QFT at the Turn of the Century: old principles with new concepts (an essay on local quantum physics)

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Abstract

Historical aspects as well as the present state of QFT are analysed from a new viewpoint whose mathematical basis is the modular theory of von Neumann algebras. Its physical consequences suggest a new symmetry concept as well as a novel way of dealing nonperturbatively with interactions. The former generalizes the infinite dimensional diffeomorphism groups of low dimensional conformal theories to infinite dimensional groups in higher dimensions, with all symmetries beyond the Poincaré group being either partially or totally "hidden" (non-pointlike). Interactions are incorporated by using the fact that the S-matrix is a relative modular invariant of the interacting- relative to the incoming- net of wedge algebras. This new point of view allows many interesting comparisions with the standard quantization approach to QFT.

1 Looking at the Past with Hindsight

To a contemporary observer the area which was once called particle physics, with QFT being its main theoretical tool, appears fractured into several loosely connected regions, ranging from the Gauge Theory of the Standard

Model, low dimensional conformal- and factorizing- models and the algebraic framework into to string theory, including its recent offspring. Each area has a special set of concepts and computational tools. Even subjects which are usually considered to be parts of QFT proper, as e.g. Lagrangian QFT and chiral conformal QFT, belong for most physicist into different drawers with hardly any interrelation. Imagine somebody who calculates radiative corrections using Lagrangian perturbation theory as a service to his experimental collegues during the week, and at the end of the week he relaxes with one of his hobbies, say chiral conformal QFT. Apart from at most some computational tricks of how to evaluate certain integrals which belong to the mathematical toolbox, there would not be much in common; in fact he would even have a hard time to justify why the same expression "QFT" appears in both of them, because the representation theoretical approach based on affine- and W-algebras has little to do with Lagrangian quantization. He would be very hard-pressed to extract any message from chiral conformal QFT, which would be valuable for his perturbative Lagrangian quantization approach.

In this essay I would like to convince the reader, that the present schism may not inevitably accompany us into the next century. There are signs of an unexpected conceptual (and perhaps even computational) development towards a new unification which could bring together most of the above areas into what one may call (lacking a more specific name and following the suggestion [1]) "**L**ocal **Q**uantum **P**hysics". As in most cases of change of paradigm, the difficult step is to liberate oneself from mighty accumulated prejudices.

Claiming to have new ideas, without confronting them carefully with the old principles underlying QFT, is in the worst case a marketing trick and at best a mathematical enrichment. Therefore it will be important to take a look at the birth of modern QFT at a time when there was still a unity: the time of renormalized perturbation theory and its successful application for the radiative corrections in QED which later on got significantly enriched by passing and extending these techniques to the standard model. Without this unified start and its impressive success, none of the above areas of QFT would have emerged, and perhaps QFT would have been a dead issue in the second half of this century.

The diverging viewpoints arose in the aftermath of perturbation theory when it became clear that one had to develop new nonperturbative concepts in order to understand strong interactions. It is very natural in such a

challenging theoretical situation to try different roads towards this goal. Indeed, one easily discerns three different ways of handling this problem which emerged at the beginning of the 60^{ies}.

Let us start with the S-matrix point of view, which later sunk into obliviousness, but in the present historical review is still of interest. Ever since the birth of QT there was the attractive message (from Heisenberg and Bohr) that, if one enters new physical territory, it is safer to look primarily at what one considers to be the indispensable observables, which are either close to the experimental hardware or at least susceptible to Gedanken-Experiments. Within the setting of the (successfully checked) Kramers-Kronig dispersion relations adapted to particle physics, the observable crown of particle physics was certainly the S-matrix. Although the Chew-Stapp bootstrap approach added some interesting aspects to the older 1943 Heisenberg attempt at a pure S-matrix theory (e.g. the crossing symmetry as an on-shell adaption of causality and its close relation to the TCP and Spin-Statistics consequences), its presentation as a theory of everything (more precisely a TOE minus gravity) was ill-conceived. Even if the nonlinear structures emerging from "maximal analyticity", unitarity and crossing symmety would have had the same formal clarity as the nonlinear Schwinger-Dyson equations, there would have still remained the excruciating task to solve such nonlinear bootstrap equations. Whereas nonperturbative models as chiral conformal QFT may be considered solutions of the Schwinger-Dyson equations, it is only due to additional physical insight into their superselected charges and their fusion laws (or representation structure) that one successfully linearizes the nonlinear structure and thus converts this "nonlinear dynamite" into a working tool, as it happened for chiral conformal field theories.

The second and certainly most widely followed approach was based on the idea that if one excepts the framework of renormalized perturbation theory and finds the right extensions of the Lagrangian quantization formalism (which was already more or less the setting in which Heisenberg and Pauli formulated in the 30^{ies} their famous contribution to QFT), one would eventually obtain a nonperturbative understanding and computational control. Most of the formalism which entered the textbooks as e.g. functional integral representations, and most gains in physical insight as embodied in gauge theories and asymptotic freedom which led up to the "standard model", we owe to this approach. Attempts to extract from Feynman graphs analytic, algebraic or combinatorial properties in the hope to find a "nonperturbative

nugget" also belong to this mode of thinking[1]. In fact the very meaning of QFT is often identified with this quantization of classical actions.

The historical origin of string theory (which we only mention in passing) has to do with the Veneziano dual model, which in turn is related to both of the aforementioned attempts. Unlike QFT, it took its most important input, namely crossing symmetry, from the S-matrix framework as an "axiomatic" input, i.e. without trying to reduce it to the more basic locality and spectral properties. In fact it was a proposal for a strong interaction S-matrix, and by an interesting observation of Virasoro, the *on-shell formalism of Veneziano could be rephrased as a kind of off-shell auxiliary field theoretic formalism in lower space-time dimension* with the living space of the S-matrix being the "target space". One of the few potentially deviating element from the spirit of QFT was the appearance of infinite towers of particles in a finite mass range. But as in Feynman's perturbation theory, one could try argue that in higher order (of unitarization, counted by genus), apart from some low lying stable particles in each charge sector, the particles in the tower become unstable (genus ≥ 2) and appear as resonances in the second Riemann sheet of the analytically continued S-matrix[2]. In this case there would be nothing "stringy" which separates the unitarized crossing symmetric dual model S-matrix from one obeying the principles of Quantum Field Theory. To arrive at the present form of string theory two more "revolutions" were necessary. As a result the origin of string theory as a trial model for the duality aspect ("nuclear democracy") of crossing symmetry was forgotten or got lost. The promise to close up the gap in understanding the latter as a on-shell manifestation of Einstein causality and an enhancement of nonperturbative knowledge about how to deal with strong interactions was not any more one of string theories main concerns, and thus the theory went (in its relation to physics as a natural science) into the "blue yonder" (to borrow some words used by Feynman in another context), driven by mathematical wanderlust.

The third approach, which we agreed to call local quantum physics, was started by physicists who either found that the rigour of canonical or path integral quantization left a lot to be desired[3] or, what will be of greater im-

[1] A nice recent illustration is the recent extraction of knot structures from the Feynman theory [2].

[2] Infinitely many particles with accumulation points at finite masses would lead to an unacceptable phase space degrees of freedom behaviour and cause a pathological thermal behaviour.

[3] Nonperturbative QFT was (and for realistic models still is) the first physical theory

portance for the main theme of these notes, they were convinced that without a new conceptual progress one will not get away from the perturbative framework.

The first step in this third approach was to realize that it would not be a bad investment to try to remove some of the "artistic" procedures of the Lagrangian quantization approach. Everybody knows that such useful ideas as canonical quantization or functional integrals cannot be taken literally, because at the end of a chain of arguments (i.e. after renormalization of the perturbative results or after taking the scaling limit in a Wilson type lattice approach) *the fields will almost never* (exception: some super-renormalizable theories) *be canonical or obey a functional integral representation.* The only relicts of these artistic inputs will be the Einstein causality (spacelike commutativity of observable fields). Another well-known obstruction against the literal use of Dirac's interaction picture in QT for the dynamics of QFT is Haag's theorem, which says that the interaction picture in a translational invariant theory is incompatible with vacuum polarization caused by interactions [1].

In order to avoid misunderstandings already at their roots, it may be helpful to state clearly that causality and localization in the sense of this essay are quantum notions. An observable is called localized e.g. in a double cone region $\mathcal{O}$ in Minkowski space (a natural family invariant under Poincaré transformations) if it commutes with all the observables "outside" (the causal disjoint $\mathcal{O}'$). Therefore the quantum localization relates inexorably inside and (causal) outside. Often it can be rephrased in terms of support properties of classical functions. Its close connection with the von Neumann algebraic view of QT [4] and the measuring process comes through the realization that Einstein causality implies an *a priory knowledge about the size of the commutant* $\mathcal{A}(\mathcal{O})'$ (the totality of all observables which are commensurable with $\mathcal{A}(\mathcal{O})$): $\mathcal{A}(\mathcal{O})' \supset \mathcal{A}(\mathcal{O}')$ for all $\mathcal{O}$. In fact in most cases for the vacuum representation one has equality of the two algebras ("Haag duality"). The only exception is spontaneous symmetry breaking. In that case one can always enlarge the algebras $\mathcal{A}(\mathcal{O})$ keeping the same Hilbert space such that the extended net is Haag dual. For an intuitive understanding one should notice that a bigger net means a lesser number of superselected charge sectors

which had to cope with a serious problem of existence within a given set of physical principles. Although physicist have learned to cover up this serious flaws semantically by saying words like "I believe that asymptoticall free couplings have a well-defined existing QFT" this situation did not change for more than 20 years.

or in case of the associated symmetry group for d=3+1 theories, a smaller (unbroken) group [31].

The Wightman framework and the LSZ formalism only used concepts which are intrinsic properties of the finite ready-made QFT and which are therefore independent of by what means it was produced. Applying this framework to perturbation theory, one obtained the finite renormalized results without infinities or the necessity to cut-off integrals for large energies. The inductive "causal perturbation" theory of Epstein and Glaser with important previous contributions of Stueckelberg, Bogoliubov and Shirkov is a good illustration. But also Schwinger knew (albeit in a less systematic way), by his point-split method, how field equations can be converted into a form so that (canonical or functional integration arguments can be avoided and) the computation remains fully finite. There is nothing surprising about this, because if it is true that causal renormalized answers have an objective and intrinsic meaning, there must be a way to obtain them without ever leaving the causal framework (i.e. without cutoffs or ad hoc regulators). Of course this is not feasible by a quantization procedure, if quantization means reference to a classical Lagrangian. For as Kramers pointed out, in this case of Lagrangian quantization, one has to lean on the old remedies of the infinite selfenergy treatment of Poincaré and Lorentz which come with the *classical particle models on top of a classical field theory.* In an LQP approach which does not refer to quantization on the other hand, there is no place for such a repair of the particle/field structure because the particle issue is fully covered (according to Wigner's particle picture) by the *representation of the Poincaré group which comes automatically with the covariant fields.* The only aspect of renormalization as far as the LQP approach is concerned is physical (re)parametrization.

After the Wightman and LSZ frameworks, an additional conceptual "toolbox" was introduced. The aim of this "Local Quantum Physics" LQP (or algebraic QFT) approach was to implement the idea that QFT had a certain similarity with representation theory of groups, if one views superselected charges as defining higher representation sectors of an observable algebra. The observable algebra corresponds to the abstract group algebra, whereas the superselection sectors correspond to the irreducible representations. This idea would have remained "abstract nonsense", without the powerful causality property which converts the observable algebra into a causal net of local observable algebras. Surprisingly, this gives enough structure to the net in order to allow for a classification of the possible representation sectors of

physical interest at least if one makes the mass gap assumption[4]. In fact it allows to reduce the representation problem to the study of endomorphisms of the algebra [1], something which is not possible in the case of group representations); an unexpected simplification in which the locality plays the essential role. In d=3+1 dimensional theories the result was more or less expected on the basis of quantization; in that case the interesting aspect consisted in the rich ingenuous way in which it was obtained: with the mass gap hypothesis one obtains local and so-called (cone-localized) topological charges (beware of confusions with the use of this word in euclidean functional integrals! Here it denotes a special class of superselection rules which have no compact localization and no description in terms of real time Lagrangian fields). In both cases the charges compose according to the fusion laws of compact group representations. In fact their carriers, the charged fields (intertwiner operators of the charged sectors with the vacuum), carry a generalized "Casimir" charge label and a generalized "magnetic quantum number" multiplicity label. The weaker spacelike cone (with a semiinfinite string as a core) localization of the topological case leads to weaker on-shell analyticity outside the standard dispersion theoretical setting of the compactly localized case. It cannot be overstressed that *the nonlocal charge carriers appear inside a perfectly local theory of observables [31].*

If the strings of the aforementioned string theory would obey the same causality and spectral properties, then they would be part of LQP and all the results and methods presented here would be applicable. But in this case there should be a *fundamental physical reason why the local theory wants these "stringy" objects* similar to the above topological charges (vis. the semiinfinite string-like localization of d=2+1 anyons and plektons). This would be a far shot from passing from point- to string-like localization on esthetical grounds or just for the sake of playing with something else. Even at the risk of repeating myself: the above semiinfinite string-like localized charge carriers exist naturally in LQP and t*heir stringlike localization is required because the principle of Einstein causality asks for its most general realization as representations of observable algebras*. Since the space-like localization cones can be made arbitrarily thin, there is still no elementary length entering the theory through the presence of these noncompact string-like objects. In a d=2+1 dimensional world, these objects would be responsible for braid

[4]Massless theories (especially abelian gauge theories) require a finer conceptual analysis on which some progress has been made recently [3]

group statistics (plektons) and if one would restrict ones interest only to compactly localized charge carriers whose scattering aspects remain within the standard analytic setting of dispersion theory, one would fall back on permutation group statistics i.e. on Fermions and Bosons with internal group symmetry i.e. one would prevent the causal theory to have its complete realization. In fact the very existence of these objects is already preempted by the structure of the model causal observable algebra. Their one particle states are described by the Wigner theory and do not reveal the stringy structure[5]; the latter only becomes visible in the new analytic structure of multiparticle states which is more complex than that of standard dispersion theory. In d=2+1 however, the "stringyness" has a more drastic consequence of leading to a change of particle statistics; instead of Fermions/Bosons one may have anyons and plektons which carry abelian or nonabelian braid group statistics.

Comparing the previous strings with those from string theory, one makes the following observation. It is very likely that string theory apart from "free strings" [5] deals with a truly nonlocal situation far removed from LQP, i.e. one which cannot be interpreted in terms of *conceptually required nonlocal objects on an underlying theory of local observables.* This situation is expected to be analogous to light-cone quantization, which in the interacting case produces nonlocal fields with no vacuum polarization whose relation to the original local fields is unknown (but there on expects at least a relation to local fields). Lacking a precise relation to a local theory, their use is limited to global features as the spectrum of particle masses, but without the knowledge whether they belong to Wigner particles in a LQP. This seems to be the present main massage from "M-theory".

The LQP in d$\leq 2+1$ [6] as well as the d=3+1 zero mass theories with Maxwell-like charges [3] offer new possibilities and requires a framework beyond the standard Lagrangian quantization which in the latter case, as well as for d=1+1 solitons [17], in fact is even wider than the one for braid group statistics.

The "revolutionary" message of LQP is on the other hand is not entirely new (and therefore not really revolutionary as it is similar to that one which led to renormalized perturbation theory): the old physical principles are maintained and even reinforced , but the conceptual framework of analyzing,

[5] The "stringy" nature in LQP never shows up in the one particle states, but only in the deviation from tensorial- and/or standard analytical- (scattering) mutiparticle structure.

interpreting and converting into computational tools changes radically. Similar to the times of renormalized perturbation theory QFT looks rejuvenated even though non of the physical principles underlying local quantum physics has been changed.

2 The Present Situation: Modular Structure of LQP

Although the LQP approach has been introduced in order to get a good hold on nonperturbative aspects, some of the power of its underlying philosophy can also be demonstrated in perturbation theory. Here one of course does not obtain different results but, as will be shown in the sequel, there is a different (less geometric and more quantum physical) interpretation. Since an account of this has been given elsewhere [4], we will be satisfied with a brief mainly verbal presentation, adapted to the present purpose.

The version of perturbation theory most close to the spirit of LQP is the causal perturbation theory of Stueckelberg, Bogoliubov, Shirkov, Epstein and Glaser. This approach was recently extended to curved space-time and gauge theory by Brunetti, Fredenhagen and Fredenhagen and Duetsch [7][8]. The interaction is implemented in Fock space via a Wick polynomial $W(x)$ in free fields associated to free particles. Since the association of free fields to particles is highly nonunique (as many as there are intertwiners between the Wigner- and the covariant representation) [9], the first question is which one should one use. The answer is: the different fields for the same (m,s) Wigner representation are just like different coordinates in differential geometry. One can use any of the infinitely many possibilities as long as one re-expresses the Wick-polynomial which defined the interaction in one “frame” in terms of the other “field-coordinates” [31]. Only if one does perturbation theory in the (euclidean) functional integral approach (i.e. by action quantization), one is restricted to a Lagrangian field, but the causal perturbation does not use (bilinear free) Lagrangians anywhere.

An elegant way to recover the uniqueness of the physics in the face of the infinitely many field coordinates is to use the concept of nets of local algebras. According the philosophy of LQP, there should be a way to avoid the fields altogether and go directly from the unique Wigner particles to the associated local net in Fock space. Indeed there is, and it uses the modular theory for

the purpose of getting a good formalism for (noncommutative, not classical as via support properties of functions or in euclidean functional integrals) localization, but these remarks are more important for our later presentations than for the causal perturbation theory at hand. For the latter we stick to fields since presently it is not known how to implement perturbative interactions without them. The most important object of causal perturbation theory is the unitary operator functional whose formal expression in Fock space is:

$$S(g) = Te^{i\int g(x)W(x)d^4x} \tag{1}$$

where $g(x)$ is a compactly supported test function which has the constant value $g(x) = g$ in a large double cone C centered around the origin and falls of to zero inside a "collar" around C in order to vanish outside a larger support double cone. As always when one is groping with mathematical meaning of formally defined objects, one may try to characterize this object by its desired properties; in this way one obtains the Bogoliubov axiomatics. And as usual in QFT, one does not know (apart from some super-renormalizable and not very interesting cases) if the axiomatics allows nontrivial solutions. Before passing to the perturbative expansion of this formal expression it is helpful to take notice of the fact that the Bogoliubov axiomatics can be successfully incorporated into the local net setting. One simply extends the above definition to (ψ denotes the free fields)

$$S(g,h) = Te^{i\int\{g(x)W(x)+h(x)\psi(x)\}d^4x} \tag{2}$$

which then allows to introduce other objects which play an important role in the formulation of causality

$$\begin{aligned} V(g,h) &\equiv S(g,h=0)^{-1}S(g,h) \qquad (3)\\ \mathit{causality} &: \quad V(g,h_1+h_2) = V(g,h_1)V(g,h_2) \\ \mathit{if}\ \operatorname{supp} h_1 &\notin \operatorname{supp} h_2 + \bar{V}_+ \end{aligned}$$

The following consideration is taken from reference [7].

The net is defined in terms of the following algebras

$$\mathcal{A}_g(\mathcal{O}) \equiv \operatorname{alg}\{V(g,h), \operatorname{supp} h \subset \mathcal{O}\} \tag{4}$$

In fact it follows from the causality that a change δg of the coupling strength g outside the supporting double cone C_{sup} of g does not change

the net $\mathcal{A}_g(\mathcal{O})$ for $\mathcal{O}$ inside C, except for a common unitary (the nets are isomorphic i.e. considered to be identical)

$$\begin{aligned} V(g+\delta g, h) \quad &= \quad AdU(g,\delta g)V(g,h) \\ \operatorname{supp}\delta g \quad &\quad \mathit{outside} \;\; C_{\sup} \end{aligned} \tag{5}$$

which changes the individual subnet members $\mathcal{A}_g(\mathcal{O}) \subset \mathcal{A}_g(C)$, but not their relative positions which characterize the net. Since C is arbitrary, the global net (inductive or universal C^*-limit) is a well-defined C^*-algebra whose construction does not require any furthergoing assumptions about the existence of an adiabatic limit. Different from the interpretation of the standard (quantization) approach, the Fock space is strictly auxiliary and only serves to link the interaction polynomial W with a concrete C^*-algebra formed by a net of local von Neumann algebras. In particular it is not to be confused with the physical states (e.g. the Fock space of the asymptotic scattering states); in fact the separation of local observable algebras and physical states on them is one of the hallmarks of LQP. According to the physical situation one may put e.g. a vacuum- or thermal- state on this algebra and recover a Hilbert space and concrete operators (i.e. the attributes of standard QT) via the canonical GNS construction.

As already mentioned, this Bogoliubov axiomatics suffers from the same global limitation as e.g. the functional integral quantization approach and can only be dealt with as a infinitesimal deformation which is the renormalized perturbation. Although the Bogoliubov approach is conceptually different from quantization, it contains an ad hoc element which separates it somewhat from LQP and practically imposes the same restrictions as for quantization: the assumption that interactions have to be implemented by a Wick-polynomial in a free field Fock space. From an intuitive point of view it is hard to imagine that in this setting one could ever liberate oneself from being restricted to an infinitesimal neighborhood of free fields. In fact the idea to couple known local fields, in order to study infinitesimal deformations around these known islands, is of a rather general nature. Its only handicap is that it does not help in the discovery of these islands (note that the point of view I present here is somewhat more pessimistic than the present renormalization[6] group philosophy), apart from some super-renormalizable

[6]Even the nontrivial exact solutions to 2-dimensional lattice models have not been found by renormalization group methods but rather by algebraic methods a la Baxter or affine algebras. Our constructive modular methods have more similarities with the latter.

situations where one can demonstrate the existence of a global theory behind the infinitesimal deformation (in lucky cases, the two are related by Borel resummation).

Continuing the present essay style, we will of course not enter the technical details[7] of renormalization which in the present context means causal extension of the time-ordered operator-valued coefficients of $S(g)$ in a power series in g and the analogous (uniquely related) retarded coefficients in the power series for:

$$A_g(x) \equiv \frac{i\delta}{\delta h(x)} V(g,h) \mid_{h=0} \tag{6}$$

which is a pointlike field associated to $\mathcal{A}_g(\mathcal{O})$. The advantage of the causal framework is that the causal extension process (causality alone would not fix the total diagonal part of the (n+1)th order in an inductive procedure from the nth order) together with the normalization conditions (≡re-normalization) takes place in one and the same reference Fock space, whereas e.g. in a functional integral approach there is no fixed quantum reference space since for each (euclidean) functional integral the reconstruction of a quantum space is by no means a foregone conclusion (not every functional integral defines a quantum theory; the criteria are very subtle and the inverse statement would also be wrong, if one leaves the area of permutation group statistics).

In order to accentuate the difference in physical concepts on the level of perturbation theory between the quantization approach and LQP somewhat more, let us briefly recall how both deal with the issue of renormalizable interactions involving higher spin particles. After almost twenty years of gauge theory I do not have to explain the meaning of Higgs mechanism for massive interacting vectormesons in the standard quantization of classical gauge theories. The reader is perhaps less familiar with the LQP approach to selfinteracting massive spin=1 particles. Some aspects of it were already alluded to in the old work of Lewellyn-Smith [10], but the recent interest in it was rekindled by some papers of Scharf in collaboration with various coauthors [11]. Adapted to the LQP framework used in the present work, one first observes that the previous method of using Wick-polynomials as interactions fails since the W involving at least one (m,s=1) field (necessarily with operator dimension $\geq 2 >$ classical dimension=1) and being at least

[7] We assume that the reader has a good knowledge of standard renormalization theory in particular in the setting of gauge theories and only emphasize those features where LQP offers a different interpretation.

trilinear must have an operator dimension ≥ 5, thus going beyond the limit set by renormalizability $\dim W = 4$. Thanks to gauge theory, we all know an emergency exit: use the BRS cohomological trick which brings the too high operator dimensions back to their classical values, do your renormalization computations in an unphysical space, and finally descend to physics at the end of the calculation. The lowering of the free field dimensions is already preempted by enlarging the Wigner space with ghost wave function in such a way that the inner product becomes milder for large momenta. It turns out that the grading of the BRS formalism follows if one wants to extend Wigner one-particle formalism to multiparticle states. The restriction to the massive vectormesons has the significant conceptual advantage that the BRS-Kugo-Ojima [12][13] formalism simplifies such that the charges Q are bilinear in the vectormeson and ghost fields. As a consequence only the formally local vectorpotential itself is unphysical, whereas the spinor matter fields are physical (i.e. unchanged in the descend to the physical cohomology space). Although the existing calculations in the causal framework are not quite complete, the following consequences cannot be seriously doubted:

- The first and second order calculations lead to the usual Yang-Mills structure for the selfcoupled vectormesons, i.e. the symmetries and the Jacobi-identity for the f_{abc} couplings result from the quantum physical consistency of the scheme. The couplings to the ghosts and to the spinor matter also follow suit.

- Without introducing an additional (this time physical) degree of freedom, it is not possible to have perturbative consistency. The simplest possibility is a scalar field (the alias Higgs field but here with vanishing vacuum expectation value); the present arguments do not completely exclude other possibilities (higher spin fields). These physical degrees of freedom cannot be preempted by extending the Wigner theory; they owe their existence to the quantum nature of the interaction. The spinor matter field ψ is physical, i.e. does not change in the cohomological descend[8].

[8] The translation to the standard gauge approach is the following: the scalar field entering through the consisteny argument corresponds to the alias Higgs field ϕ but now with vanishing vacuum expectation, and the matter field ψ corresponds to the alias composite $\psi \cdot \phi$.

Although the arguments are not entirely waterproof as a result of some computational gaps, even the most ardent gauge theory fans would be shocked if besides the gauge theory involving his "fattened vectormesons" (by the Higgs mechanism) there would exist another renormalizable interaction between massive spin=1 fields. Of course in view of the particle content of the electroweak coupling, the scalar (perturbative!) consistency companion of massive vectormesons merits a more precise future consideration in the present framework. Note however that the latter puts the relation classical/quantum field theory in the right order. The quantum side is the more fundamental one, and since together with renormalizability it seems to be unique (without the need of a gauge principle!) it is the highly ambiguous classical side which need to be told through the Bohr correspondence principle to follow the gauge principle. The opposite, i.e. the "quantization gauge principle" is not wrong, especially if it is not considered as a selection principle between several competing possibilities, but rather as a mnemotechnical device which selects an efficient computational recipe without getting bogged down with more fundamental matters. In this spirit the terminology "gauge theory" continues to make sense. The present method also brings into the open, that although there is no intrinsic physical meaning to the Higgs condensates, the Schwinger "charge screening" has a good intrinsic physical meaning [14][31].

The better known abelian massive $s = 1$ theory is in some sense an exception; in that case there exists another massive version without employing the BRS like cohomological extension, which is similar to the Gupta-Bleuler method of QED and often refereed to as the Stueckelberg extension. This leads to another massive theory in which the e.g. spinor matter field is renormalizable but unphysical or, after an operator gauge transformation involving the so-called Stueckelberg field, physical but nonrenormalizable (only the ψ-field, not the whole theory) i.e. in that "unitary gauge" the momentum space the ψ-correlation functions have an ever increasing (with perturbative order) power behavior. The conjecture (unchecked) would be of course that this theory is some sort of massive QED i.e. without suffering the charge screening phase transition.

The present LQP induced consistency point of view would only amount to a different interpretation of a well-studied subject if it would not give rise to new ideas which remain hidden (or are difficult to access) from the quantization approach. Such is the particle structure of the zero mass vectormesons i.e. the state space of QED and QCD. In that case there is no obvious

reference space since scattering theory cannot be invoked. A direct attack would lead to cohomological BRS operators Q with interacting (trilinear and higher) contributions [8] and an ever changing position of the physical cohomology space (depending on the perturbative order). The construction of the physical space is turned difficult by additional infrared problems which are not only outside the scope of scattering theory but also outside of perturbation theory. The good conceptual understanding of the massive theory with a renormalized physical spinor field (in the standard gauge approach the renormalized covariant ψ is not physical) suggests to try to approach the zero mass theory as a limit of the massive one. This cannot be done naively, because e.g. in the abelian case the point-like off-shell physical field as it stands cannot approach the unscreened charged field since the latter is known to be not pointlike but rather has a noncompact stringlike extension. Also the alias scalar Higgs field does not decuple naively. But having the decoupling problem of the latter interrelated with the emergence of a stringlike physical charge-carrying field should give a good hint of what (stringlike) modification is necessary in order to have a finite physical zero mass limit. Buchholz succeeded in the Schwinger model to illustrate the emergence of a stringlike charged state in the zero mass scaling limit. For the realistic case we have to leave this problem to the future. The same comment applies to the question if the cohomological extension idea could produce renormalizable interactions involving fields of higher spin>1 (which are perhaps not covered by the gauge quantization formalism [4]).

The cohomological method from the point of view of LQP is not less mysterious than from the standard gauge-theoretical quantization point of view. This begs the answer to the question about its deeper physical origin, in particular a more poignant answer to the question: why can one not stay throughout the whole procedure in a physical space and use a Wick-product formalism? A hint for the answer comes if one tries in the abelian massive case to construct a vectormeson field in the physical cohomology space. One finds by integrating up the field strength $F_{\mu\nu}$ that the cohomology space *can only accommodate a nonlocal semiinfinite stringlike extended vectorpotential* which than has the classical operator dimension 1. The zero mass situation points into the same direction of stringlike localized vectorpotentials and this can be seen already on the level of the Wigner theory: the photon does not admit (as a result of the noncompact structure of the Wigner little group) a covariant $D^{(\frac{1}{2},\frac{1}{2})}$ vector-like description in the unextended physical Wigner

space [9]; the best one can do is to introduce a semiinfinite stringlike potential which transforms under Lorentz-boosts in an affine manner (i.e. with an additional additive "gauge" term which remains localized in a wedge). The presence of these noncompact objects in an LQP approach to the free photon net can be seen in the breakdown of Haag duality for multiply connected spacetime regions [1] [31]. Although the observable part of the theory remains local, the interaction is transmitted by slightly nonlocal objects in such a way that the interaction cannot be described by standard local Wick polynomials of free fields. It is presently not known how to deal with such a situation of nonpointlike objects in a perturbation theory.

In LQP where one thinks in terms of nets of algebras, the issue of short distance behavior versus existence problems is much more subtle and almost hidden. For those nets which are generated by local fields (probably all physically admissable nets), on may think of a "minimal field coordinate", i.e. a generating field with the smallest possible operator dimension. It is not very palatable to think of a tight connection between the operator dimension of this field and the existence of the net. If the standard renormalizability (and existence) criterion is limited to perturbation theory or more generally to Lagrangian quantization, should be expected to be decidable in a LQP setting.

Let us now get to the proper area of LQP: *Symmetry.* There the shortcoming of the Lagrangian quantization approach has been always visible, although most pragmatically minded physicist accommodated themselves with this situation. The issue of internal symmetries ever since the time of Heisenberg's isospin had been handled by choosing field multiplets which transform linearly under compact group transformations (without asking about the physical origin), and spacetime symmetries were imposed in a kinematic fashion through Poincaré covariance of the fields without any visible relation to the algebraic structure of QFT. Attempts to obtain nontrivial "marriage" between the two failed [16], apart from the physically rather concocted and accidental (but mathematically natural and rich) supersymmetry. But it was just the approach to supersymmetry in the spririt of algebraic QFT by Haag Lopuszanski und Sohnius [1] through which many QFT physicist became aware of the existence of those algebraic methods.

Both aspects have been significantly changed through LQP. There internal symmetries emerge together with (particle) statistics from the endomorphism structure of the observables [1]. In four dimensions, at least with the mass gap assumption, the possibilities are exhausted by Fermions and

Bosons with compact group symmetries. In d=2+1 spacetime dimensions with the same mass gap assumptions the possibilities are exhausted by braid group statistics[9]. The ensuing "plektons" (with their abelian version being anyons) have transformation properties which "blur" the sharp distinction between inner and outer symmetries which one meets in d=3+1 theories. Finally the particle statistics concepts in massive d=1+1 is insufficient and has to be replaced by the concept of (half-space) solitons [17]. The endomorphism methods of LQP in the pursuit of the statistics/internal symmetry issue have a significant overlap with the V. Jones subfactor theory. This is coming out particularly strong in the case of braid group statistics where the Jones inclusion index is the square of the statistical dimension.

More recently there have been considerable advances in the understanding of the interrelation between algebraic structure and spacetime symmetries. Since I believe that this is presently the most promising new LQP research line, I will be more detailed in its description. This development started with Bisognano's and Wichmann's observation that the causal localization of the fields in the Wightman theory allow to unravel the modular structure of the associated von Neumann algebras belonging to the wedges. The wedge regions are the same as those introduced into classical gravity by Rindler and later used for the simplest illustration of the Hawking temperature by Unruh. Also the relation to the Bisognano-Wichmann issue is by no means new; it has been thoroughly discussed in a paper by Sewell [18]. What is more of recent vintage is the converse: the reconstruction of spacetime attributes (e.g. Poincaré symmetry) from modular properties of special von Neumann algebras. But first let us define what we mean by modular structure. Each von Neumann algebra in "standard form" i.e. a weakly closed operator algebra $\mathcal{A}$ together with a cyclic and separating vector Ω permits the definition of an unbounded closed Tomita involution S :

$$SA\Omega = A^*\Omega \tag{7}$$

whose polar decomposition $S = J\Delta^{\frac{1}{2}}$ yields the two modular objects: the

[9]As a side remark one may add that the very braid group statistics, even in the absence of additional interactions ("free"" plektons and anyons), requires the presence of vacuum polarizazion contributions in addition to the mass shell component of e.g. state vectors obtained by applying one string-localized anyon operator to the vacuum and this does not go away in the nonrelativistic limit (no anyonic Fock space). A nonrelavistic theory which preserves the relativistic anyonic spin-statistics connection remains therefore a QFT and is not a QM [4].

modular conjugation J which transforms $\mathcal{A}$ into its von Neumann commutant $\mathcal{A}'$

$$J\mathcal{A}J = \mathcal{A}' \tag{8}$$

and the unbounded modular operator Δ which defines via

$$\sigma_t(A) = \Delta^{it} A \Delta^{-it} \tag{9}$$

an automorphism of the algebra. With the help of unbounded involutory (on its domain) Tomita S one can define real closed subspaces H_R

$$H_R = \{\psi \in H \mid S\psi = \psi\} \tag{10}$$

Their complex superposition $H_R + iH_R$ is dense in H and forms, if equipped with the graph norm of S a Hilbert space in its own right.

This theory entered physics (or more correctly, a special version was independently discovered by physicists [1]) via the analysis of thermal states. For this reason one of the characteristic properties of this automorphism with respect to the state is called the KMS (Kubo-Martin-Schwinger) property because these authors introduced it in another context. Bisognano and Wichmann succeeded to identify these modular objects J, σ_t for the wedge algebra $\mathcal{A}(W)$ in a Wightman field theory in which case the J conjugation was related to the all important TCP conjugation and σ_t to the Lorentz boost which leaves W invariant. The prerequisites "cyclic and separating" were well-known to be fulfilled for all subalgebras which belonged to localization regions $\mathcal{O}$ with a nontrivial causal complement as a result of the Reeh-Schlieder theorem which proved the denseness[10] of $\mathcal{A}(\mathcal{O})\Omega$ as well as the absence of annihilators in $\mathcal{A}(\mathcal{O})$ of Ω, where the latter is the vacuum vector. The modular objects exist for all Reeh-Schlieder regions, but to understand their physical interpretation has remained a difficult task. In conformally

[10] This local denseness property of QFT, which in a more popular version is sometimes referred to as the "particle behind the moon" argument, was and still is often the course of misunderstandings since it seems to set relativistic localization radically apart from the wave function localization of QM. The resolution of this paradox is that in order to approximate a particle state behind the moon (and the compensating antiparticle in some other remote region), one needs an infinite amount of energy and that the physically attainable localization has finite energy i.e. is a phase space localization. There is a big difference in phase space localization: in QM it leads to the finite degrees of freedom per phase space volume, whereas in LQP this set of states is a "nuclear set" (i.e. mildly infinite) [1].

invariant theories the use of the conformal reflection allowed to extend the determination of the modular quantities for double cones and the forward light cone [1]. The most detailed results were obtained for chiral conformal theories; with the exception of disconnected intervals on the light cone one obtained full knowledge.

This begged the question if one could use the modular theory for the inverse purpose of extracting e.g. geometrical structures as spacetime symmetries from the positions of algebras in the net (in analogy of algebraizing the internal symmetries through endomorphisms). This path was opened by Borchers [19] and followed up by Wiesbrock [20] and several other authors [21]. Again the most complete results were obtained in chiral conformal QFT where two von Neumann algebras (type III_1 factors) in a special position were shown to carry the information about the full Moebius group and to permit the (re)construction of the full chiral net. With mild additional assumptions such a reconstruction of the full covariant net in terms of a finite set of suitably chosen algebras in controlled relative position [22]. Two interrelated concepts for a pair of von Neumann algebras turned out to be useful in this pursuit: modular inclusions and modular intersections.

Let us pause for a moment and try to understand the importance of these development for QFT. The modular theory only works for sufficiently noncommutative algebras. In fact it demands more noncommutativity than "noncommutative geometry" because the latter still permits commutative examples whereas the former does not (maximally abelian algebras are excluded by LQP). One is penetrating with this machinery real time local quantum physics at the extreme opposite end of "quantization" i.e. one is dealing with local quantum theory in its purest most intrinsic form. The crucial question is whether this new conceptual framework allows a nonperturbative understanding of interactions. Before I will address this important point I would like to add some surprising results to the above symmetry discussion [22]. It is well-known that chiral conformal field theory enjoy a much larger algebraic symmetry in form of the diffeomorphism group which is however not reflected as an invariance of the vacuum state. In the standard approach to chiral theory based on the "Lie–field structure" [4] of the energy momentum tensor whose Fourier components are the generators of the Virasoro algebra. From the LQP viewpoint the important question is whether these diffeomorphisms have also a modular origin and if affirmative, what subalgebra and state gives rise to these diffeomorphisms. Looking at the "lowest"(n=2) of

the covering satellites of the Moebius group

$$z \to \sqrt{\frac{a+bz^2}{c+dz^2}}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SU(1,1) \tag{11}$$

the candidate for the dilation subgroup rewritten in the more common Cartesian coordinate reads $x \to \frac{1-x^2}{2\lambda} + sign(x)\sqrt{1+\left(\frac{1-x^2}{2\lambda^2}\right)^2}$. Let us look at how this transformation acts on the simplest of all chiral theories: the one-component Weyl algebra (or the U(1) current algebra). If the scalar product associated with the Moebius invariant vacuum is (using again Cartesian coordinates)

$$\langle f, g \rangle = \int\int \frac{f(x)g(y)}{(x-y+i\varepsilon)^2} dxdy = \omega(If, g) \tag{12}$$

where

$$\begin{aligned} (If)(x) &= \int \frac{-1}{(x-y+i\varepsilon)^2} dy \\ \omega(f,g) &= \int f(x)g'(x)dx \end{aligned} \tag{13}$$

the ω being the symplectic form which defines the Weyl C^*-algebra. It is now fairly easy to verify that

$$\begin{aligned} \langle f, g \rangle_2 &\equiv \omega(I_2 f, g) \\ I_2 &\equiv \Gamma^{-1} I \, \Gamma \\ (\Gamma f)(x) &\equiv f\left(\frac{x}{2} + sign(x)\sqrt{1+\left(\frac{x}{2}\right)^2}\right) \end{aligned} \tag{14}$$

defines an inner product which is invariant under the action of the n=2 covering of the Moebius group

$$\begin{aligned} f(x) &\to (Uf)(x) \\ U &\equiv \Gamma^{-1} V \Gamma, \quad V \in SL(2) \\ (Vf)(x) &= f\left(\frac{\alpha x + \beta}{\gamma x + \delta}\right) \end{aligned} \tag{15}$$

With the present calculation (and its generalization to the nth-covering) indicating (for n=2) the modular origin of the geometric diffeomorphisms of

chiral conformal QFT with the algebra being n-component i.e. consisting of n disjoint intervals (those intervals which are mapped by Γ into the half-line), and the state being different from the vacuum but invariant under the n-fold covering, we still have make sure that the covering of the dilation really is the modular group for this situation. This we do by verifying that the state fulfills the KMS condition for the $U(\lambda)$ covering dilation

$$\begin{aligned} \lim_{\lambda \uparrow 2\pi i} \langle U(\lambda) f, g \rangle_2 &= \lim_{\lambda \uparrow 2\pi i} \langle V(\lambda) \Gamma f, \Gamma g \rangle \\ &= \langle \Gamma g, \Gamma f \rangle = \langle g, f \rangle_2 \end{aligned} \tag{16}$$

This is the KMS condition for a quasi-free state on the Weyl algebra given by the two-point function (14). With this result we have achieved two goals. On the one hand we have an access to the diffeomorphism group from a more general viewpoint than the one postulating the existence of an energy momentum tensor for a chiral net. On the other hand, and this may be the more important lesson, the modular origin of the infinite dimensional diffeomorphism group together with the observation that all chiral subalgebras (either for single or disjoint intervals) together with suitable "geometric states" exhaust the geometric diffeomorphisms makes it hard to resist the temptation to consider the universal modular group $\mathcal{G}_{\mathrm{mod}}$ i.e. the group generated by the modular groups of all space-time regions as the generalization of the diffeomorphism group of chiral conformal QFT to general QFT. This may be turn out to be the most useful generalization of Lie-group theory to infinite dimensions which goes beyond the affine- or W-algebra and is relevant for higher dimensional theories and which is still in the spirit of V. Kac [23].

In fact our picture receives additional support from a very interesting recent paper by Buchholz et al. [24]. There is an important distinction between the chiral and higher dimensional theories: for the latter only the Poincaré subgroup (which is spanned by the wedge algebras) acts as local diffeomorphism. All the other infinitely many transformation of $\mathcal{G}_{\mathrm{mod}}$ act in a nonpointlike way or represent "hidden symmetries", as we will say from now on. By this we do not mean something mysterious, but only that it is hidden if one uses the quantization methods of Lagrangian QFT. Such situations have been previously envisaged [25].

In order to understand the hidden symmetry situation, let us look at three different examples but before a reminder of two important mathematical definitions is helpful.

and Theorem (Wiesbrock, Borchers) Let $\mathcal{N} \subset \mathcal{M}$ be two von Neumann

algebras with a common cyclic and separating vector Ω, and (J, Δ) be the modular object associated to $(\mathcal{M}, \Omega)$. Assume furthermore that the "inclusion is modular"(more precisely +modular) i.e. that

$$\Delta_{\mathcal{M}}^{it} \mathcal{N} \Delta_{\mathcal{M}}^{-it} \subset \mathcal{N}\,, \quad t \geq 0 \tag{17}$$

Then there exists a unitary group $U(a)$ with positive generator $H, U(a) = e^{-iaH}$ fulfilling

$$\begin{aligned} U(a)\mathcal{M}U(-a) &\subset \mathcal{M}, \quad a \geq 0 \\ U(a)\Omega &= \Omega \\ \mathcal{N} &= U(1)\mathcal{M}U(-1) \end{aligned} \tag{18}$$

The second definition is that of modular intersection.

(Wiesbrock, Borchers) We say a pair $(\mathcal{N}, \mathcal{M}, \Omega)$ has modular intersection abbreviated by mis (more precisely +msi) if the following two conditions hold

$$\begin{aligned} I) \quad ((\mathcal{N} \cap \mathcal{M}) &\subset \mathcal{N}, \Omega), \ ((\mathcal{N} \cap \mathcal{M}) \subset \mathcal{M}, \Omega) \\ & are \ \text{mod}.\ inclusions \end{aligned} \tag{19}$$

$$II) \qquad J_{\mathcal{N}}(s - \lim_{t \to \infty} \Delta_{\mathcal{N}}^{-it} \Delta_{\mathcal{M}}^{it}) J_{\mathcal{N}} = s - \lim_{t \to \infty} \Delta_{\mathcal{M}}^{-it} \Delta_{\mathcal{N}}^{it}. \tag{20}$$

The main theorem for such pairs of algebras is
Let $(\mathcal{N}, \mathcal{M}, \Omega)$ have mis. Then

$$\begin{aligned} &a)\ \frac{1}{2\pi}(ln(\Delta_{\mathcal{N}}) - ln(\Delta_{\mathcal{M}})) \\ &is\ essentially\ selfadjoint \end{aligned} \tag{21}$$

Denote $U(a), a \in \mathbf{R}$, the unitary group on $\mathcal{H}$ with generator $\frac{1}{2\pi}(ln(\Delta_{\mathcal{N}}) - ln(\Delta_{\mathcal{M}}))^{-}$. Then

$$\begin{aligned} b)\ \Delta_{\mathcal{M}}^{-it} U(a) \Delta_{\mathcal{M}}^{it} &= \Delta_{\mathcal{N}}^{it} U(a) \Delta_{\mathcal{N}}^{-it} = U(e^{-2\pi t} a), \ \ \forall t, a \in R \\ c)\ J_{\mathcal{M}} U(a) J_{\mathcal{M}} &= J_{\mathcal{N}} U(a) J_{\mathcal{N}} = U(-a), \ \ \forall a \in R \\ d)\ \mathcal{N} &= U(1)\mathcal{M}U(-1). \end{aligned} \tag{22}$$

The application of this theorem to the situation of two intersecting wedges spanned by three independent light like vectors l_1, l_2, l_3 lead to the following theorem [26][4]

Let $\mathcal{V}$ be the set of regions in $\mathbf{R}^{1,2}$ containing the wedges $W[l_1,l_2], W[l_1,l_3]$ and which is closed under

a) Lorentz boosting with $\Lambda_{12}(t), \Lambda_{13}(s)$,

b) intersection

c) (causal) union

d) translation in l_1 direction

e) causal complement

Then $\Delta^{it}_{W[l_1,l_2]\cap W[l_1,l_3]}$ maps sets in $\mathcal{V}$ onto sets in $\mathcal{V}$ in a well computable way and is in fact a "hidden symmetry" which acts inside $\mathcal{V}$ geometric (in the sense of mapping subregions into subregions, since we are here dealing with nets and not with pointlike fields).

In fact it can be shown that this hidden symmetry extends the geometric subgroup $G(4)$ of the 6-parametric Poincaré group in d=2+1. This subgroup is the biggest geometric group which we can associate with two wedges. It consists of the longitudinal group associated with one of the wedges consisting of the two light ray translations and the wedge associated Lorentz-boost and in addition the Wigner light-like little group in the common light like direction. In terms of the infinitesimal generators we have

$$G(4) : P_{\pm},\ M_{t,x},\ G = \frac{1}{\sqrt{2}}\left(M_{t,x} + M_{t,y}\right) \tag{23}$$

The fact that G behaves just like the velocity generator of the Galilei group with P_+ the "central mass" and P_- the "nonrelativistic Hamiltonian" is the reason why we chose this notation (there are actually two copies $G_{\pm}(4)$ which are reflection-related). The full d=2+1 Poincaré group and the net can be generated from 3 algebras in a specific modular position.

All the previous theorems and statements have their counterparts in d=3+1 with the wegde-affiliated extended Galileian group being an 8-parametric subgroup $G(8)$ of the Poincaré group.

Now let us look at another example [27] of a hidden symmetry which is closely related to the previous one, even though at first sight it does not appear like it. Start from a chiral conformal net with a translation group KMS state Ω_β with temperature β instead of a vacuum state. In that case the von Neumann algebra of the full line $\mathcal{M}$ is a hyperfinite type III_1 algebra and the translational generator has two-sided spectrum. The commutant $\mathcal{M}'$ represents a "shadow" world i.e. a copy of $\mathcal{M}$. Consider now the action of the one-sided translation $T(a)$ on the halfline algebra $\mathcal{N}$. From the fact that it compresses $\mathcal{N}$ into itself together with its modular role with respect to

($\mathcal{M}$,Ω_β), we conclude that $\mathcal{N} \subset \mathcal{M}$ is modular and therefore there must exist another translation group $U(\alpha)$ with a positive generator $\sim \ln \Delta_{\mathcal{N}} - \ln \Delta_{\mathcal{M}}$ which together with the modular operator for $\mathcal{N}$, $\Delta_{\mathcal{N}}^{it}$ forms a group isomorphic to dilation+translation. This is the looked-for hidden symmetry generator. For one sign of α it compresses $\mathcal{N}$ into itself and for the other it spreads $\mathcal{N}$ into $\mathcal{M}$. On $\mathcal{M}$ it acts geometrically only for one sign whereas the other sign leads to a nongeometric spread into the shadow world $\mathcal{M}'$ (since the latter is nongeometric). In fact $\mathcal{N}$ can be represented as an mis. of $\mathcal{M}$ and $\cup_\alpha adU(\alpha)\mathcal{M}$.

Another illustration of a nongeometric hidden symmetry comes from the modular group of the double cone algebra $\mathcal{A}(\mathcal{C})$ generated from a massive free field. Since in this case we only were able to construct the modular group for the pair $(\mathcal{A}(\mathcal{C}), \Omega_{m=0})$ i.e. for the "wrong" (zero mass) vacuum, we prefer to return to this example in the outlook.

The domains of the Tomita operators $S(\mathcal{O})$ are related to the ranges of $\mathcal{A}(\mathcal{O})\Omega$ and probably also to the Wightman domain restricted to $\mathcal{O}$ [29]. In the chiral conformal theories it follows from the work of Fredenhagen and Joerss [28] that the field state vectors

$$A(f)\Omega = \int A(x)f(x)dx\Omega \tag{24}$$

(with the $f's$ having compact support) generate an irreducible representation of the infinite parametric universal modular group $\mathcal{G}_{\mathrm{mod}}$. Since local field state vectors are in a one-to-one relation with the fields $A(f)$, this observation leads to an exiting conjecture

(Fredenhagen) The pointlike local (Wightman) fields are the generators of the irreducible representations of the infinite Lie-groups $\mathcal{G}_{\mathrm{mod}}$ and vice versa each irreducible representation of .$\mathcal{G}_{\mathrm{mod}}$ defines a local field.

This conjecture if true, would attribute a much deeper role to fields than envisaged in their role as generators of local algebras. In fact they would re-obtain some of the important geometric role they had in the quantization approach but this time it comes from the modular structure which is the most basic aspect of LQP. Even if this turns out to be not true for all Wightman fields, it certainly holds for all fields which are constructed by modular theory [29][4]. Besides the chiral conformal fields this (see the following remarks) includes the factorizing models. In some sense the universal modular group $\mathcal{G}_{\mathrm{mod}}$ is like the diffeomorphism group. Of course as always, the devil lies in the details. If one defines this group as being generated from the one-

parametric the modular groups for all localization and all physical states in one folium, then it is too large to be useful. The analogy with the chiral diffeomorphism group suggests to choose only one state per algebra which should be in some intuitive sense the "most geometrical one". Perhaps some hint from the recently introduced condition of geometric modular action [24] could turn out to be helpful to find the definition for the smallest $\mathcal{G}_{\mathrm{mod}}$ which is closest to the chiral diffeomorphisms.

One should compare the above conjecture to the momentum space Wigner creation and annihilation operators which create the irreducible representation of the Poincaré group. So in some loose way of speaking the Wigner representations are on-shell and the irreducible representations of $\mathcal{G}_{\mathrm{mod}}$ are off-shell.

At this point it is profitable to reflect on the general notion of symmetry in physics and mathematics. All mathematicians and some physicist know that symmetry and the implementation by groups was obtained by Galois in his study of inclusions of fields into extended fields (constructed by adjoining roots of polynomial equations): $\mathcal{A} \subset \mathcal{A}_{ext}$. More than 150 years later, Vaughn Jones and other mathematicians placed the symmetry concept into the new realm of inclusions of noncommutative algebras. They found that an interesting new situation arises, if they study a von Neumann subfactor of a von Neumann factor algebra: $\mathcal{N} \subset \mathcal{M}$, such that this inclusion pocesses a conditional expectation. In fact their findings could be linked up with the older endomorphism studies of the physicist in connection with charge superselection rules in algebraic QFT and the origin of internal symmetries [1][31]. The (noncompact) spacetime symmetries however do not follow from the algebraic structure by these methods. Rather one has to study another kind of inclusions which are far removed from the Jones inclusions in the sense that they posses no conditional expectations. In that case the guiding principle (see the two previous definitions) is modular theory and one has to replace the Jones inclusions by modular inclusions.

Let us finally give some hints of how to deal with interactions in the modular spirit [29]. The first step is the realization that the S-matrix S_{sc} is a relative modular invariant for wedges referring to the interacting theory and its free (incoming) companion

$$J = S_{sc} J_{in} \tag{25}$$

and as such it is independent of what wedge W was used for its definition.

Note that the modular groups do not have a nontrivial relative modular invariant since the Poincaré group is "blind" against interactions.

The eigenspaces $H_R(W)$ define the spatial aspect of modular wedge localization. For factorizing systems this eigenvalue equation can be decomposed into its n-particle components

$$S\,|\psi\rangle^{(n)} = |\psi\rangle^{(n)} \tag{26}$$

which in terms of momentum space (momentum rapidity) read as

$$\psi(\theta_n + i\pi, ..., \theta_1 + i\pi) = \prod_{i<j} S^{(2)}(\theta_i - \theta_j)\overline{\psi(\theta_n, ..., \theta_1)} \tag{27}$$

i.e. the value of a strip-analytic function at the upper rim is given by the complex conjugate with the S-matrix intervening and defining a kind of Hilbert problem. The space of solutions is most conveniently described in terms of a basis which is different from the standard $a^{\#}$-basis by applying so called Zamolodchikov-Faddeev operators $Z^*(\theta)$ to the vacuum. In the simplest case of a theory with only one kind of selfconjugate particle (which we choose for pedagogical reasons), this algebra has the form:

$$\begin{aligned} Z^{\#}(\theta_1)Z^{\#}(\theta_2) &= S_{sc}(\theta_1 - \theta_2)Z^{\#}(\theta_2)Z^{\#}(\theta_1),\ Z^{\#} \equiv Z \ or\ Z^* \\ Z(\theta_1)Z^*(\theta_2) &= S_{sc}^{-1}(\theta_1 - \theta_2)Z^*(\theta_2)Z(\theta_1) + \delta(\theta_1 - \theta_2) \end{aligned} \tag{28}$$

Fourier transformation to x space gives the following on-shell (they obey the free field equation) nonlocal operators:

$$\begin{aligned} Z(x) &= \frac{1}{2\pi}\int_C e^{-ipx}Z^*(\theta), \quad p = m(ch\theta, sh\theta) \\ Z(\theta) &\equiv Z^*(\theta + i\pi) \end{aligned} \tag{29}$$

The Wick-ordering for such fields is done in the same way as in the standard case; the only difference is the occurrence of θ-dependent c-numbers under commutation. We expect that these operators simplify the characterization of interacting localized states. Indeed vectors of the form:

$$\begin{aligned} \left|\psi^{(n)}\right\rangle &= A_{\psi^{(n)}}\Omega \\ A_{\psi^{(n)}} &\equiv \int_C \psi^{(n)}(\theta_n, \theta_{n-1}, ...\theta_1) : Z^*(\theta_1)...Z^*(\theta_n) : \end{aligned} \tag{30}$$

If we chose $\psi^{(n)}$ according to (27), we have solved the equation for the interacting wedge localized subspace $H_R^{(n)}(W)$:

$$S\left|\psi^{(n)}\right\rangle = \left|\psi^{(n)}\right\rangle, \ S = J\Delta^{\frac{1}{2}}, \ J = S_{sc}^{*}J_0 \tag{31}$$

The general solution can be written as:

$$\psi^{(n)}(\theta_n, ..., \theta_1) = F(\theta_n, ..., \theta_1)\psi_0^{(n)}(\theta_n, ..., \theta_1) \tag{32}$$

The step from localized subspaces $H_R(W)$ to localized algebras $\mathcal{A}(W)$ is nontrivial if the S-matrix has a nontrivial rapidity dependence. Only in models as the Ising or Federbush model which have constant S-matrices the Z-F fields (29) are also the generators of the wedge algebras. For this and other related problems we refer the reader to [31][4].

3 A Peek at 21^{st} Century Local Quantum Physics

Peeks at the future consist mostly of personal expectations and, if one looks at the many attempts during the last two decades in this direction which turned out to remain unfulfilled promises, on gets a little bit discouraged. In fact one sometimes has the impression that even the presence is not any more what it used to be, namely an open situation which can be entered by young gifted researchers without being overly concerned about the continuation of their carriers. There is nothing more counterproductive for new insights than to have half-baked theories around for several decades. Just imagine what labyrinth we would have entered already in the twenties, if the Bohr-Sommerfeld theory would not have developed rather swiftly into QM. The longer such unfinished theories are around, the stronger becomes the prejudices against anybody who starts to asks critical questions.

In view of the fact that LQP has left the area once covered by "Axiomatic QFT" behind, and instead of just rigorously reformulating what we learned from perturbative QFT, offers a variety of new concepts and constructive methods which in most cases were not (and cannot be) abstracted from the Lagrangian quantization approach (and asks critical questions), one dares to hope that particle physicist become more aware of these results. In my view this could lead to an extremely fruitful dialogue between the present framework and other approaches involving modern and exciting ideas.

My conviction that the present framework will have a rich future stems primarily from the fact that the intrinsic logic of LQP is so strong and convincing that it appears to me as a much safer guide than that obtained from

the quantization approach. Whereas the canonical formalism, the interaction picture, the formalism of time-ordering etc. can (and has been) be used outside of relativistic QFT, the modular approach is *totally specific for real time LQP*. In fact it is the only truly noncommutative entrance into QFT which came really from physics (rather than physical illustrations of mathematical concepts as done e.g. with noncommutative geometry). Admittedly, it is an area, which because of its strong conceptual roots and demanding mathematical apparatus is not easy to enter; neither does the subject render itself to fast publications. But in compensation, even if progress at times is very slow, it carries a conceptual profoundness and mathematical solidity which, if coupled with the belief in the guiding power of physical principles (especially through times of crisis), is hard to substitute.

Let me distinguish between goals, the ones which I expect to be reached in the coming years and those which I would place into the more distant future.

One of the short range goals is to refine the modular localization method for the construction of factorizing d=1+1 models by improving the mathematical control of the modular objects for e.g. double cones with the aim to get a more profound understanding for the vacuum polarization caused by compact localization+interaction and more generally about the operator content of local operators in terms of the asymptotic particle creation- and annihilation- operators (or more elegantly in terms of the closely related Faddeev-Zamolodchikov operators). A closely related subject is the use of the same constructive modular method to complete the insufficient knowledge contained in chiral conformal exchange algebras [4]. The aim here is to incorporate chiral conformal field theory into the main body of QFT where one first classifies the statistics and superselected charges and then constructs its simplest (free) carriers (and later couples them to describe interactions). The field statistics can be classified with the method of computing tracial states ("Markov traces") on the braid group B_∞, a method elaborated for the permutation group S_∞ in the famous DHR papers [1] and then found in the vastly more general context of the subfactor theory by V. Jones and H. Wenzl [32]. This includes a good portion of braid group statistics and leads to quantized (in the sense of discrete) statistics parameters (statistical dimensions and phases). It is believed that chiral conformal theory permits no genuine interactions and that e.g. the field theory of W-algebras is uniquely fixed in terms of the superselected charges and their plektonic statistics. Therefore a special case of this general aim would be to derive the Kac-Friedan-Qiu-

Shenker quantization of the energy-momentum algebra from the statistics quantization by computing the composite energy-momentum tensor in terms of the "free" plektonic fields. Needless to say that this has not been carried out.

The fact that the braid group analysis applies word for word to massive d=2+1 theories gives rise to the physically important problem of actually constructing "free" plektonic fields (with anyons belonging to the abelian braid group statistics). Here "free" means that the contribution of the scattering matrix contribution S_{sc} in the modular relation between the interacting and incoming modular wedge involutions

$$J = S_{stat} S_{sc} J_0 \tag{33}$$

is trivial $S_{sc} = 1$ and the only nontrivial part is due to the spatially constant statistical Klein-factor S_{stat}. The physical importance of such a construction results from the fact that it could classify the spin-statistics possibilities of condensed matter quasiparticles in layers in a similar vein as the standard d=3+1 spin-statistics analysis leads to the classification of standard particles and (nonrelativistic) quasiparticles. There is of course a big difference in the two cases in that in the d=3+1 case one finds generators (the standard free fields) without vacuum polarization and with the Fock space structure as a reference space whereas genuine d=2+1 plektons different from Fermions/Bosons are always accompanied by vacuum polarization and semiinfinite stringlike extension. In practical terms this means that the nonrelativistic limit in the latter case cannot be reduced to QM, but remains a nonrelativistic field theory. This is probably the deeper reason why the Leinaas-Myrheim-Wilszek program only succeeded to deform the spin in two-particle systems (with the help of the Aharonov-Bohm formalism), but was unable to maintain the cluster decomposition properties which would consistently assign the right statistics to the multiparticle system. The Chern-Simons program, at least being better disposed in this respect, also did not succeed to provide a field theory for anyons. My explanation for this lack of success is that braid group statistics field theory is less commutative than one with permutation group statistics, and that for such situations the method of quantization via euclidean functional integrals becomes too artistic (i.e. in need of too many special repair rules) for being still managable by quantum field theoretical pragmatism.

In this connection some words about the position of topological field theory and LQP are in order. Already in the DHR work on superselection

charges in QFT, there was the so-called intertwiner calculus, i.e. the choice of a representative endomorphism for each charge sector and the infinite set of reduction intertwiners which are part the composition theory of endomorphisms. As shown by DHR in their series of papers from the beginning of the 70^{ies} [1], the algebra generated by these intertwiners have a natural faithful tracial state by which it is converted into a hyperfinite type II_1 combinatorial algebra (algebra of "tangles"). This algebra contained in particular a representation of the permutation group S_∞. In more recent times [6] this was generalized to the braid group situation B_∞. By working with an algebraic compactification of the observable algebra (which is natural in the presence of physical braid group statistics), the intertwiner algebra includes the 3-manifold invariants [30][31][11]. This way of abstracting the topological field theory from LQP has the advantage that it comes equipped with a physical interpretation. On the other hand if one obtains it by imposing rules on formal Chern-Simons functional integrals a la Witten, or follows the triangulation approach [33], a physical interpretation is difficult, as a result of the totally global nature of the theory. Although from LQP one obtains the Jones knot theory and the Witten 3-mf. invariants together with physical interpretation (see appendix of [30] and [31]), it is nevertheless an interesting question to ask whether there is an LQP version of the Witten method i.e. an understanding in terms of putting states on Weyl-like algebras (which are suggested by canonical quantization of the Chern-Simons theory in a convenient gauge which kill the spacetime "meat" and only leave the combinatorial "bones" [31]). The idea is that one can bridge the Witten approach with the subfactor approach of Jones, the combinatorial approach of Turaev and Viro and that of the LQP intertwiner algebras by putting *singular states* on appropriatly extended Weyl algebras of semiinfinite string-like localized one-forms (with a distiction beween angular exact and closed forms) is very tempting [31] but has not been carried out.

Another aspect of chiral conformal theory whose understanding in the LQP scheme is expected to be extremely beneficial for the higher dimensional cases is that of the "modular" (used in another sense than in this paper)

[11] Note that the 3-mf. *is not the spacetime on which the real time field theory lives* (in the sense of localization), but rather belongs to that area in which internal and external symmetries get inexorably intermingled and which in higher dimensional theories was subject to the No-Go theorems preventing such nontrivial intertwining. Thus these invariants could manifest themselves in scattering properties of plektons with all geni appearing at the same time (as in d=3+1 all partial waves appear simultaneously).

identities between thermal correlation function with the Gibbs Hamiltonian being the positive rigid rotation operator L_0. These relations involves the Verlinde matrix S which diagonalizes the fusion of superselection charges and Verlinde's argument is purely geometrical and not quantum field theoretical. The LQP interest arises from the analogy of that relation with the so-called Nelson-Symanzik symmetry between space and time for bosonic theories in case that the thermal theory is also enclosed in a spatial box. But the chiral (as well as the plektonic d=2+1) situation is much more noncommutative, so that one would be outside the setting of functional integrals representations.

Another not too distant goal is the understanding of entropy within the LQP setting. The recent so-called AdS- conformal QFT equivalence may be taken as a suggestion that in addition to the more classical gravitational entropy of Bekenstein and Hawking, there my be a purely quantum entropy of algebraic origin. This would be somewhat reminiscent of the relation between the Hawking temperature related to a classical (Killing vector) horizon and the thermal aspect of localization [29]. It is well-known that one can generalize the von Neumann definition of entropy to type I (and type II) algebras but not to type III. In some intuitive sense they contain too many degrees of freedom. There is a very general idea of how to approximate the local hyperfinite type III_1 of LQP by type I (the type of algebra of ordinary QM): the "split property" [1]. In the case of a wedge algebra in d=1+1 one would take the opposite wedge and create a "collar" by shifting it from the first one by a space distance a. It turns out that there is a canonical choice and therefore a natural modular operator $\Delta_a = e^{-K_a}$ with a natural a-dependent "Hamiltonian" K_a with a finite entropy if it is of trace class. For higher dimensional theories one has to work with double cones instead of wedges [1]. Of course on physical grounds one expects it to diverge with the surface of the wedge [4]. But one also expects at least a logarithmic divergence of the entropy surface density in the limit of $a \to 0$ as a result of increase of degrees of freedom. Suppose that our QFT is 2-dimensional conformal. In that case the canonical association of K_a to wedge situation and the fact that the limiting modular group is a subgroup of the conformal group suggests that entropical property of K_a should be determined by the energy momentum tensor of the model and hence be proportional to the central extension constant c. In order to obtain a finite expression one would have to take ratios of entropies for different localization regions. The proportionality with c would harmonize with arguments of a quite different nature [34]. The higher dimensional massive case of e.g. a double cone algebra could be connected to the previous

one by juxtapositioning two arguments: the rotational symmetry (including that of the modular group for the double cone region) would reduce the problem to two dimensions and the geometric behavior near the boundary like that of the massless limit theory could make the leading divergent term in $a \to 0$ be that of the massless limit. Taking the optimistic viewpoint that all these intuitive arguments are borne out by future calculations, one would then learn that those entropical properties which are presently attributed to string theory and/or special curved space situations are in fact, like the localization temperature, generic properties of (nonperturbative) LQP. Again LQP suggests many new ideas but not always a speedy solution.

Let us now look at some further out problems. The most important aspect in the application of LQP to particle physics is the understanding of higher dimensional interacting theories. In that case one does not know either the S-matrix nor any off-shell properties. To think about a trial S-matrix, which is Poincaré and TCP invariant and fulfills a weak form of cluster properties, but lacks the analytic and crossing properties resulting from locality, is not difficult to implement by examples. One could speculate on using such an S-matrix in a kind of iterative Hartree Fock off-shell modular localization procedure, but lacking a clear physical guidance one cannot avoid running into a similar quagmire as with e.g. iterative approaches of the Schwinger-Dyson equations. Perhaps an idea which was inherent in the old string theory may be a way out. I am thinking about the alleged (but not really understood) mysterious relation between a higher dimensional on-shell S-matrix theory and low dimensional off-shell field theory as it is implicit in Virasoro's analysis of the Veneziano S-matrix. This may be a way of obtaining very good trial S-matrices for the constructive modular approach. It is somewhat unfortunate that string theory has only a forward gear (the previous old-fashioned remark is of no interest to a string theorist); it is the first physical theory which seems to be incapable to trace back with (historical) hindsight, because at the important cross roads it created too much revolutionary dust.

Looking back at the birth of renormalization theory and its stunning predictive success, one may ask whether the paradigm change of LQP could have a predictive power and where would one expect it. Due to the more subtle conceptual and mathematically unaccustomed situation one would not expect such a swift progress. I would think that the first place where this may occur is the classification of "free" (quasi) particles in condensed matter physics. As we have argued, even in its nonrelativistic version there

is no reason that a braid group statistics setting which is faithful with a plektonic version of the Spin-Statistics theorem should fit into QM since vacuum polarization of the "plektons" is necessary structural property to uphold braid group statistics. The modular localization method is capable to cope with situations in which the "free" multiparticle structure is more subtle than just (symmetrized or anti-symmetrized) tensor products, i.e. which are outside the usual Fock space. QM even if one uses Aharonov-Bohm interactions cannot do this, and the Chern-Simons approach is too artistic for being able to use its suggestive power in order to extract the correct picture including the nontrivial vacuum polarization aspects. Even if nature refuses to show us the nonrelativistic carriers of these new properties (e.g. in the fractional Quantum Hall effect or High T_c-Superconductivity), we still are condemned to understand these phenomena as theoretical laboratories for difficult situations e.g. quark confinement and other nonunderstood ideas in d=3+1 particle physics.

A change of paradigm should also have manifestations in general philosophy (not necessarily related to natural sciences). The "Weltbild" underlying classical physics and quantum theory is Newtonian in the sense that reality comes about by filling a spacetime manifold with a material content. But the way LQP thinks about reality is different and closer to Leibnitz theory of "monades". A single monade has no discernable properties and by itself cannot create reality. Rather one needs relations between monades in order to achieve that. If one replaces monades by hyperfinite type III_1 von Neumann algebras and the relations by those of these algebras in the net (Jones inclusions, modular inclusions etc.) one obtains an interesting analogy in which the algebras are similar to points in geometry. This is because like points, these algebras have no individuality (if you have seen one, you know them all). However as in any analogy, there are limitations. For example the von Neumann factors may mutually contain each other which leads to situations considerably richer than those which one can obtain with sets of points.

In an intellectually honest essay, one should also mention those things which are outside its scope. One such area is "Quantum Gravity". Although LQP through its modular structure makes an interesting relation between the spacetime as a mere indexing device of nets and physical space time as an arena on which (hidden) symmetry groups act, it does not fully elevate spacetime into the quantum arena. Even if one subjects the living space and the spacetime symmetry group to the noncommutative geometrical structure

arising from a quasi-classical interpretation of Einstein's equations together with an additional mild requirement [35] (limits on arbitrarily small black holes), one obtains at best a net with a nonabelian indexing. Quantum Gravity should not be describable in terms of indexed nets at all, and hence be outside the LQP framework. It is not clear to me whether string theory is revolutionary enough ("crazy enough", using the expressions by Pauli and Feynman) to achieve that. It is confusing to read in articles about string theory that it is on the one hand a quantum gravity theory, but at the same time obeys the principles of local quantum physics. *A resolution of more than twenty years old agony of whether string theory is LQP in disguise or a new conceptual framework on its way towards q.g. is overdue,* see also the remarks in the last section of [37], where one finds the statement that the occurrence of the first alternative would be the "ultimate irony". Most people (including myself) hope that this may not happen, i.e. that string theory may not be forced to return all the way back to its field theoretic origins.

A cheap formal to obtain algebras which do not contain localizable subalgebras is to take a chiral model and average its Moebius invariant correlation functions with a Fuchsian subgroup of the Moebius group. In that case the original net of intervals on the circle gets completely delocalized and the so produced algebra ceases to be an indexible net. Causality and locality are lost in the algebraic structure (which is not unreasonable since in Quantum Gravity one should not have a knowledge of spacelike which is, like the vacuum, a global notion), but one could think that there are special states on this algebra through which it is brought back at least infinitesimally. In most contemporary work Quantum Gravity means euclidean curved space time QFT. In my view the distance to the rather well understood QFT in CST is so immense, that it is presently easier to say what Quantum Gravity is not, rather than what it really is.

Another area for which LQP can only offer (physically) destructive ideas is supersymmetry. None of the symmetry arguments (neither the endomorphism theory of inner symmetries nor the modular group approach to $\mathcal{G}_{\mathrm{mod}}$) points to it. From the LQP approach it appears like an accidental symmetry. This is also borne out by all known solvable models, the best known case being the conformal tricritical Ising model. In all those cases the full content of the model can be understood without ever mentioning supersymmetry; it is just there without influencing the charge content or the short distance properties of the model as compared to neighbouring non-supersymmetric

models in the same family (conserved quantities of course always retain their canonical dimensions). Accidental symmetries should be unstable under e.g. thermal excitation.. Indeed, whereas all other symmetries (inner and outer) suffer a spontaneous breaking with a clear intrinsic signal of the thermal theory that there is indeed a spontaneously broken symmetry, the supersymmetry suffers a total "collapse" [36] without any intrinsic sign left by a broken symmetry. This indicates that if the perturbative claims about supersymmetric gauge theory have a nonperturbative counterpart and nature really yields results which resemble those of supersymmetry, there may be something else behind these observations. The setting in which we studied the LQP version of gauge theory in the previous section should be a good framework for clarifying this issue. LQP views new ideas in particle physics as an enlargement of the range of validity of old principles (perhaps Quantum Gravity as an exception) and therefore prefers discoveries to inventions. Since the strings of string theory and the supersymmetry are resisting such an interpretation, LQP has a hard time to cope with these inventions. In this respect it seems not to be much different from nature, except that a *theoretical* physicist should use his conceptual instruments and leave nature to experimentalists.

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