fig. 9 we show projection of the $(\Delta m^2, \sin^2 2\theta)$ regions of small mixing solutions onto D/N-asymmetry - slope plot which illustrates the correlation [22]. The correlation is solar model dependent. For the model BP95 [23]
Figure 8. The day-night asymmetry - spectrum distortion plot. The distortion is characterized by the mean kinetic energy deviation. In panel (b) the regions show the map of the small (S) and large (L) mixing solutions at 95 % C. L. in the mass-mixing plane (panel(a)). (From [21].)

the regions corresponding to two channels of conversion are well separated. However in the models with smaller boron neutrino flux (see e.g. [9], [10]) both the slope and the D/N asymmetry become smaller and the two regions overlap. The identification of solutions (using this correlation) will be difficult. Notice that for small original boron neutrino flux the D/N asymmetry is negative in whole region of the $\nu_{e} - \nu_{s}$ solution and in part of the $\nu_{e} - \nu_{\mu}$ region. This is related to the fact, that for a small flux a required oscillation suppression should be weak, so that the survival probability $P_{\odot} > 1/2$ (see (9)). Moreover, due to additional contribution from $\nu_{\mu}$ the $\nu_{e} - \nu_{\mu}$ solution requires stronger suppression. The two solutions can be also distinguished by measurements of the neutral current effect in SNO.

Figure 9. The slope - D/N-asymmetry plot [22]. The regions of predictions of small mixing MSW solutions: $\nu_{e} - \nu_{\mu}$ (bold lines), $\nu_{e} - \nu_{s}$ (thin lines). Solid lines correspond to solar model BP95, dashed lines are for BP95 model with diminished (by factor 0.7) boron neutrino flux.

5. Conclusion

Oscillations of neutrinos crossing the core of Earth can be parametrically enhanced. This leads to appearance of the parametric peak in the oscillation probability as function of neutrino energy. The parametric enhancement can be relevant for solar and atmospheric neutrinos as well as for neutrinos from supernova. Strong enhancement of the regeneration probability for solar neutrinos which cross the core is due to the parametric resonance.

Solution of the solar neutrino problem should be considered in wider particle physics context which allows one to explain, e.g., the atmospheric neutrino problem. Under certain conditions the
two problems “decouple” and the solution is still reduced to simple two neutrino case. However, in a number of schemes one gets modification of the simple two neutrino effect. This can manifest as complicated distortion of the neutrino (and the recoil electron) energy spectrum and also can lead to a peculiar change of the flavor composition of the solar neutrino flux with energy.

Precise measurements of spectrum can reveal physics “beyond the solar neutrino problem”. One possibility is the Planck mass suppressed couplings of neutrinos with particles from the hidden sector.

Different solutions of the solar neutrino problem lead to specific correlations between the spectrum distortion and time variations of fluxes. This can be used to distinguish solutions.

Recent experimental data and new calculations of the fluxes require smaller oscillation effects (smaller mixing angles etc.), so that the identification of the solution becomes more difficult.

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FLUX-DEPENDENT INFORMATION
[ SKam(374 d) + Kam + Cl + Ga total neutrino rates ]

\( \delta m^2 \) (eV\(^2\))

95% C.L.

\( \sin^22\theta/\cos2\theta \)

S = small mixing MSW
L = large mixing MSW

NO OSC.

FLUX-INDEPENDENT INFORMATION
[ SuperKamiokande day-night and energy spectra ]

\( \frac{N-D}{N+D} \)

\( \frac{\Delta\langle T \rangle}{\langle T \rangle} \times 10^{-2} \)

Skam 374 days (±1σ total)
Skam 306 days (±1σ total)

NO OSC.