PERSISTENCE OF COLLECTIVE FLUCTUATIONS IN N-BODY META-EQUILIBRIUM GRAVITATING AND PLASMA SYSTEMS

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Persistence of collective fluctuations in N-Body meta-equilibrium gravitating and plasma systems

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Starting from a meta-equilibrium state (in the Vlasov limit), the time scale of the fluctuations exhibited by systems of one-dimensional charged particles is computed. This study is given both for plasma and gravitational systems. The use of the multiple water bag model allows an analytical treatment for both collective and individual modes. These results are compared with those obtained by numerical simulations of N-body systems. Finally, it is numerically shown that collective effects are responsible of the long time scale of phase-space holes structures.

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I. INTRODUCTION

The evolution of plasma or gravitational systems is usually divided in two parts. In a first step, collective effects drive the system toward a meta-equilibrium state (that is an equilibrium in the Vlasov limit) and in a second step, the collisional effects thermalize slowly the distribution function. This scenario has been used by many authors to explain the relaxation of objects like galaxies for which the evolution is driven by the sole collective effects – the universe being too young to be affected by collisional effects. After the model of violent relaxation proposed by Lynden-Bell [1], a lot of works have studied this first step both from a theoretical and numerical point of view, see for example Gurzadyan [2], Miller [3], Reidl [4], Yamashiro [5] and more recently Tsuchiya [6].

The aim of this paper is to look at the evolution of system after this first time for both plasma and gravitating systems. Then, the distribution function is a meta-equilibrium. It can be noticed that the first step of evolution involving the sole collective effects do not allow the system to reach exactly a meta-equilibrium state. Eulerian simulations of gravitating systems [7] have shown the formation of holes in phase space which remains unchanged for the entire simulation time and prevents the system to reach completely such a meta-equilibrium. Moreover, in the plasma case, these holes are responsible of the stopping of the Landau damping. It will be seen hereafter that these structures are strong enough to resist to individual effects.

All distributions of the sole energy are solution of the Vlasov equation. To compute the time scale and the frequency spectrum of the fluctuations, we need to solve the Vlasov equation linearized around this meta-equilibrium. Here, a fundamental difference appears between plasma and gravitating systems. While this problem is easily solved by a double Fourier-Laplace transform in $x$ and $t$ (the Landau treatment) in the homogeneous plasma case, the gravitational problem implies a treatment of the inhomogeneous equilibria which, strangely, has not been very much addressed. Then, in order to perform analytical treatment (at least in a first step), the water-bag model is adopted hereafter: in such model [8] the distribution function is constant by steps between two contours defined by a function $a_{\pm}(x,t)$, where the subscript $\pm$ refers respectively to upper (+) and lower (-) values of the contour.

Consequently, plasma and gravitational systems exhibit very different behavior and indeed analytical treatment. The comparison of the results is interesting but not obvious since in the plasma homogeneous infinite case, we deal with a continuous spectrum of wave numbers (each with a resonance frequency), while, in the gravitational case, we have a discrete spectrum of eigen frequencies.

The paper is organized as follow. After this introduction, Sect. II is devoted to the analytical treatment allowed by the multiple water-bag (abbreviated MWB hereafter) model both for plasma and gravitational case. Section III gives numerical result for the plasma and Sect. IV for the gravitational case. In Sect. V the stability of structures initially dug out in the phase space distribution of the gravitational case (holes) is numerically studied. Sect. VI gives our conclusions.
II. THE MULTIPLE WATER-BAG MODEL – LINEARIZED EQUATION AROUND EQUILIBRIUM

In order to obtain analytical results, we will limit this work to one dimensional system and a very simple distribution function.

The simplest distribution function \( f(x,v,t) \) one can think about is the one which has a single value \( A \) in a delimited area of the space phase. This model, called water-bag, has been introduced in [8] by De Pack. He, and after him many authors, noticed that, with this simple model, analytical treatment can be performed [9]. An extension of this model is the multiple water-bag which presents several areas of constant value, each delimited by a “bag”. The MWB model can be obtained by the discretization of a continuous distribution function. Nevertheless it introduces discontinuities and the connection of the physical properties of the continuous distribution function and the discretized one deserves a careful treatment [10].

Here, to describe the meta equilibrium, we will take a MWB, function of the energy alone (see Fig.1). Then each bag is delimited by two contours, symmetric with respects to the \( x \) (space) axis, with, on the two contours of bag \( i \), a given energy \( \epsilon_i \). The velocity of a particle moving on the border of bag number \( i \) is \( \pm a_i(x) \) with

\[
\epsilon_i = \frac{1}{2} ma_i^2(x) + m\phi(x),
\]

where \( \phi(x) \) is the potential created by all the bags at point \( x \).

![Diagram](https://via.placeholder.com/150)

FIG. 1. (a) MWB model, (b) Cut off of Fig.1a along the O'O axis.

Equation (1) depends on the square of the velocity \( a_i \), this is the reason of the symmetry with respect to \( x \). Let us call \( x_{st} \) the value for which the bag closes (because of the absence of neutralizing species, the bags always close in the one dimensional gravitational case). For this value, we have : \( a_i(x_{st}) = 0 \) that is :

\[
\epsilon_i = m\phi(x_{st}).
\]

Now, let us perturb the MWB equilibrium. If we keep in mind the picture of the particle following the borders of the \( i^{th} \) bag, its two velocities \( V_{i\pm}(x,t) \) obey the equations :

\[
\frac{\partial V_{i\pm}}{\partial t} + V_{i\pm} \frac{\partial V_{i\pm}}{\partial x} = E,
\]

where

\[
V_{i+} = a_i + v_{i+} \quad \text{for the upper border in the phase space plane},
\]

\[
V_{i-} = -a_i + v_{i-} \quad \text{for the lower border}.
\]

The field \( E \) reads

\[
E = E_0 + E_1,
\]

where \( E_0 \) is the field created by the unperturbed MWB equilibrium and \( E_1 \) is the correction at first order for the perturbed one. In the plasma case, \( E_0 = 0 \), and in the gravitational case it is given by the Poisson law :

\[
\frac{dE_0(x)}{dx} = -4\pi G \sum_i A_i 2a_i(x),
\]

where the summation involves all the bags not yet closed at point \( x \) (see Fig. 1b). The linearization of (3) gives for the two perturbed velocities of bag \( i \) :
\frac{\partial v_+}{\partial t} + \frac{\partial}{\partial x}(a_i v_{i+}) = E_1 , \tag{6}

\frac{\partial v_-}{\partial t} - \frac{\partial}{\partial x}(a_i v_{i-}) = E_1 , \tag{7}

where \( E_1(x, t) \) is the sum of the partial fields \( E_{1i}(x, t) \) created by the particles of bag \( i \):

\[ E_1 = \sum_i E_{1i} , \tag{8} \]

and

\[ \frac{\partial E_{1i}}{\partial x} = -4\pi G A_i (v_{i+} - v_{i-}) . \tag{9} \]

In addition we will constrain the perturbed field \( E_{1i} \) to be equal to zero at \( x = \pm x_{si} \). It means that the points \( \pm x_{si} \) are held fixed. Obviously \( E_{1i} = 0 \) for \( |x| > x_{si} \).

Calculating the difference between equations (6) and (7), and with the help of (9), we find, after integrating on \( x \):

\[ \frac{\partial E_{1i}}{\partial t} = -4\pi G A_i a_i (v_{i+} + v_{i-}) . \tag{10} \]

Substituting \( v_{i+} \), \( v_{i-} \) by (9) and (10) into the result of (6)+(7), we obtain, after Fourier transform on \( t \), for each bag \( i \):

\[ a_i \frac{\partial}{\partial x} a_i \frac{\partial}{\partial x} E_{1i}(\omega, x) + \omega^2 E_{1i}(\omega, x) = -\omega_j^2 E_1(\omega, x) , \tag{11} \]

where \( \omega_j^2 \) is the Jean’s frequency associated to the bag \( i \). \( \omega_j^2 \) reads

\[ \omega_j^2(x) = 8\pi G A_i a_i(x) = 4\pi G n_i(x) . \tag{12} \]

Equation (11) provides for each bag, an equation connecting \( E_{1i} \) to \( E_1 \), while (8) will give the dispersion relation. For example, in the usual homogeneous plasma case, \( a_i(x) = a_i \), and we can now take the Fourier transform on \( x \) of (11) to obtain:

\[ E_{1i} = \frac{\omega_{pi}^2}{\omega^2 - k^2 a_i^2} E_1 . \tag{13} \]

The plasma frequency \( \omega_{pi}^2 \) replaces the Jean frequency, with, formally \( \omega_{pi}^2 = -\omega_j^2 \), because of the change of sign in the Poisson law. And, indeed, equation (8) and (13) give the dispersion relation:

\[ \sum_i \frac{\omega_{pi}^2}{\omega^2 - k^2 a_i^2} = 1 , \tag{14} \]

which can be deduced from the general formulæ.

In the gravitational case, the eigenvalues \( \omega \) of (11) are found with the constraint \( E_{1i}(\pm x_{si}) = 0 \).

It can be noticed that, if the velocity perturbation of the bags is taken independent of the time \( t \), then \( \omega = 0 \) is a solution of (11). This marginal mode corresponds to a translation at constant velocity of the bags. The argument reads as follow. Deriving twice with respect to \( x \) (1), one obtains:

\[ \frac{d}{dx} \left( a_i \frac{d}{dx} a_i \right) + \frac{d^2 \phi}{dx^2} = 0 . \tag{15} \]

Supposing the entire system moving at velocity \( v \), the field \( E(x, t) \) reads at \( t = \Delta t \):

\[ E(x, \Delta t) = E(x - v\Delta t, 0) = E(x, 0) - \frac{dE_0(x)}{dx} v\Delta t \tag{16} \]

where the second term of the right hand side is the perturbed field \( E_1 \) defined by (4). Now, taking (5) and (8) into account (15) becomes:
\[
a_i \frac{d}{dx} (a_i \frac{d}{dx} E_{3i}) = -\omega_{ji}^2 E_i
\]

which is indeed (11) with \( \omega = 0 \). Consequently, \( \omega = 0 \) corresponds to a translation of the entire system at constant velocity.

On the other hand, the period of rotation \( T \) of a particle of energy \( \epsilon_A \) in the potential \( \phi \) of the unperturbed multiple water-bag, is given by

\[
T = 2\sqrt{2} \int_0^{x_A} \frac{dx}{\sqrt{\epsilon_A/m - \phi(x)}}
\]

where \( x_A \) is the position of the particle when its velocity is equal to zero.

As already mentioned, the restriction to one-dimensional system allows to obtain the relatively simple system of equations as given by (8) and (11). Another interesting point is, still for 1D system, the existence of an exact code. As 1D particles are infinite plane sheets, each creating a field which is a constant. Consequently the total field is piecewise constant and depends only on the relative positions of the particles. Each one experiences an uniform acceleration as long as it does not cross its neighbors, then it experiences a new field and a new accelerated motion. The program calculates the time at which crossings between two particles takes place and keeps the position order relation between them. To have more precision, refer to [11]. The crucial property of this code is to be exact and with no error introduced, except the round-off errors due to the finite number of bits treated by the computer.

Consequently, this code will take into account all the effects, the individual as well as the collective ones. Moreover it will give precisely the individual modes.

### III. PLASMA CASE

In the plasma case, the space translation invariance (i.e., homogeneous character of the meta equilibrium [*]) allows to push the analytical treatment much further. The well-known classical first order theory in the graininess parameter \( g = (n\lambda_D)^{-1} \), where \( n \) is the density and \( \lambda_D \) is the Debye length, of the system uses both Laplace transform in time and Fourier transform in space to calculate the fluctuating field. The treatment is achieved supposing the independence of each wavenumber \( k \), in agreement with the property of the Vlasov equation linearized around an infinite homogeneous meta equilibrium. The result shows a collective behavior around the resonance frequency for \( k\lambda_D \ll 1 \). These resonances are given by the dispersion relation which reads for the Maxwellian distribution function (in the limit of small \( k \))

\[
\omega_k^2 = \omega_p^2 + 3k^2V_T^2
\]

where \( \omega_p \) is the plasma frequency and \( V_T \) the thermal velocity.

In the simple one bag case the dispersion relation (which has been recovered by the previous calculation, see (14)) reads

\[
\omega_k^2 = \omega_p^2 + k^2a^2
\]

where \( \pm a \) are the velocities delimiting the border of the bag. This relation shows that collective fluctuations will not happen in this case because the excited waves correspond to a phase velocity \( (a^2 + \omega_p^2/k^2)^{1/2} \) larger than \( a \) and no particles have a velocity larger then \( a \). This important difference between a water-bag and a Maxwellian distribution concerning the level of excitation of small \( k \) spectrum of the charge density has been studied and checked by numerical simulation in [12]. In order to exhibit some collective fluctuations we must consider at least two bags. In this case the dispersion relation is given by (14) for \( i = 1, 2 \). Figure 2, which gives \( \epsilon \) function of \( \omega/k \), indicates that the collective contribution is given by particles with velocities in the range \( [a_2, a_1] \), that is particles which belong to the outer bag (in phase space). The role and the importance of this second pole is clearly exhibited by the double water bag, but similar results can be obtained with two electron plasmas at very different temperatures or with mixture of electrons, positive and negative ions.
The level of excitation of a wave is proportional to $1 - v_G/v_\phi$, where $v_G$ is the group velocity and $v_\phi$ is the phase velocity [13]. This expression computed for the second pole (inside the water-bag) shows that this level function of $k$ goes to a maximum. The length of the system will be chosen in order to have a fundamental wave number not too far from this maximum. Fortunately, this will give rather small systems, which desamphasize the role of the Landau pole (which as a matter of fact is not excited, but also not damped by the multiple water bag) allowing a better study of the second pole. Moreover, the computational effort will not be too heavy and more attention can be paid to the graininess parameter value.

Figures 3 show the snapshots of the evolution of the distribution function taken at $\omega_p t = 0, 250, 500, 1000$ for a system which length $L$ equal $10\lambda_D$ and containing 1000 particles (ions and electrons) by Debye length; this corresponds to a grain. It must be noticed that Balestic [14] has proved that no global evolution due to graininess can take place on time $\omega_p t$ proportional to $n\lambda_D$ in the one-dimensional case, and that the overall distribution will thermalize on time $\omega_p t$ proportional to $(n\lambda_D)^2$ [15]. On the other hand, test particles can relax in time $(n\lambda_D)\omega_p^{-1}$ (see [16]); these results have been numerically confirmed in the case of the water-bag. In the double water-bag case, an undamped pole exists, with a phase velocity located in the outer bag. Consequently, in the regular fluctuations theory, an infinite level of excitation is present at this frequency. Of course, neglected phenomena (second order in graininess factor for example) will bring a finite limit but we should observe a quick destruction of a beam of particles located initially at this velocity. Turning to numerical simulation, we first observe that the global distribution function indeed does not change during the time of order $n\lambda_D \sim 1000$ (see Fig. 3). On the other hand, we show on Fig. 4 the evolution of 6 beams of particles which belong to the distribution but which are labeled in order to follow their motions; in order to have a better insight, the number of particles is now of $n\lambda_D = 2000$. Two of them represent the particles at the border of the inner bag, two the particles at the border of the outer bag and two are formed with these particles which excite the fluctuations supported by the outer poles. It is clear from Fig. 4 that these two beams are quickly (around $\omega_p t = 40$) affected by these collective fluctuations. Let us point out that we are at the limit of validity of the usual first order in $g$ theory since the regular collisional term (first order in $g$) exhibits singularities for the two beams with velocities equal to the phase velocity of the second pole (shown on Fig. 4). Indeed in time now much shorter than $n\lambda_D \omega_p^{-1}$ (20 to 40$\omega_p^{-1}$ while $n\lambda_D = 2000$) we see a strong destruction of these two beams while the others are much less affected.

![FIG. 2. Dielectric coefficient function of the frequency for a double water-bag distribution function.](image)

![FIG. 3. Time evolution of a double water-bag with $n\lambda_D = 1000$ and $L = 10\lambda_D$. The velocity distribution function and the representation in phase space are given.](image)
The density Fourier transform on both space and time variables given Fig. 5a,b and c, shows that the collective fluctuations are mostly supported by the largest wavelength allowed by the periodic system (that is $k = 2\pi/L$). The other modes obtained with larger wavelengths are indeed present but less excited (see, for example, Fig. 5b and c which give the cases $k = 4\pi/L$ and $k = 5\pi/L$ respectively). Figure 5a shows that, in fact, the Landau pole is excited at a very low level. It is initially excited and remains for the duration of the simulation because it exhibits no dumping [12].

The rapid destruction of perturbed equilibrium system in the Vlasov limit has also been observed in the case of a Lorentzian velocity distribution function for which the energy diverges on small $k$ because of a large number of particles in the tail. This indicates the non physical character of both distributions which have a too slow decrease in $v$ of the velocity distribution or which have no particles of high velocities at all but presents a sharp cut-off.

Nevertheless, in the last case, it is an excellent model which allows analytical treatment as can be seen Sect. II and numerically confirmed in next section. Finally, the double water-bag model is a good approximation of a two electronic population plasma having two different temperatures - the landau pole of the law temperature population exciting the particules which belong to the high temperature population.
IV. GRAVITATIONAL CASE

The change of sign in the interaction gives very different dynamical properties between plasma and gravitational systems; for example, the virial presents very large and periodic oscillations in the gravitational case while, in the plasma, it looks like a noise (see Fig. 6a and b). Actually, the absence of neutralizing species is the point that prevents to adopt a similar analytical treatment for both systems. Moreover, the neutralizing background is needed to treat the Jeans’s instability which requires an infinite medium [17].

![Graph](image)

**FIG. 6.** Virial function of time for (a) the plasma and (b) the gravitational case.

The lack of spatial homogeneity prevents the independence between the different wave numbers and imposes a proper mode analysis. As already mentioned, the calculus of such modes is easily done when we restrict to the one-dimensional multiple water-bag distribution function. Nevertheless, even in that case, the determination of the modes become more and more difficult as the number of bags $N$ increases since $N$ coupled equations must be solved. System (11), (8) has been numerically solved for the four first modes both for the single water-bag and a double water-bag. In that last case we have two independents parameters $A_2/A_1$ and $c_2/c_1$ where $A_i$ is the height of bag $i$ and $c_i$ is the energy of the border of bag $i$ (the bags are numbered from outside to inside). A shoot method, coupled with a runge-kutta scheme of order 4 [18], gives for the first collective mode, which are alternatively even and odd, respectively $\omega = 0; \omega = 0.7; \omega = 1.1; \omega = 1.5$ for the single water-bag and $\omega = 0; \omega = 0.8; \omega = 1.2; \omega = 1.7$ for the double water-bag with $A_2 = A_1$ and $c_2 = c_1/2$. For values of $\omega$ large enough, the Jean’s frequency $\omega_J$ does not play any role and the solutions are those of a string with fixed end points.

As already mentioned the first mode $\omega = 0$ is a marginal mode corresponding to an overall translation of the bags. In our numerical experiments, The initial conditions are chosen such that the total impulsion is zero and, consequently, this mode will not be excited.

On the other hand, the period of rotation of the particles in the field of the unperturbed meta-equilibrium gives the individual modes. The oscillation frequencies vary from $\omega = 0.43$ at the center to $\omega = 0.39$ at the border for the single water-bag and from $\omega = 0.49$ to $\omega = 0.42$ in the double water-bag case.

The system numerically simulated contains $N$ particles of equal mass $m$ uniformly distributed inside the phase space contour of energy $e_\epsilon$ with the phase-space density $A_i$. The normalization is such that the total mass $mN$ equals 1 and $e_\epsilon$, the maximum energy, equals 1. With this normalization and taking $4\pi G = 1$, the square of the Jean’s frequency $\omega_J^2$ defined as $z = 0$ is equal to $3/16$ for the single water-bag and $3 (\sqrt{2} + 1)/8 (2\sqrt{2} + 1)$ for the double water-bag with $A_2/A_1 = 1$ and $c_2 = c_1/2$.

Figures 7 show the snapshots of the evolution of a single water-bag for respectively the overall distribution, the population of initially high energy particles and the population of initially low energy particles. The first column shows that no global change happen and the water-bag character of the distribution is conserved on the duration of the simulation. On the other hand, the two last columns show that the two populations mix but keep a strong cohesion. This process is very far from the idea of a smooth diffusion. Indeed, the Fourier transform of the field given Fig. 8 exhibits the collective and individual modes (including the harmonics) theoretically determined before and the biggest mode is the first collective mode at $\omega = 0.7$. In order to have a better statistic and to have both even and odd contributions, the field is collected for 10 positions equally spaced out from 0 to $2\pi$ and the averaged value taken around the mean value is given Fig. 8.
FIG. 7. Gravitational case: Meta-equilibrium single water-bag with $N = 5000$ particles. Time evolution of respectively the overall distribution (first column), the $N/2$ particles of initial low energy (second column) and $N/2$ particles of initial high energy (third column).

FIG. 8. (a) Time evolution and (b) time frequency spectrum of the averaged gravitational field (around the mean value) taken each $0.23885$ from $x = 0$ to $x = 2.3885$ for the water-bag case containing $N = 10000$ particles. The field is sampled from $t = 0$ to $t = 2000$ each 0.05. In Fig. 8b, the arrows indicate the collective modes, while the grey area give the range occupied by the individual modes.

The same kind of diagnostic can be obtained with the double water-bag and Fig. 9 gives the time Fourier transform of the field collected in the same conditions as for the single water-bag. Also in that case, collective and individual modes, including the harmonics of the individual excitations are present and dominated by the first collective mode at $\omega \sim 0.8$. 

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FIG. 9. (a) Time evolution and (b) time frequency spectrum of the averaged gravitational field (around the mean value) taken each .2586 from \( x = 0 \) to \( x = 2.586 \) for the double water-bag case containing \( N = 10000 \) particles. The field is sampled from \( t = 0 \) to \( t = 2000 \) each 0.05. In Fig. 9b, the arrows indicate the collective modes, while the grey area give the range of the individual modes.

For these two distribution functions, the single and double water-bag, the numerical results indicate that collective modes are highly excited by the grainy character of the distribution function describing the systems.

V. STABILITY OF HOLES IN PHASE-SPACE

Numerical simulations of systems outside equilibrium show that they do not relax toward an equilibrium: they develop arms which carry the excess of kinetic energy and go around an empty zone of phase space. This process, which seems to be systematic, creates holes in phase space which remains for the duration of the simulation and prevent the system to reach a complete meta-equilibrium. Nevertheless, taking a time average allow to obtain a distribution of the alone energy [7]. Moreover, numerical simulations reveal that the number of holes and their positions are closely related to the initial shape of the distribution function.

In order to study the behavior of this structure, a hole is created ab initio in the water bag equilibrium [19]. Figures 10 show the evolution of the hole for nearly 130 rotations of the system. The hole is still present at the end of the simulation and it resists to the differential rotation of the particles which are localized at its border. In order to have a better insight of the behavior of this hole, it is initially filled with “test particles” which experience the field of the other but do not contribute to the field. Figures 11a,b,c and d give respectively, the kinetic energy of the particles of the systems, its Fourier transform, the kinetic energy of the test particles and its Fourier transform. The modes excited Fig. 11d are the individual modes, indicating that the holes rotates at the same velocity than the particles. Moreover this holes triggered the collective mode as can be seen Fig. 11b.
FIG. 10. Time evolution in phase space of a water-bag containing $N = 5000$ particles, in which a hole is created ab initio, for almost 130 rotations of the system.
FIG. 11. (a) Total kinetic energy $K_{Tot}$ function of time for the system represented Fig. 10, (b) frequency spectrum of $K_{Tot}$, (c) Kinetic energy $K_{Test}$ function of time for the test particles filling initially the hole, (d) frequency spectrum of $K_{Test}$. In Fig. 11b and 11d, the arrow indicates the collective modes, while the grey area gives the range of the individual modes calculated for the complete equilibrium water-bag (with no holes).

To show the long time scale of the hole, we go back to the complete water bag equilibrium and follow the particles which are initially localized in the area of the previous hole. These particles are just "labeled" and have the same physical properties as the others. Figure 12 show that the differential rotation between particles of low and high energy stretches the area occupied by these labeled particles. Nevertheless, the presence of another effect can be detected because the stretching is not complete.

![Image of phase space evolution](image)

FIG. 12. Time evolution in phase space of a population of particles which belong to the water-bag represented in Fig. 7 and initially localized in the same area as the hole of Fig. 10.

Finally, we numerically study the behavior of symmetric structures. Starting from a symmetric distribution function, the Eulerian simulations (for which the Vlasov equation is directly integrated) show the formation of an even number of symmetric holes (two in the simulation given by Mineau [7]). In our case, the particle description of the system breaks the Liouville invariant and the holes may even disappear. The stability of two symmetric holes initially digged in the single water-bag is numerically studied. Figures 13 show the evolution of test particles initially localized in these holes. It reveal that the symmetry rapidly breaks: one of the hole goes to the center while the other goes near the border of the bag. Then, as already mentioned, their periods of rotation, given by the particles of its border, change and they rapidly merge to form a single hole.
FIG. 13. Time evolution in phase space of a water-bag containing $N = 5000$ particles, in which two symmetric holes are created ab initio, for almost 130 rotations of the system.

VI. CONCLUSION

This paper gives both analytical and numerical approaches of the determination of collective modes in the gravitational and plasma systems. These approaches are possible because restriction to 1D systems and on the multiple water bag model. It must be pointed out that the theory of plasma assumes a neutral medium that is, most of the time, a uniform motionless background. Infinite gravitational system also need a neutralizing background, but this system is unstable under Jeans instability and clusters into subsystems the dimension of which is of the order of the Jeans length [20]. Here, the model studied is not infinite and does not need such an unphysical background.

N-body numerical simulations confirm pretty well the theoretical results and show that the gravitational system present very strong collective behavior in a certain sense stronger than in the plasma case.

These collective effects triggered by the grainy nature of our system explain the very strange behavior of labeled population, a fact already mentionned in Luwel and Severne [19].

Moreover, numerical simulations show that holes are structures which certainly play an important role in one dimensional systems, a fact already noticed in the plasma case. With an initial hole, the water-bag keeps almost its shape for a time large compared to the time necessary to destroy the same area filled with particles.

* For plasma in presence of external potential or in inhomogeneous magnetic field, inhomogeneous equilibrium do exist. They should be treated as the gravitational equilibrium and exhibits discret eigenfrequencies.
