INTRODUCTION

The standard model of particle physics is symmetric under the discrete transformation $CPT$. A theoretical approach is provided of deriving $CPT$-invariant field theories and interactions at the level of the field equations. For recent work on this topic, see Refs. [1] and [2]. The authors of these papers show that the $CPT$-invariance of the field equations is preserved under the discrete transformation $CPT$. This approach has the advantage of being independent of the specific choice of gauge symmetry, and therefore it can be applied to a wide range of physical systems.
At the level of the standard model, protons and antiprotons are composite particles formed as bound states of quarks and antiquarks, respectively. Possible CPT- and Lorentz-violating effects in the standard-model extension appear as perturbations involving the basic fields [3]. A distinct set of quantities is assigned to each quark flavor, and suitable combinations of these determine the CPT- and Lorentz-violating features of the proton. For our present investigation of protons and antiprotons in a Penning trap, it suffices to work within the usual effective theory in which the protons and antiprotons are regarded as basic fermions described by a four-component Dirac quantum field with dynamics governed by a minimally coupled lagrangian. Based on the quantities for the fundamental particles, we introduce effective quantities controlling possible CPT- and Lorentz-breaking effects for the proton. The lagrangian is taken to be the standard one for proton-antiproton quantum electrodynamics but extended to include possible small CPT- and Lorentz-violating terms. Further details are given in Ref. [6].

For the Penning-trap experiments of interest, the dominant contributions to the energy spectrum arise from the interaction of the proton or antiproton with the constant magnetic field of the trap. The quadrupole electric and other fields generate smaller effects. In a perturbative calculation, the dominant corrections due to CPT- and Lorentz-violating effects can therefore be obtained by considering a constant uniform magnetic field only. Since the signals of interest are energy-level shifts rather than transition probabilities, it suffices to use relativistic Landau-level wave functions as the unperturbed basis set and to calculate within first-order perturbation theory. However, the unperturbed energy levels must be taken as the Landau levels shifted by an anomaly term and other corrections.

To lowest order in the fine-structure constant, the perturbative hamiltonian \( H_{\text{pert}}^p \) for a proton of mass \( m \) is

\[
H_{\text{pert}}^p = a_0^p \gamma^0 \gamma^\mu - b_0^p \gamma^0 \gamma^\mu - c_{0,0}^p m \gamma^0 + \frac{i}{2}(c_{0,1}^p D^j + c_{0,2}^p D^j) \gamma^0 \gamma^j - \frac{i}{2} D^j c_{1,0}^p \gamma^0 \gamma^j + \frac{i}{2} D^j c_{2,0}^p \gamma^0 \gamma^j + \frac{1}{2} H^p \sigma^{\mu \nu} \gamma_{\mu \nu} ,
\]

where \( D^j \) is the appropriate covariant derivative. The quantities \( a_0^p, b_0^p, H^p, c_{0,0}^p, c_{0,1}^p, c_{0,2}^p, c_{1,0}^p, c_{2,0}^p, d_{0,0}^p, d_{0,1}^p, d_{0,2}^p, d_{1,0}^p, d_{1,2}^p, d_{2,0}^p, d_{2,1}^p, d_{2,2}^p \) are effective couplings for the proton and antiproton arising in the standard-model extension. We note that the terms involving \( a_0^p, b_0^p \) break CPT while those involving \( H^p, c_{0,0}^p, c_{0,1}^p, c_{0,2}^p, d_{0,0}^p, d_{0,1}^p, d_{0,2}^p \) preserve it, and that Lorentz invariance is broken by all five terms.

For the antiproton, the hamiltonian can be found via charge conjugation. However, it should be taken into account that experimental procedures for replacing particles with antiparticles in Penning traps typically reverse the electric field but leave unchanged the magnetic field.

We denote the proton energy levels without CPT- and Lorentz-violating perturbations by \( E_{n,s}^p \), where \( n \) is the principal quantum number and \( s \) is the spin. The proton cyclotron and anomaly frequencies are defined as

\[
\omega_c = E_{n+1}^p - E_{n+1}^p , \quad \omega_a = E_{n-1}^p - E_{1+n}^p .
\]

From the CPT theorem, these frequencies have the same values as those of the antiproton.

We can calculate energy corrections in first-order perturbation theory for the CPT- and Lorentz-breaking couplings. The corrected cyclotron and anomaly frequencies can then be found. Calculations at leading order in the CPT- and Lorentz-breaking quantities, in the electromagnetic fields, and in the fine-structure constant give the results

\[
\omega_c^p = \omega_a^p \approx (1 - c_{0,0}^p - c_{0,1}^p - c_{0,2}^p) \omega_c ,
\]

\[
\omega_a^p \approx \omega_a + 2b_0^p - 2d_{0,0}^p m_p - 2H^p , \quad \omega_a^p \approx \omega_a - 2b_0^p - 2d_{0,0}^p m_p - 2H^p ,
\]

for the modified frequencies. In these expressions, \( \omega_c \) and \( \omega_a \) are the unperturbed frequencies of Eq. (3).

**ANOMALOUS MAGNETIC MOMENTS**

Currently, the antiproton magnetic moment is known to a precision of 3 parts in \( 10^3 \) from experiments with exotic atoms [7]. In principle, the anomalous magnetic moments of protons and antiprotons could be measured in Penning traps. A comparison of the experimental ratios \( 2\omega_c^p/\omega_c^p \) and \( 2\omega_a^p/\omega_a^p \) would then provide a sharp test of CPT and Lorentz violation. The possibility of such experiments has received some attention in the literature [8,9].

The sensitivity of possible future \( g - 2 \) experiments to CPT and Lorentz violation can be investigated using the present theoretical framework. We find that the proton-antiproton differences at leading order for the cyclotron and anomaly frequencies are

\[
\Delta \omega_c^p \equiv \omega_c^p - \omega_a^p = 0 , \quad \Delta \omega_a^p \equiv \omega_a^p - \omega_a^p = 4b_0^p .
\]

The leading-order signal for CPT breaking is thus an anomaly-frequency difference. Denoting the exact physical energy levels with possible CPT violation by \( E_{n,s}^p \) and \( E_{n,s}^p \), the corresponding figure of merit providing a well-defined measure of the violation is
\[ r_{\mu \nu}^p \equiv \frac{|\Delta \omega _{\mu \nu}^p|}{\omega _{\mu \nu}^p} \]

where the weak-field, zero-momentum limit is understood. We find

\[ r_{\mu \nu}^p \approx |\Delta \omega _{\mu \nu}^p|/2m_p \approx |2\Delta \omega _{\mu \nu}^p|/m_p \]

within the present theoretical framework.

Assuming an experiment could measure \( \omega _{\mu}^p \) and \( \omega _{\nu}^p \) with a resolution of the order of \( 2\pi \times (1 \text{ Hz}) \) and assuming equality of \( \omega _{\mu}^p, \omega _{\nu}^p \) is observed to one part in \( 10^8 \), a bound of \( |\Delta \omega _{\mu \nu}^p| \leq 10^{-15} \text{ eV} \) becomes possible. The corresponding estimated bound on the figure of merit \( r_{\mu \nu}^p \) is

\[ r_{\mu \nu}^p \lesssim 10^{-23} \]

The estimate shows the promise held by this type of experiment to tightly bound CPT violation in a baryon system. The standard-model extension has also been applied to neutral mesons \([10]\) and leptons \([11]\).

Measurements of diurnal variations in the anomaly frequency could also place bounds on a combination of couplings in the standard-model extension. An estimate of one part in \( 10^{-33} \) has been made for a suitable figure of merit \([6]\).

**CHARGE-TO-MASS RATIOS**

Penning-trap experiments confining single protons and antiprotons can provide precision comparisons of their cyclotron frequencies \([2]\), yielding the limit \( |\Delta \omega _{\mu \nu}^p|/\omega _{\mu \nu}^p \leq 10^{-9} \). Equation (1) gives the corresponding conventional figure of merit \( r_{\mu \nu}^p \) and its bound.

Within the present theoretical framework, the perturbed proton and antiproton cyclotron frequencies are given in Eq. (4). Both are independent of leading-order CPT-violating quantities. As the cyclotron frequencies are unshifted even if CPT is broken, a comparison of these frequencies would represent an inappropriate measure of CPT violation in the context of the present theory. For example, the figure of merit \( r_{q/m}^p \) in Eq. (1), which is proportional to the frequency difference \( \Delta \omega _{\mu \nu}^p \), may vanish even though explicit CPT violation occurs in the standard-model extension.

The effect of the Lorentz-breaking but CPT-preserving couplings is to induce identical shifts in the proton and antiproton cyclotron frequencies. This indicates that the frequency difference \( \Delta \omega _{\mu \nu}^p \) would also be an inappropriate measure of Lorentz violation in the present theoretical context.

Another possible experimental signal is the occurrence of diurnal variations in the cyclotron frequencies, which could be induced by the Earth’s rotation during an experiment. Such variations would arise in the present standard-model extension from the dependence of the cyclotron frequencies on the components \( |c_{\mu 1}^p + c_{\mu 2}^p| \) of \( c_{\mu \nu}^p \).

A suitable theoretical figure of merit can be introduced by defining for the proton

\[ \Delta \omega _{\mu \nu}^p \equiv \frac{|c_{\mu 1}^p + c_{\mu 2}^p|}{\omega _{\mu \nu}^p} \]

It is the amplitude \( r_{\mu \nu}^p \) of periodic fluctuations in \( \Delta \omega _{\mu \nu}^p \). We find

\[ r_{\mu \nu}^p \approx |c_{\mu 1}^p + c_{\mu 2}^p|/m_p \]

in the comoving Earth frame. The appearance of \( \omega _{\mu}^p \) implies that the value of this figure of merit depends on the magnetic field.

A crude upper bound on \( r_{\mu \nu}^p \) can be obtained from the data in Ref. [2], which represent alternate measurements of proton and antiproton cyclotron frequencies \( \omega _{\mu}^p, \omega _{\nu}^p \) over a 12-hour period. The slow drifts in these frequencies are confined to a band of width about \( 2\pi \times (2 \text{ Hz}) \). This suggests a bound on a possible diurnal variation in \( r_{\mu \nu}^p \) arising from the contribution proportional to \( |c_{\mu 1}^p + c_{\mu 2}^p| \), given by

\[ r_{\mu \nu}^p \lesssim 10^{-24} \]

Diurnal fluctuations in the antiproton cyclotron frequency could be analyzed similarly.
EXPERIMENTS WITH HYDROGEN IONS

The precision of proton-antiproton cyclotron-frequency comparisons is limited by the need to reverse the electric field each time the other species is loaded in the trap [2]. A recent experiment by Gabrielse and coworkers [12] has addressed this issue by comparing antiproton cyclotron frequencies with those of an $H^-$ ion instead of a proton. The electric field is fixed throughout the experiment, and the magnetic-field variation between measurements is reduced due to the rapid interchange possible between simultaneously trapped hydrogen ions and antiprotons. The expected theoretical value of the difference $\Delta \omega_c^{H^-} \equiv \omega_c^{H^-} - \omega_c^e$ can be obtained in the context of conventional quantum theory using known values of the electron mass and the $H^-$ binding energy. Comparison of this theoretical value with the experimental result for $\Delta \omega_c^{H^-}$ is expected to provide a symmetry test with a ten-fold improvement on the previous test [2].

The theoretical analysis of this experiment within the present theoretical framework requires a description of the electromagnetic interactions of the hydrogen ion in a Penning trap in the presence of possible CPT and Lorentz violation. The hydrogen ion is treated as a charged composite fermion of mass $m_{H^-}$, so its electromagnetic interactions can be discussed within an effective spinor electrodynamics producing a modified hamiltonian of the form (2), but with a different set of CPT- and Lorentz-violating couplings. The modified cyclotron frequency is then calculated as for the proton-antiproton case. All the effective CPT- and Lorentz-breaking couplings for a hydrogen ion are determined by appropriate combinations of the corresponding quantities for its constituent proton and electrons. Lowest-order perturbation theory can be used to find approximations to these relationships, giving expressions involving the proton-antiproton couplings as well as electron-positron couplings $a^e_{\mu\nu}$, $b^e_{\mu\nu}$, $H^e_{\mu\nu}$, $c^e_{\mu\nu}$, and $d^e_{\mu\nu}$.

Subject to the approximations above, the component $\Delta \omega_{c,\text{th}}$ of $\Delta \omega_c^{H^-}$ that is determined theoretically to arise purely from CPT- and Lorentz-violating effects is

$$\Delta \omega_{c,\text{th}} \approx (e_{\parallel\parallel}^0 + e_{\parallel 1}^0 + e_{22}^0)(\omega_c - \omega_c^{H^-}) - \frac{2m_e}{m_p}(e_{11}^e + e_{12}^e - e_{22}^e - e_{11}^0 + e_{22}^0)\omega_c^{H^-}.$$  \hspace{1cm} (13)

Again, $\omega_c$ is the proton-antiproton cyclotron frequency in the absence of CPT or Lorentz perturbations. This result implies that in the context of this theory the experiment constrains a combination of Lorentz-violating but CPT-preserving quantities, including $e_{\parallel\parallel}^0$ and $e_{22}^0$. The latter would be inaccessible through the experiments with cyclotron or anomaly frequencies. In addition, this experiment does not look for diurnal variations in the cyclotron frequency, which means potential systematics associated with diurnal field drifts are eliminated.

The definition of a model-independent figure of merit follows from considerations similar to those leading to the figures of merit defined above. We define the quantity

$$\Delta \omega_{c}^{H^-} \equiv \left| \frac{\omega_{c}^{H^-} - \omega_{c}^{H^-}_0}{2\omega_{c}^{H^-}} \right| \left| \frac{\omega_{c}^{H^-} - \omega_{c}^{H^-}_0}{2\omega_{c}^{H^-}} \right|.$$  \hspace{1cm} (14)

As given here, $\Delta \omega_{c}^{H^-}$ is nonzero even if CPT and Lorentz symmetry is preserved. To arrive at a measure that vanishes in the exact symmetry limit, we remove from the hydrogen-ion terms in $\Delta \omega_{c}^{H^-}$ the conventional contributions arising from the differences between the $H^-$ ion and a proton: the masses of the two electrons and the binding energy. The result is an appropriate figure of merit for Lorentz violation, denoted by $\Delta \omega_{c}^{H^-}$.

Estimating a precision of one part in $10^{18}$ in measurements of the ratio $|\Delta \omega_{c}^{H^-}|/\omega_{c}^{H^-}$, we estimate an experimentally attainable bound of $\omega_{c}^{H^-} \lesssim 10^{-25}$. Indeed, the Gabrielse experiment [12] placed a bound of

$$\omega_{c}^{H^-} \lesssim 4 \times 10^{-26}$$  \hspace{1cm} (15)

on this figure of merit.

CONCLUSIONS

We have used a general theoretical framework based on an extension of the standard model and quantum electrodynamics to establish and investigate possible signals of CPT and Lorentz breaking in Penning-trap experiments with protons and antiprotons. We have looked for leading-order limits arising from precision measurements of anomaly and cyclotron frequencies.

Sharp tests of CPT symmetry would be possible in experiments comparing anomaly frequencies. We have introduced appropriate figures of merit with attainable bounds of approximately $10^{-25}$ for a plausible experiment with protons and antiprotons.
In contrast, comparative measurements of cyclotron frequencies for protons and antiprotons are insensitive to leading-order effects from CPT breaking within the present framework. However, diurnal variations of cyclotron frequencies and comparisons of cyclotron frequencies for hydrogen ions and antiprotons are affected by different CPT-preserving Lorentz-violating quantities. These experiments could generate bounds on various dimensionless figures of merit at the level of $10^{-24}$ in the proton-antiproton system, and $10^{-25}$ using the $H^-$-antiproton system.

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