


Introduction

A model for the X-ray emission from the CMB relics

Abstract

A model for the non-thermal X-ray emission from the CMB relics is presented. The model is based on the assumption that the CMB relics are heated by the interaction of the CMB with the cosmic microwave background radiation. The model is tested by comparing the observed X-ray emission with the predicted emission from the CMB relics. The agreement between the observed and predicted emission is good, which supports the model. The model predicts that the X-ray emission from the CMB relics should be detectable by future X-ray telescopes. The implications of this model for the understanding of the early universe and the evolution of the universe are discussed.
2. KELVIN-HELMHOLTZ INSTABILITY OF THE SHEAR ZONE

For the purpose of a stability analysis, we model the equilibrium state of the equatorial shear zone as a uniform plane-parallel slab of unmagnetized relativistic fluid, with thickness $2H$ in the $z-$direction, moving with velocity $u_e$ with respect to a stationary background medium. The exterior fluid also has a relativistic equation of state, as well as containing a uniform magnetic field oriented perpendicular to the flow velocity, $B_0$. The assumption of a negligible magnetic field in the equatorial zone is motivated by its identification with the neutral sheet in the pulsar wind. We parameterize the field strength in the high-latitude region by its beta parameter with respect to the external gas pressure, $\beta \equiv 8\pi P_{\infty} / B_0^2$. Pressure balance in the $z-$direction then requires the pressure in the unmagnetized slab to satisfy $P_{\infty} = P_{\infty} + B_0^2 / 8\pi$. Note that the particle densities in both fluids are irrelevant since the inertia is dominated by the relativistic energy density.

The Kelvin-Helmholtz (K-H) instability for a slab geometry with various physical conditions has been discussed extensively in the literature (e.g., Gill 1965; Ferrari, Massaglia, & Trussoni 1982; Payne & Cohn 1985; Hardee & Norman 1988; Hardee et al. 1992). Since the plane-parallel model is intended to approximate axisymmetric radial flow, we consider only modes propagating parallel to the flow velocity. Given perturbations of the form $f(z) \exp[\sqrt{\omega - k_x z}]$, it is straightforward to derive the dispersion relation for the specified conditions. Defining the dimensionless quantities $w \equiv \omega H / c$, $\nu \equiv k c / \omega$, and

\[
\begin{align*}
\kappa_1 &= \frac{3(1 - \nu u^2) - (\nu - u)^2}{1 - u^2}, \\
\kappa_2 &= \frac{2(1 + 2\beta)}{(3 + 2\beta)}, \\
A &= \frac{(1 - u^2)(2\beta + 1)}{2(\beta + 1)(1 - \nu u)^2 \kappa_2},
\end{align*}
\]

we obtain

\[
\tan (w \kappa_1) = -A \kappa_1
\]

for the modes that are antisymmetric (AS) about $z = 0$ (kink modes), and $\cos (w \kappa_1) = A \kappa_1$ for the symmetric (S) or pinch modes. The form of the dispersion relation given above is analogous to that introduced by Gill (1965) and used by many subsequent authors. For example, to relate our variables to the nonrelativistic magnetohydrodynamic limit studied by Hardee et al. (1992), we make the following identifications with their parameters $\beta_{\infty}, \beta_{\infty}, \chi_{\infty}, \chi_{\infty}, k_1 \rightarrow \beta_{\infty} H / w$, $k_2 \rightarrow i \chi_{\infty} H / H$, $A \rightarrow - (\chi_{\infty} H) / (\chi_{\infty} H / \kappa_2)$. To satisfy boundary conditions at $|z| \rightarrow \infty$, the real part of $w \kappa_2$ must be positive. Note that special relativistic effects are included in the derivation.

The mode structure predicted by eqs. (1)-(4) is qualitatively similar to that found in nonrelativistic treatments of slab jets, both with and without magnetic fields (Hardee & Norman 1988; Hardee et al. 1992). We are interested in the spatial growth of modes with real driving frequencies, so we set $\omega$ (and $w$) real and $k = k_R + ik_I$, $k_I > 0$ then corresponds to a growing mode. In addition to the fundamental AS and S modes which are unstable at all flow velocities, internal reflection modes appear when the flow is sufficiently supersonic (and magnetosupersonic; see Hardee et al. 1992). However, these latter modes have phase and group velocities that exceed the effective sound speeds in both the unmagnetized $(c / \sqrt{3})$ and magnetized $(c / \sqrt{3})$ regions. If Tanvir et al. (1997) are correct that the wisps move outward at speeds $\lesssim c / 3$, then the pattern speed is highly subsonic.

We will assume (based on the appearance of the wisps; see § 3) that we are dealing with the fundamental AS mode in the long-wavelength limit, $|k H| < 1$. We can then approximate the dispersion relation by $A + w \approx 0$. These modes correspond to side-to-side displacements — ripples — of the slab. Since the $c$-folding length of the instability, $k_R^{-1}$, is a monotonically decreasing function of $\omega$, all wavelengths shorter than a certain value will have reached nonlinearity at any given radius in the wind. We will associate the wisps with the longest wavelength (i.e., lowest frequency) modes that have room to grow, since these are expected to dominate the large-scale structure of the ripples. Thus, the dominant modes have $k_t \sim \sim c / r$, where $r$ is the radius at which the ripples appear.

Phase velocities $v_p = \omega / k R \equiv \nu \kappa_1$ of very-long-wavelength modes vary as $v_p \propto (k H / H)^{1 / 2} \propto \omega^{1 / 3}$, i.e., they decrease with increasing wavelength and decreasing frequency. The observed pattern speed of a weakly unstable linear wave packet is the group velocity, $\partial \omega / \partial k R$, but the physical significance of this quantity is ill-defined for modes with $k_t \sim k R$, as is the case here. Moreover, we are considering modes that have grown to nonlinear amplitudes, by which point the pattern speeds may have changed. In their numerical simulations of nonrelativistic slab jets, Norman & Hardee (1988) find that the linear phase velocity gives a more accurate estimate of the nonlinear pattern speed than does the group velocity, and we will adopt this assumption here. Therefore, if the high-latitude wind has decelerated to $c / 3$ at the point at which we see the wisps, then the equatorial wind would have to be traveling with speed $uc$ significantly faster than $c / 3$ (from a relativistic point of view) in order to obtain the observed pattern speeds of $\sim c / 3$. Note that the standard spherical shocked wind model for the Crab Nebula (Rees & Gunn 1974) predicts decreasing subsonic velocities $\sim (c / 3)(r / \rho)^{-1 / 2}$ outside the shock radius, $r_s$. (The exact radial dependence is sensitive to the two-dimensional flow pattern.) Even if the equatorial wind speed significantly exceeded $c / 3$, it would not necessarily have to be highly supersonic (i.e., $> c / \sqrt{3} \sim 0.58c$). Figure 1 shows $k R H$ (solid lines) and $k H R$ (dashed lines) as a function of slab velocity $u$, for negligible $\beta \rightarrow \infty$ (light lines) magnetic fields, with the phase velocity fixed at $1 / 3$. The magnetic field strength has little effect on the curves. Note also that $k H$ is already quite small ($< 0.15$) for transonic flow ($u = 0.58$), and decreases steadily for higher flow velocities.

Setting $\nu \kappa_1 = 1 / 3$ corresponds to the case in which the outflow speeds of the high-latitude wind can be neglected. If the high-latitude wind were still moving outward at a sizable fraction of $c / 3$ in the wisp region, then similar net pattern speeds could be attained with a slower equatorial wind. In this case, it would be easier to obtain very small values of $k \theta H$ which are desirable for producing sharp, intense wisp features (see § 3). Note, however,
that the limiting case of an equatorial wind moving only slightly faster than the high-latitude wind is not conducive to producing wisps with the observed spacing through K-H instability, since the modes would be advected outward before they had time to grow.

At the end of the epoch studied by Tanvir et al. (1997), there were two bright wisps roughly 70” and 150” from the pulsar. If we argue that the wavelength 2π/kR of the dominant wisp mode is approximately r/2 and that this corresponds to ~ 6 growth lengths (consistent with fact that k1 ≈ k2), then we have kRr ≈ 4π. From the above estimates based on the theory of K-H instability, we expect kRr ≈ 0.1 for u ≥ 0.6, implying that the modes potentially have aspect ratios r/R ≈ 1.6. The ratio can be much larger if the equatorial wind is faster than 0.6c or if the speed of the high-latitude wind is taken into account. These values are in the range required to obtain the observed narrowness and brightness contrasts of the wisps from viewing angle effects, as we shall now describe.

3. NARROW, BRIGHT FEATURES FROM A RIPPLED SHEET

Two of the most striking features of the wisps are their narrowness and their brightness contrast with respect to the local background. Assuming that the wisps are cylindrical filaments, Hester et al. (1995) estimated their equipartition (i.e., minimum) pressures at 28–54 times the mean equipartition pressure in the nebula (P_eq) ~ 7 × 10^{-3} dyne cm^{-2}; Trimble 1982). If the wisps were highly overpressured with respect to the local medium, then they would have to be out of dynamical equilibrium with their surroundings, i.e., they would have to be shocks. In our observing frame these putative strong shocks are traveling downstream at ~ c/3, but in the fluid frame they would have to be traveling upstream at somewhat more than the sound speed, c/3. Applying the relativistic velocity addition law, this would imply that the equatorial wind would have to be traveling outward at highly supersonic speed, ≥ 0.8c. This is difficult to reconcile with the hypothesis that the wisps exist in the postshock region where the wind is subsonic or, at best, mildly supersonic.

Even if the equatorial wind is highly supersonic in the wisp region, it is hard to imagine what would create a set of shocks that always move outward at ~ c/3. Suppose the high-latitude wind went through a shock at r_s, but the equatorial wind punched through and remained supersonic. The sudden jump in pressure would drive oblique shocks into the equatorial wind as it crossed r_s with the obliquity adjusted so that the shock fronts track the position of r_s. Thus, these shocks would not move systematically outward unless r_s also moved systematically outward, which seems unlikely. Similarly, internal shocks caused by intrinsic fluctuations in the wind should exhibit both inward and outward motions.

Fortunately, the high surface brightnesses can be explained without resorting to large overpressures. First, the mean pressure in the Crab may well be several times larger than the equipartition pressure. Moreover, the local pressure in the center of the nebula may be several times the nebular mean, due to magnetic tension effects (Kennel & Coroniti 1984; Begelman & Li 1992). A simple theory of the wind termination shock gives the pressure just downstream from the shock as P_s = 9 × 10^{-6}(L_u/3) × 10^{38} erg s^{-1}/(r_s/0.1 pc)^{-2} dyne cm^{-2}, where L_u is the wind power. Recall that we are associating the wisps with K-H modes that grow downstream from the shock. Since the inner wisp is situated at ~ 0.1 pc, the actual radius of the shock might be 2–3 times smaller, suggesting that the ambient pressure in the wisp region could be as much as ~ 4–10 times larger than (P_eq). Second, Doppler boosting will cause one to overestimate the equipartition pressure in the approaching wisps by a factor ~ D^{1.5}, where D is the Doppler factor (Heinz & Begelman 1997). For wind speeds ~ 0.5c and viewing angles ~ 30°, this leads to a further decrease in the intrinsic equipartition pressure by a factor ~ 2.

The remaining factor ~ 3–6 in inferred overpressure can be explained away if we interpret the wisps not as cylindrical filaments but as thin radiating sheets, viewed nearly edge-on. This geometry can arise naturally from the nonlinear development of antisymmetric Kelvin-Helmholtz modes in a thin equatorial wind, provided that kRr is sufficiently small and the amplitude of the ripples grows sufficiently large. If the equatorial wind has uniform emissivity and the depth of our line of sight through the wind is 2πr/R ≫ H, then the inferred equipartition pressure goes down by a factor ~ c/2. Thus, a depth-to-width ratio of c/2 ~ 10–30 is probably sufficient to explain the observed surface brightness in the wisps. These ratios are consistent with the predictions of the Kelvin-Helmholtz instability, provided that the viewing geometry is favorable.

This simple model can account not only for the brightness enhancement in the wisps, but also for their narrowness and their apparent confinement to a pair of arcs along the near and far sides of the wavefront. To illustrate this, we consider a ripple axisymmetric slab with upper and lower surfaces (at some fixed time) given by

$$ z_\pm = H(\pm 1 + a \sin ks) $$

where $s^2 = (x^2 + y^2)^{1/2}$ in terms of Cartesian coordinates defined in the equatorial plane of the slab. Now suppose we observe the slab along a line of sight in the $x' - z'$ plane, at an angle $\theta$ ~ 30° from the slab’s equator. We define observer’s coordinates (x, y, z) according to

$$ x' = x \cos \theta - y \sin \theta, \quad y' = y, \quad z' = z \sin \theta, \quad (x, y, z) \text{ lie in the sky-plane and } z \text{ denotes position along the line of sight.} $$

Our line of sight enters and exits the slab at

$$ z_{\pm} = H \cos \theta \left[ 1 \pm \sin \left( k r_{\|} - \frac{k x \cos \theta}{r_{\|}} z_{\pm} \right) \right], $$

where $r_{\|} = (x'^2 + y'^2)^{1/2}$ and we have neglected terms of second- and higher order in $z_{\pm}/r_{\|}$. If the slab radiates uniformly, its brightness is proportional to the length of the line of sight between the entrance and exit points, ($z_+ - z_-$). Since the unperturbed slab ($a = 0$) has uniform brightness 2Hcsc $\theta$, we can quantify the enhancement due to rippling through the multiplicative factor

$$ \Delta \equiv (z_+ - z_-) \sin \theta/2H. $$

Defining the quantities $s \equiv (z_+ + z_-) \sin \theta/2H$ and $q \equiv kH \cos \theta/(r_{\|} s \sin \theta)$, we can determine $\Delta$ from the following pair of equations:

$$ \Delta = 1 - a \sin(q \Delta) \cos(k r_{\|} - q s), $$

$$ s = a \cos(q \Delta) \sin(k r_{\|} - q s). $$
For fixed $a$ and $q$ (corresponding to a fixed position angle on the sky, relative to the center of the slab), maximum values of $\Delta$ occur for $s = 0$ and $\sin k\eta = 0$, and satisfy $\Delta_m = \pm a \sin (q \Delta_m)$. For a given value of $\Delta_m$, there are an infinite number of contours in the $a - q$ plane which have identical shapes but are displaced from one another according to $q_n(a) = q_0(a) + n(\pi / \Delta_m)$, where $n$ is an integer. Figure 2 shows a set of contours corresponding to $q_0$, which is the relevant branch if the wisps have simple brightness contours with a single maximum at $y = 0$. For ripples of fixed amplitude $a$, the maximum possible enhancement, $\Delta = a + 1$, occurs for $\cos (qa + q) = 0$.

From the definition of $q$, we see that the maximum value of $q$ for fixed $x$ occurs at $y = 0$. Thus, if the wisps are to be brightest on the leading and trailing sides of the wavefront (i.e., at $y = 0$), we must have $\partial \Delta_m / \partial q > 0$. For the $q_0$ branch, this corresponds to the condition $kB \cos \theta < \pi / (2a + 2)$. Figure 3 shows a representative surface brightness plot for this case.

Narrower structures with higher brightness contrasts are possible if the emissivity of the equatorial zone is non-uniform. In particular, we might expect the slab to be edge-brightened, since the magnetic field strength will decline towards the center of the slab (since it reverses sign in this region), and the acceleration of relativistic electrons might also be greatest in the regions of strong shear and large field gradients. If the emissivity is confined to a fraction $\xi$ of the slab’s thickness, then the maximum surface brightness contrasts will be enhanced by an additional factor of $\sim \xi^{-1}$. We would also expect to see a doubling of the wisp structures, for which there may be evidence in the WFPC2 image presented by Hester et al. (1995; Fig. 8).

4. DISCUSSION

I have proposed that the moving wisps near the center of the Crab Nebula can be interpreted as long-wavelength Kelvin-Helmholtz instabilities in the equatorial zone of the shocked pulsar wind. This interpretation explains why the wisp pattern persists over time and moves outward from the pulsar, as revealed in recent observations (e.g., Tanvir et al. 1997), instead of remaining stationary or oscillating in position. It also explains the apparent confinement of the wisps to a plane oriented perpendicular to the inferred pulsar rotation axis. The observed pattern speeds of $\lesssim c / 3$ are consistent with equatorial outflow velocities of $\gtrsim 0.6c$, shearing against a high-latitude flow with speeds of much less than $c / 3$. If the speed of the high-latitude flow is taken into account, constraints on the equatorial flow speed are relaxed somewhat.

The wisps’ appearance as narrow, high-intensity arcs of emission need not imply that they are dissipative structures. They could simply be geometric artifacts resulting from the projection of our line of sight through the ripples, radiating slab. Such an interpretation is workable provided that the dominant K-H modes have sufficiently long wavelengths ($kB \ll 0.05$) and large amplitudes. At such large amplitudes, nonlinear effects (secondary instabilities, etc.) may be important, and the actual shapes of the ripples are likely to be far from the simple sine wave I assumed in § 3 for illustrative purposes. Dissipative processes (e.g., second-order Fermi acceleration, reconnection) could be associated with the complex nonlinear flow so one cannot rule out the possibility that localized dissipation contributes to the appearance of the wisps. To reconcile the observed spacing of the wisps with the geometric model for their appearance would require the sheet to be thin indeed, $H / r \lesssim 0.01$, where $r$ is the radial distance of the wisps from the pulsar. Note that our model for the Kelvin-Helmholtz instability is simplified by assuming a “top-hat” velocity profile rather than a smooth gradient, but this detail should not be important for such small values of $kB$.

A more substantial worry is the stringent requirement on the slab's emissivity compared to that of its surroundings. To avoid the wisps’ being drowned out by a bright background, the equatorial zone’s intrinsic emissivity would have to be many times larger than that of the high-latitude regions, since the latter occupy so much more volume. At
Fig. 2.— Contours of maximum brightness enhancement due to viewing geometry through a uniformly emitting rippled slab, in the \( a - q \) plane. \( a \) is the amplitude of the ripples in units of slab thickness \( H \) and \( q \) is the product of scaled wavenumber \( kH \) with geometric factors. Contours are labeled with the value of the brightness enhancement factor, \( \Delta_m \).

Fig. 3.— Surface brightness map of a rippled slab with \( kH = 0.05, a = 15 \), and \( \theta = 30^\circ \). Displacements on the sky are scaled in units of \( kH \) and only one quadrant is shown. The nested contours correspond to brightness enhancement factors of 10 and 15 relative to the background; contrast can be increased by decreasing \( kH \) and increasing \( a \). Compare the morphology to Figures 1 and 2 in Tanvir et al. (1997) and Figure 8 in Hester et al. (1995).
present. I have no explanation for why this should be the case, although one might suspect that particle acceleration is especially efficient near the neutral sheet, where the magnetic fields are highly dynamical and reconnection may be occurring.

I have tentatively identified the equatorial zone with the "neutral sheet" across which the direction of the toroidal magnetic field reverses. The physical thickness of the shear layer ($H$) is presumably determined by physical processes occurring in the field-reversal region. Why there should be shear between the equatorial zone and the high-latitude regions of the wind, especially downstream of the main wind shock where one might expect both regions to be decelerated to $c/3$? One possibility is that the equatorial zone has a higher ram pressure upstream of the shock, because of uncertain physical processes near the base of the pulsar wind. (Note that the ram pressure has components due to both the particle flux and the Poynting flux.) When the high-latitude wind shocks it slowed down to $\sim c/3$ and subsequently decelerates further, but the plasma in the equatorial zone pulses through the shock radius with little change in velocity. The equatorial zone then comes into pressure equilibrium with the high-latitude shocked wind through a system of oblique shocks, which slows it somewhat but still can leave substantial shear.

It is also possible for the equatorial wind to accelerate relative to the high-latitude wind, even if both regions initially co-move at $\sim c/3$ after passing through a strong shock upstream of the wisps. The reason is that the high-latitude wind is more highly magnetized than the plasma in the equatorial zone where, effectively, the magnetic field vanishes. If the high-latitude wind maintains a well-organized toroidal field downstream of the shock, then the total pressure (gas + magnetic) in the decelerating high-latitude wind will actually decrease with radius, due to the effects of magnetic tension (Begelman & Li 1992). The equatorial zone will try to maintain pressure balance with the high-latitude wind but, because its magnetization is much smaller, it will respond to the pressure gradient by accelerating. The bulk Lorentz factor in adiabatic flow of an unmagnetized relativistic fluid varies as $\rho_{\text{rest}}^{-1/4}$. Thus, in order for the equatorial zone to accelerate from $v_0 = 1/3$ to $v = 0.5, 0.6, or 0.7$, the pressure would have to drop to $0.71, 0.52, or 0.33$ of its value immediately outside the shock. Such pressure drops are possible provided that the magnetic pressure in the high-latitude wind is not much smaller than the gas pressure (Begelman & Li 1992).

A near-equipartition magnetic field in the wisp region is seriously at odds with widely accepted theoretical models of the Crab Nebula (Rees & Gunn 1974; Kennel & Coroniti 1984; Emmering & Chevalier 1987), which imply that the magnetic pressure in the wisp region is only a couple of percent of the gas pressure. (This is consistent with a ratio of Poynting flux to kinetic energy flux in the pre-shock wind of $\lesssim 0.003$.) However, this inference depends critically on the assumption that an axisymmetric toroidal field structure is maintained throughout the nebula. Begelman (1998) has pointed out that such a field geometry is highly unstable, and that the rearrangement of the field will seriously weaken any theoretical upper limits on the wind's magnetization. One cannot rule out the possibility that the shocked wind is sufficiently magnetized to sustain a modest pressure drop, before reaching radii beyond which instabilities wash out further pressure gradients.

Because of the many uncertainties regarding the structure of the Crab pulsar wind and its nebular environment, it may prove difficult to devise definitive observational tests of the model presented in this paper. However, if the model is correct qualitatively, we expect the following observational trends:

1. An inverse relationship between wisp spacing and pattern speed. If the wisps represent the nonlinear development of the Kelvin-Helmholtz instability, we should not be surprised to find fluctuations in the wisp spacing and pattern speed, even if the wind and main shock system are relatively steady. Since the wavelength and phase velocity are inversely correlated for long-wavelength K-H modes (with $v_p \propto \lambda^{-1/2}$ asymptotically), there should be a qualitatively similar relationship between wisp spacing and pattern speed. This is probably the most distinctive prediction of this model, compared to others. (For example, the pattern speed of magnetosonic waves should be independent of wavelength.)

2. Possible lack of evidence for localized dissipation in the wisps. If the wisps were dissipative structures, one might expect to see evidence of fresh particle acceleration (e.g., flattening of the synchrotron spectrum) associated with the brightest features. If the wisps are instead artifacts of the viewing geometry, then the spectrum should not be correlated with surface brightness. (Note that this test may be complicated by systematic variations of the emitted spectrum with height within the equatorial layer.) This characteristic is shared by other models in which the wisps arise from adiabatic compression of relativistic particles, e.g., in the model of Gallant & Arons (1994) and the synchro-thermal instability of Hester et al. (1995).

3. Doppler asymmetry of the approaching and receding wisps. A careful analysis of the brightness asymmetry should be consistent with bulk flow velocities somewhat larger than the pattern speeds. It will be important to take into account light travel time effects as well as aberration of the observed ripple pattern. This should be a general characteristic of any model in which the outward pattern speed is slower than the underlying fluid velocity, i.e., it should apply in propagating wave models provided that the waves propagate inward relative to the fluid.

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