The dyadospere of black holes and gamma-ray bursts
of the mass-energy of an extreme rotating black hole can be stored as rotational energy and gedanken experiments have been conceived to extract such energy (see e. g. Ruffini & Wheeler 1971). In the case of black holes endowed with an electromagnetic field, it follows from Eqs. (1-3) that up to 50% of the mass energy of an extreme EMBH with \( \frac{Q_{\text{max}}}{m} = r_+c^2/\sqrt{G} \) can be stored in its electromagnetic field. It is appropriate to recall that even in the case of an extreme EMBH the charge to mass ratio is \( \sim 10^{18} \) smaller then the typical charge to mass ratio found in nuclear matter, owing to the different strength and range of the nuclear and gravitational interactions. In other words it is enough to have a difference of one quantum of charge per \( 10^{18} \) nucleons in the collapsing matter for an EMBH to be extreme.

By applying the classic work of Heisenberg & Euler (1931) to EMBH’s, as reformulated in a relativistic-invariant form by Schwinger (1951), Damour & Ruffini (1975) showed that a large fraction of the energy of an EMBH can be extracted by pair creation. This energy extraction process only works for EMBH black holes with \( M_\text{r} < 10^8M_\odot \). They also claimed that such an energy source might lead to a natural explanation for GRBs and for ultra high energy cosmic rays.

The general considerations presented in Damour & Ruffini (1975) are correct. However, that work has an underlying assumption which only surfaces in the very last formula: that the pairs created in the process of vacuum polarization are absorbed by the EMBH. That view is, however, fundamentally modified by the introduction of the novel concept of the dyadosphere of an EMBH (Ruffini 1998, Preparata, et al. 1998) and by the considerations given below.

For reasons of simplicity we use the case of a non-rotating Reissner-Nordstrom EMBH to illustrate the basic gravitational-electrodynamical process. The case of a rotating Kerr-Newmann black hole, namely an EMBH with angular momentum, will be considered elsewhere by Damour and Ruffini (in preparation).

Introducing the dimensionless parameters \( \mu = \frac{M}{M_\odot} > 3.2, \xi = \frac{Q}{Q_{\text{max}}} \leq 1 \), the horizon radius may be expressed as

\[
r_+ = \frac{G M}{c^2} \left[ 1 + \sqrt{1 - \frac{Q^2}{GM^2}} \right] = 1.47 \cdot 10^5 \mu (1 + \sqrt{1 - \xi^2}) \text{ cm.}
\]

Outside the horizon the electromagnetic field measured by an orthonormal tetrad at rest at a given radius \( r \) in the Boyer-Lindquist coordinate system (see e.g. Ruffini 1978) has only one nonvanishing radial component \( E = \frac{Q}{r^2} \). We can evaluate the radius at which the electric field reaches the critical value \( E_c = \frac{m c^2}{e} \) defined by Heisenberg and Euler, where \( m \) is the mass and \( e \) the charge of the electron. We can then express (Ruffini 1998, Preparata et al. 1998) the radius of the dyadosphere in terms of the

![Fig. 1. Allowed range of variability of \( \mu = \frac{M}{M_\odot} \) and \( \xi = \frac{Q}{Q_{\text{max}}} \) for which the dyadosphere exists. The allowed region lies below the curve shown in the figure, see text.](image)

Planck charge \( q_\text{P} = (\hbar c)^{\frac{3}{2}} \) and the Planck mass \( m_\text{P} = (\hbar c)^{\frac{3}{2}} \) in the form

\[
r_{\text{ds}} = \left( \frac{\hbar}{mc} \right)^{\frac{3}{2}} \left( \frac{GM}{c^2} \right)^{\frac{1}{2}} \left( \frac{m_\text{P}}{m} \right)^{\frac{1}{2}} \left( \frac{e}{q_\text{P}} \right)^{\frac{1}{2}} \left( \frac{Q}{\sqrt{GM}} \right)^{\frac{3}{2}} = 1.12 \cdot 10^8 \sqrt{\mu \xi} \text{ cm},
\]

which clearly illustrates the hybrid gravitational and quantum nature of this quantity. The dyadosphere extends over the radial interval \( r_+ \leq r \leq r_{\text{ds}} \). It is important to note that the radius of the dyadosphere is maximum for the extreme value \( \xi = 1 \) and that the dyadosphere exists for EMBH’s with mass from the upper limit for neutron stars at \( \sim 3.2 M_\odot \) all the way up to a maximum mass of \( 6 \cdot 10^8 M_\odot \); correspondingly smaller values of the maximum mass are obtained for smaller values of \( \xi \), as indicated in Fig. 1 where we plot the range of \( \xi \) values for which vacuum polarization can occur for selected values of the EMBH mass. The range of variability of \( \xi \) is subject to the inequality \( \xi_{\text{min}} \leq \xi \leq 1 \) where \( \xi_{\text{min}} \) is implicitly defined by

\[
\mu = 6 \cdot 10^5 \frac{\xi_{\text{min}}}{(1 + \sqrt{1 - \xi_{\text{min}}^2})^2}.
\]

The electromagnetic field, which decreases inversely with the mass, never becomes critical for EMBH’s with mass larger than the maximum value stated above.

The density and energy of pairs created in the dyadosphere can be modeled (Preparata et al. 1998) by imagining the dyadosphere to be a sequence of concentric thin shell spherical capacitors at successive values of the radius
The density $n_{e^+ \gamma^-}(r)$ as a function of the radial coordinate for EMBH’s of $10 M_\odot$ corresponding to the charge parameter values $\xi = 1$ (upper curve) and $\xi = 0.1$ (lower curve).

$r$, each of thickness $\lambda \sim O(\frac{\lambda}{r_{ds}})$ and charge $\Delta Q(r)$ given by

$$\Delta Q(r) = Q \left[ 1 - \left( \frac{r}{r_{ds}} \right)^2 \right]. \quad (7)$$

On a very short time scale $O(\frac{\lambda}{r_{ds}})$, the QED vacuum dielectric breakdown produces a number density of pairs $n_{e^+ \gamma^-}(r)$ such that $E(r) \propto \frac{e}{\epsilon_0}$. This approximate equality simply means that the $e^+ e^-$ pair creation exponentially decreases when the initial electric field is screened to the critical value. It is important to emphasize that the first layer of thickness $\lambda$ outside the horizon produces a number of pairs sufficient to reduce the charge of the EMBH to $Q_c = \frac{e}{\epsilon_0} r_{ds}^2$, its critical value in the sense of Heisenberg and Euler. The total number of pairs actually created in the dyadosphere is very much larger than the number captured by the black hole, since it is amplified by the factor $(r_{ds} - r_+)/\lambda$.

The density of pairs as a function of the radius is then given by

$$n_{e^+ \gamma^-}(r) = \frac{1}{4\pi \epsilon_0^2} \left( \frac{Qmc}{\epsilon_0^2} \right) \left[ 1 - \left( \frac{r}{r_{ds}} \right)^2 \right]. \quad (8)$$

In Figs. 2 and 3, we plot the density of pairs for $10 M_\odot$ and $10^5 M_\odot$ EMBH for selected values of $\xi$. These two values of the mass were chosen to be representative of objects typical of the galactic population or for the nuclei of galaxies compatible with our upper limit of the maximum mass of $6 \cdot 10^5 M_\odot$.

We are now in a position to compute the total number of pairs $N_{\text{pair}}$ created in the dyadosphere and from a knowledge of the electrostatic energy density in each shell, the energy density of created pairs as a function of the radial coordinate and the total energy $E_{\text{pair}}^{\text{tot}}$ in the pairs. Finally we can estimate the total energy extracted by the pair creation process in EMBH’s of different masses for selected values of $\xi$ and compare with the values with the maximum extractable energy given by the above formula for black holes (see Eqs. (1) and (3)). This comparison shows that the efficiency sharply decreases as one reaches the maximum value of the EMBH mass permitting vacuum polarization, while the efficiency approaches 100% in the low mass limit (Preparata et al. 1998).

Due to the very large pair density given by Eq. (8) and to the sizes of the cross-sections for the process $e^+ e^- \leftrightarrow \gamma + \gamma$, the system is expected to thermalize to a plasma configuration for which

$$N_{e^+} = N_{\gamma^-} = N_{\gamma} = N_{\text{pair}} \quad \text{(9)}$$

and reach an average temperature

$$kT_{\text{a}} = \frac{E_{\text{pair}}^{\text{tot}}}{3N_{\text{pair}} \cdot 2.7}. \quad \text{(10)}$$

where $k$ is Boltzmann’s constant. The average energy per pair $\frac{E_{\text{pair}}^{\text{tot}}}{N_{\text{pair}}}$ is shown as a function of the EMBH mass for selected values of the charge parameter $\xi$ in Fig. 4.

As shown by Ruffini et al. (1998) the further evolution of this plasma leads to a relativistic expansion, $e^+ e^-$ annihilation and an enormous pair-electromagnetic-pulse “P.E.M. pulse”. By introducing a variety of models based on relativistic hydrodynamical equations, it has been shown that the dyadosphere of the EMBH reaches
relativistic expansion with a relativistic gamma factor 100 - 1000 within seconds.

If this basic scenario is confirmed by observations, other fundamental questions should be investigated to understand the origin of the dyadosphere. Some preliminary work along these lines has already been done by Wilson (1975, 1977) who has shown how relativistic magnetohydrodynamical processes occurring in the accreting material around an already formed black hole lead naturally to very effective charge separation and to reaching the critical value of the charge given by the limiting value of Eq. (3) on a timescale of $10^2 - 10^3 \text{GM}/c^2$. These studies show the very clear tendency of powerful processes of charge separation and of magnetosphere formation to occur in accretion processes. They cannot, however, be simply extrapolated to the formation of the dyadosphere. The time scale of the dyadosphere discharge is of the order of $10^{-13}$ sec (for a detailed discussion including relativistic effects, see Jantzen & Ruffini 1998). Such a time scale is much shorter than the characteristic magnetohydrodynamical time scales. We expect that the formation of the dyadosphere should only occur during the gravitational collapse itself and in the process of formation of the EMBH, with the formation of a charge depleted region with an electric field sufficient to polarize the vacuum. A complementary aspect dealing with the extermination and equipartition of the electromagnetic energy in a gravitating rotating object advanced in Ruffini & Treves (1973) should also now be considered again. Modeling such a problem seems to be extremely difficult at the present time, also in absence of detailed observations narrowing down the values of the fundamental parameters involved. Confirmation of the basic predictions of the dyadosphere model by observations of gamma ray bursts and their afterglow would provide motivation and essential information to attack such a difficult problem.

It goes without saying that the Heisenberg & Euler (1931) process of vacuum polarization considered here and in Damour & Ruffini (1975) has nothing to do with the evaporation of black holes considered by Hawking et al. (1974) either from a qualitative or quantitative point of view. The effective temperature of black hole evaporation given by Hawking for a $10^{58}$ black hole is $T \sim 6.2 \times 10^{-5} K^*$, which implies a black hole lifetime of $\tau \sim 10^{66}$ years and an energy flux of $10^{-24}$ ergs/sec, while a $10^{58}$ black hole has a Hawking temperature of $T \sim 6.2 \times 10^{-13} K^*$ and lifetime of $\tau \sim 10^{58}$ years with an energy flux of $10^{-53}$ ergs/sec!

References

Heisenberg W., Euler H., 1931, Zeits. Phys. 69 742.
Hawking S.W., 1974, Nat. 288 30.
Ruffini R., 1998, at the XIIXth Yamada Conference on “Black Holes and High-Energy Astrophysics” ed. H. Sato. Univ. Acad. Press, Tokyo, the concept of the dyadosphere (from the Greek word duois-duais for pairs) was presented there.
Schwing J., 1951, Phys. Rev. 82 664.