On the relation of the gravitino mass and the GUT parameters

V.I. Tkach\textsuperscript{a}, J.J. Rosales \textsuperscript{a,b}, and J. Torres \textsuperscript{a}\textsuperscript{†}

\textsuperscript{a}Instituto de Física de la Universidad de Guanajuato, Apantado Postal E-143, C.P. 37150, León, Guanajuato, México
\textsuperscript{b} Ingeniería en computación, Universidad del Bajío, Av. Universidad s/n Col. Lomas del Sol, León, Gto., México

Abstract

In this article we consider the local supersymmetry breaking and the broken SU(5) symmetry permissible by dilaton vacuum configuration in supergravity theories. We establish the parameter relation of spontaneous breaking of supersymmetry and of the GUT theory.

PACS: 04.65.+e, 11.30.Qc, 12.60.Jv.
The structure of the effective N=1 supergravity theory cannot provide the small vacuum energy, this explanation may be only under the construction of the quantum gravity theory, possibly in the frame of superstring theories. The question about spontaneous supersymmetry breaking and the arising of small scale mass $10^2 \sim 10^3$ Gev probably can be determined from the supergravity theory or from their effective supergravity theory, these possibilities are intensively discussed in the literature. Quadratically divergent one-loop corrections to finite contributions of the effective potential $m_{3/2}^2M_{pl}^2$ could destabilize the hierarchy $m_{3/2} << M_{pl}$. Moreover, the $m_{3/2}^2M_{pl}^2$ contributions to the vacuum energy cannot be cancelled by symmetry breaking phenomena occurring at much lower energy scales. Possibly this realization will be understood in the future theory with hidden symmetry leading to the vacuum energy $m_{3/2}^2M_{pl}^2$ elimination [1]. Nowadays the most natural candidate for such theories is the heterotic superstring [2].

In all the models of spontaneous breaking of local supersymmetry an additional intermediate mass scale $M_{\text{hidd}}$ is introduced, so that $M_{\text{w}} << M_{\text{hidd}} << M_{pl}$ in order to have mass value for the gravitino in the range $10^2 \sim 10^3$ Gev, acceptable from the point of view of phenomenology, as well as from the point of view of quadratical divergences in the action for the absent fields in the minimal supersymmetric extension of the standard model (MSSM). While $M_{hidd}$ is a new mass scale coinciding with $M_{GUT}$ for the geometrical hierarchy case $10^{10}$ Gev [3,4], the effective string theory with observable sector $SU(3) \otimes SU(2) \otimes U(1)$ and the hidden mass scale sector $10^{13}$ Gev [5–8] are not coincided with $M_{GUT}$.

The problem of the vanishing vacuum energy in the classical level is determined by the so-called no-scale supergravity models [7], however there is not stable minimum of the vacuum without flat direction [5,7,9]. In the case of the effective $N = 1$ supergravity theories we have difficulties with the supersymmetry breaking in the moduli direction (hidden sector) in the minimum of the potential [9,10].

In our previous works [11,12] it was shown, that for spatially homogeneous part of fields in the supergravity theories interacting with matter fields there is a vacuum configuration invariant under $n = 2$ local conformal supersymmetry. This supersymmetry is a subgroup of
the four-dimensional space-time supersymmetry. As the requirements of the local conformal supersymmetry are not so hard as the requirements of the space-time supersymmetry, then the new possibilities in research of spontaneous supersymmetry breaking arise.

The purpose of this letter is to provide a mechanism, which naturally generates a small scale of the order $10^2 \sim 10^3$ Gev. It is done without additional intermediate mass scale parameter. In this case we will have a stable minimum of the potential with zero energy in the classical level corresponding to tree-level approximations; not breaking SU(5), not breaking supersymmetry and two minima with supersymmetry breaking, and SU(5) breaking in the phases $SU(3) \otimes SU(2) \otimes U(1)$ and $SU(4) \otimes U(1)$ states. The gravitino mass in the state $SU(4) \otimes U(1)$ will have a value in two orders less than $m_{3/2}$ in the states with symmetry of the standard model, and the mass $m_{3/2}(SU(3) \otimes SU(2) \otimes U(1))$ is defined by a constant self-interaction $\alpha_{GUT} = \frac{1}{26}$ and the GUT mass $M_{GUT}$. The construction of the spontaneous breaking supersymmetry mechanism is related to the existence of vacuum states in supergravity and the effective theory of supergravity invariant under the local conformal supersymmetry (which is a subgroup of $d = 4, N = 1$ supergravity) [12].

The effective scalar field potential in the local conformal supersymmetry corresponding to the potential is given by $V_{eff} = V_F + V_D$ [12].

$$V_F = \frac{e^{\alpha G}}{\kappa^4}[\alpha^2 G_A G^A G_D G_D - 3],$$

where $\kappa = \frac{1}{M_{pl}}$ and $M_{pl} = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18}$ Gev. is the reduced mass Planck. The scalar field potential (1) depends on the real gauge invariant Kähler function $G(z^A, \bar{z}^\dot{A}) = K(z^A, \bar{z}^\dot{A}) + \log|g(z^A)|^2$, where $K(z^A, \bar{z}^\dot{A})$ is the Kähler potential whose second derivatives determine the kinetic terms for the fields in the chiral supermultiplets and $g(z^A)$ is the superpotential. $\alpha$ is an arbitrary parameter, which is not fixed by conformal supersymmetry, and as it will be shown in this work it plays the role of the dilaton coupling constant [13]. Derivatives of the Kähler function are denoted by $\frac{\partial G}{\partial z^A} = G_A, \frac{\partial G}{\partial \bar{z}^\dot{A}} = G_{\dot{A}}$, and the Kähler metric is $G_{A\dot{B}} = G_{\dot{B}A} = K_{A\dot{B}} = K_{\dot{B}A}$. The inverse Kähler metric $G^{AB}$, such as $G^{AB}G^{BC} = \delta^C_A$, can be used to define $G^A \equiv G^{AB}G_B$ and $G^B \equiv G_A G^{AB}$. In our notation
repeated indices are summed, unless otherwise stated. Note, that in contrast with global supersymmetry the \( F \)-term part of the effective scalar potential in (1) is not positive semi-definite in general. Therefore, it allows to have spontaneously supersymmetry breaking with vanishing classical vacuum energy, unlike in global supersymmetry.

In order to discuss the implication of spontaneous supersymmetry breaking we need to display the potential (1) in terms of the auxiliary fields \( F_A \) of the matter supermultiplets

\[
V_F = \frac{1}{\kappa^2} F_A \bar{F}^A - \frac{3}{\kappa^4} e^{\alpha G(z^A, \bar{z}^\bar{A})},
\]

where \( F_A \) has the following form

\[
F_A = \frac{\alpha}{\kappa} e^{\frac{z^A}{2}} G(z^A, \bar{z}^\bar{A}) G_A(z^A, \bar{z}^\bar{A}).
\]

The local supersymmetry is spontaneously broken if the auxiliary fields (3) of the matter supermultiplets get non-vanishing vacuum expectation values. According to our assumption at the minimum the potential (2) is \( V_F(z^A, \bar{z}^\bar{A}) = 0 \), but \( <F_A> = F_A(z_0, \bar{z}_0) \neq 0 \) and, thus, the supersymmetry is broken. The measure of this breakdown is the gravitino mass \( m_{3/2} \), which in our case is given by [12]

\[
m_{3/2} = \frac{1}{\kappa} e^{\frac{z^A}{2}} G(z^A, \bar{z}^\bar{A}) = e^{\frac{\alpha G(z^0_0, z^0_0)}{2}} M_{pl},
\]

which depends on the vacuum expectation values \( z^A_0 = <z^A> \) of the scalar fields of the theory determined by the condition of minimum vacuum energy. For convenience in the following we shall also classify the fields as \( z^A \equiv (S, z^a) \), where \( S \) stands for the dilaton field, while \( z^a \) for the spatially homogeneous chiral fields. So, the conditions for the accurately spontaneous supersymmetry breaking with vanishing vacuum energy at the classical level is very simple if we take \( \alpha = \sqrt{3} \)

\[
\frac{\partial V_F}{\partial z^a}(S, \bar{S}, z^a_0, \bar{z}^\bar{a}_0) = 0, \quad V_F(S_0, \bar{S}, z^a_0, \bar{z}^\bar{a}_0) = 0, \quad F_S(S_0, z^a_0) \neq 0,
\]

where \( S_0, z^a_0 \) are the absolute minima. The first condition implies the existence of a minimum, the second condition implies the vanishing cosmological constant, and the non-vanishing F-term implies the spontaneously supersymmetry breaking. We take the Kähler function as
$$G(S, z^a) = - \log(S + \bar{S}) + \frac{\kappa^2}{2} \bar{z}^a z^b + \log\{|\frac{\kappa^3}{2} g(z^a)|^2\}. \quad (6)$$

After substitution of (6) into (1) the effective potential becomes

$$V_F(S, z^a) = \frac{e^{\alpha G}}{\kappa^4} [\alpha^2 G S G^{SS} G_S + \alpha^2 G z^a G \bar{z}^b z^a G z^a - 3]. \quad (7)$$

Now, we will consider the SU(5) theory. The scale where the unified gauge symmetry is broken is described by a mass parameter $M_{\text{GUT}}$. Hence, the minimal choice of a superpotential is written as

$$\tilde{g}(z^a) = \tilde{g}(\Sigma) = \frac{1}{3} \text{Tr} \Sigma^3 + \frac{M_{\text{GUT}}}{2} \text{Tr} \Sigma^2, \quad (8)$$

where $g(z^a) = \lambda \tilde{g}(z^a)$ and $\Sigma$ is the adjoint representation 24 of SU(5) and $\lambda$ is a self-interaction coupling constant. In our analysis of the broken SU(5) GUT we will consider only the part of the supersymmetric potential [14] depending on $z^a = \Sigma_y^a \equiv \Sigma$, which is the adjoint representation 24 of SU(5), as the minimum of the scalar fields potential is achieved when the vacuum expectation values of the other left-handed multiplets are vanished. We consider the case when $g(S, z^a) = g(z^a)$. The condition $\partial_{\Sigma} g(z) = 0$ shows, that there is a minimum of the potential $V(\Sigma)$ for the global supersymmetry inclusively in the presence of the D-term in the effective potential, if $(\Sigma^a_y)_{\text{diag}}$ possesses one of the following vacuum expectation values $<\Sigma>$:

$$(i)0, \quad (ii)\frac{1}{3} M_{\text{GUT}}(1,1,1,1,-4), \quad (iii)M_{\text{GUT}}(2,2,2,-3,-3) \quad (9)$$

and the vacuum energy values of all other components of $\Sigma_y^a$ are zero [4,14]. Thus, solution (i) does not break SU(5), while (ii) breaks SU(5) gauge group into $SU(4) \times U(1)$, and solution (iii) breaks the gauge group into $SU(3) \times SU(2) \times U(1)$. The supersymmetric self-interaction fields $\Sigma$ are constructed in such form, that in the mass scale of the grand unification $M_{\text{GUT}}$ the broken symmetry of SU(5) takes place.

The contributions of D-terms in the effective potential preserve their forms and for the local conformal supersymmetry have the standard form

5
\[ V_D = \frac{1}{2\alpha} (Re f^{-1})^{ij} (G_a(T_i)\bar{z}^a)(G_{\bar{a}}(T_{\bar{a}})\bar{z}^\bar{a}), \]  

(10)

the functions \( f_{ij} \) in this case have the form \( f_{ij} = \delta_{ij} S \), and in particular the gauge coupling constant of SU(5) is done by \( g^2_{GUT} = \langle S \rangle \). In the analysis of the effective potential \( V_{eff} = V_F + V_D \) the stationary points corresponding to the minimum of \( V_D \) term can be ignored because of the condition \( G_{z^a} = 0 \), and this permits the analysis only for the \( V_F \) term.

Deriving \( G(S, z) \) with respect to dilaton field in (7) we get

\[
G_S = \frac{\partial G}{\partial S} = -\frac{1}{S + \bar{S}}, \quad G_{\bar{S}} = \frac{\partial G}{\partial \bar{S}} = -\frac{1}{S + \bar{S}}, \quad G_{SS} = \frac{1}{(S + \bar{S})^2},
\]

(11)

and after substituting them again into (7) the potential becomes

\[ V_F = \frac{3}{\kappa^4} e^{\sqrt{3}G} G_{z^a} G_{\bar{z}^\bar{a}} G_z. \]

(12)

We see, that if \( G_{z^a} = 0 \) for any \( z_0 = \langle \Sigma_0 \rangle \) then \( V_F \equiv 0 \), while modification \( \frac{\partial g(z)}{\partial z^a} = 0 \) in \( G_{z^a} = \frac{\partial g(z)}{\partial z^a} + \kappa^2 z^a = 0 \), which leads to small correction in vacuum value \( \langle \Sigma \rangle \) (9).

So, we compute the condition of the stationary points in the dilaton direction, i.e. \( \frac{\partial V_{eff}}{\partial S} = 0 \), we obtain

\[ \frac{\partial V_F}{\partial S} = \frac{3\sqrt{3}}{\kappa^4} e^{\sqrt{3}G} \{ G_{z^a} G_{\bar{z}^\bar{a}} G_z \} G_S = 0, \]

(13)

and

\[ \frac{\partial V_F}{\partial \bar{S}} = \frac{3\sqrt{3}}{\kappa^4} e^{\sqrt{3}G} \{ G_{z^a} G_{\bar{z}^\bar{a}} G_z \} G_{\bar{S}} = 0. \]

(14)

The conditions of the stationary points in the dilaton direction are \( \frac{\partial V_F}{\partial S} = \frac{\partial V_F}{\partial \bar{S}} = 0 \) and can be satisfied in two different ways: as \( G_{z^a} \neq G_{\bar{z}^\bar{a}} \neq 0 \), then \( G_S = G_{\bar{S}} = 0 \), and therefore \( F_S = 0 \) and the supersymmetry is not broken in the dilaton direction, on the other hand, if \( G_S \neq 0, G_{\bar{S}} \neq 0 \) and \( G_{z^a} = G_{\bar{z}^\bar{a}} = 0 \) then, we have broken supersymmetry in the dilaton direction. Therefore, state with not broken SU(5) (9) although \( G_S \neq 0 \), as soon
as \( \tilde{g}(z_0') = \tilde{g}(\Sigma_0) = 0 \) is equal to zero, and \( F_S \) is defined by (3), then \( < F_S > = 0 \) and the supersymmetry is not broken. The states with broken \( SU(5) \) into \( SU(4) \otimes U(1) \) and \( SU(3) \otimes SU(2) \otimes U(1) \) and \( < F_S > \neq 0 \) have broken supersymmetry. Minimizing the Eq. (7) with respect to the chiral fields we have

\[
\frac{\partial V_F}{\partial z} = 3 \kappa^4 e^{\sqrt{3}G} G_x G^x \left[ \sqrt{3} \left( G_z \right)^2 + G_{zz} \right] = 0, \tag{15}
\]

\[
\frac{\partial V_F}{\partial \bar{z}} = 3 \kappa^4 e^{\sqrt{3}G} G_x \left[ \sqrt{3} \left( G_{\bar{z}} \right)^2 + G_{\bar{z}z} \right] = 0. \tag{16}
\]

The minimization of the potential (7) requires that \( G_z = 0 \) and, therefore, in the classical level the energy is equal to zero. In the case when \( G_z \neq 0 \) and \( G_{\bar{z}} \neq 0 \) there are conditions for (15,16) with \( V_F > 0 \), therefore, we get

\[
\sqrt{3} \left( G_z \right)^2 + G_{zz} = 0, \quad \sqrt{3} \left( G_{\bar{z}} \right)^2 + G_{\bar{z}z} = 0, \tag{17}
\]

but in this case to find the stationary points corresponding to maximum we need to considerate the \( V_D \) contribution (of the D-term) (10) in the relations (17). Neglecting \( \frac{M^2_{\text{GUT}}}{M_{\text{pl}}} \) corrections to the solutions of the equation \( G_z = 0 \), included in (9) the three solutions for \( \tilde{g}(z') \) in (8) are

\[
\tilde{g}(z_0') = (0, \frac{10}{27} M^3_{\text{GUT}}, 5 M^3_{\text{GUT}}). \tag{18}
\]

The third solution corresponds to the \( SU(5) \) breaking into \( SU(3) \otimes SU(2) \otimes U(1) \) and the second solution corresponds to \( SU(4) \otimes U(1) \) state. Then, in this case the gravitino mass in \( SU(3) \otimes SU(2) \otimes U(1) \) state is

\[
m_{3/2} = \frac{1}{\kappa} e^{\sqrt{3}K(S_0, z_0')} |\tilde{g}(z_0')|^{\sqrt{3}} = \frac{1}{(2S_R)^{s/2}} \left( \frac{5}{2} \lambda \right)^{\sqrt{3}} \left( \frac{M_{\text{GUT}}}{M_{\text{pl}}} \right)^{3\sqrt{3}}, \tag{19}
\]

where \( S + \bar{S} = 2ReS = 2S_R \) and \( ReS = S_R \). The gravitino mass \( m_{3/2} \) is not fixed at the classical minimum, but it is a function of the dilaton field \( S \) parametrizing the flat dilaton directon with the conditions \( G_S \neq 0 \) and \( G_{\bar{S}} \neq 0 \) in our case. The value \( S_0 = < S > \) in the minimum with \( V_F = 0 \) is not fixed by Kähler function.
In order to have stable minimum in the dilaton direction we modify the superpotential
\[ g(S, z^a) = g(S)\tilde{g}(z^a). \]
The requirement of vanishing vacuum energy imposes a non-trivial constraints on the structure of dilaton sector of the theory
\[ \Pi \equiv G_S G^S - 1 = 0, \quad at \quad S = S_0, \quad (20) \]
and the constraint
\[ \partial_S \Pi = 0, \quad at \quad S = S_0, \quad (21) \]
which is necessary to preserve the stationary points conditions \( G_{z^a} = 0 \) in the observable directions. In this case we have the conditions following from (20)
\[ \partial_S g(S)\partial_S \tilde{g}(\tilde{S}) = \frac{g(S)\partial_S \tilde{g}(\tilde{S})}{S + \tilde{S}} + \frac{\tilde{g}(\tilde{S})\partial_S g(S)}{S + \tilde{S}}, \quad (22) \]
these conditions and (21) are necessary to find the points \( S_0 \) of the stable minimum with \( V(S_0, z^a_0) = 0 \), which will be defined by parameters of the superpotential \( g(S) \) and for good parameter values \( < S_R > \) may be \( < S_R > = 2 \), \( i.e \) corresponding to gauge coupling with value \( \alpha_{GUT} \sim \frac{1}{26} \) at the GUT mass scale \( M_{GUT} \sim 10^{16} \) Gev. The superpotential value in the dilaton direction in this point defines the magnitude of the coupling constant \( \lambda \) of self-interaction 24 multiplet
\[ |g(S_0)| = \lambda. \quad (23) \]
The most direct way to define \( < S_R > = 2 \) value through gauge group hidden sector in the effective superstring theory is including a sector with moduli field direction \( T_i \) and the superpotential \( g_{np}(S, T) = g(S)h(T) \). In this case the Kähler potential has the form [8,9]
\[ K(S, T, z^a) = -\log(S + \tilde{S}) - 3\log(T + \tilde{T}) + \frac{K^2}{2} z^a\tilde{z}^a, \quad (24) \]
then the constraints (20,21) become
\[ \Pi \equiv G_S G^S + G_T G^T - 1 = 0, \quad |g_{np}(S_0, T_0)| = \lambda, \quad (25) \]
\[ \partial_S \Pi = \partial_T \Pi = 0, \quad \text{at} \quad S = S_0, \quad T = T_0, \quad (26) \]

these constraints are imposed only in the hidden sector direction, and \( S_0, T_0 \) are defined by parameters of the superpotential in the hidden sector \([8, 9]\), if we consider the self-dual points of the modular space contribution \([9]\), then we have the following gravitino mass in the \( SU(3) \otimes SU(2) \otimes U(1) \) state

\[
m_{3/2} = \left( \frac{5 \pi \frac{1}{2} \lambda}{2^3} \right) \sqrt{3} \left( \frac{\alpha_{\text{GUT}}}{M_{\text{pl}}} \right)^{\frac{3}{2}} \left( \frac{M_{\text{GUT}}}{M_{\text{pl}}} \right)^{3/2} M_{\text{pl}}, \quad (27)\]

for \( M_{\text{GUT}} \sim 10^{16} \) GeV value and \( \alpha_{\text{GUT}} \sim \frac{1}{80} \) under \( \frac{1}{80} \leq \lambda \leq \frac{1}{20} \) we obtain \( 10^2 \leq m_{3/2} < 10^3 \) GeV. Note the circumstance which is due to the \((18)\): the gravitino mass in the state \( SU(4) \otimes U(1) \) is related with the gravitino mass in \( SU(3) \otimes SU(2) \otimes U(1) \) states by relation (for equality values \( \alpha_{\text{GUT}}, \lambda \) parameters defined by the hidden sector)

\[
m_{3/2}(SU(4) \otimes U(1)) = \left( \frac{10}{2^7 / 5} \right)^{\sqrt{3}} m_{3/2}(SU(3) \otimes SU(2) \otimes U(1)). \quad (28)\]

So, in the case of the hidden sector model with \( SU(N) \) gauge group the superpotential \( g_{np} \) is given by \( g(S) = -Ne^{-\frac{32 \pi^2}{3} S} \) and \( h(T) = (32\pi e)^{-1} \eta^{-6}(T) \), where \( \eta(T) \) is Dedekin eta function \([9, 10]\). The coupling constant \( \lambda \) in this case is defined by the magnitude of superpotential \( g_{np}(S_0, T_0) \) and can be exponentially small, so we cannot exclude from consideration the case of arbitrary parameter \( \alpha \) values (including \( \alpha = 1 \) value). In that case the constraints \((25, 26)\) on the hidden sector take the forms

\[
\Pi(\alpha, S, T) = \alpha^2 G_S G^S + \alpha^2 G_T G^T - 3 = 0, \quad |g(S_0, T_0)| = \lambda, \quad (29)
\[
\partial_S \Pi = \partial_T \Pi = 0, \quad \text{at} \quad S = S_o, \quad T = T_o.
\]

These constraints give us the stable minimum with vanishing vacuum energy (at tree-level) without flat direction and with \( < F_S > \neq 0, < F_T > \neq 0 \) we have broken supersymmetry in the moduli direction and the broken states \( SU(5) \), \( < F_{\gamma} > = 0 \). Then, in the state with symmetry \( SU(3) \otimes SU(2) \otimes U(1) \) of the standard model we have the following gravitino mass relation

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\[ m_{3/2} = \frac{\lambda^\alpha}{\langle T_R \rangle^{1/2}} (5\pi \alpha_{GUT})^{1/2} \left( \frac{M_{GUT}}{M_{pl}} \right)^\alpha M_{pl}, \tag{30} \]

and the gravitino mass depends on the gauge group of the hidden sector in the effective supergravity theory.

**Acknowledgments:**

We are grateful to I. Bandos, I. Lyanzuridi, L. Marsheva, O. Obregón, A. Pashnev and J. Socorro for their interest in this paper. This work was supported in part by CONACyT grant 3898P-E9608.
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