Observation of three-photon Greenberger-Horne-Zeilinger entanglement

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We present the experimental observation of polarization entanglement for three spatially separated photons. Such states of more than two entangled particles, known as GHZ states, play a crucial role in fundamental tests of quantum mechanics versus local realism and in many quantum information and quantum computation schemes. Our experimental arrangement is such that we start with two pairs of entangled photons and register one photon in a way that any information as to which pair it belongs to is erased. The registered events at the detectors for the remaining three photons then exhibit the desired GHZ correlations.

Ever since the seminal work of Einstein, Podolsky and Rosen [1] there has been a quest for generating entanglement between quantum particles. Although two-particle entanglements have long been demonstrated experimentally [2,3], the preparation of entanglement between three or more particles remains an experimental challenge. Proposals have been made for experiments with photons [4] and atoms [5], and three nuclear spins within a single molecule have been prepared such that they locally exhibit three-particle correlations [6]. However, until now there has been no experiment which demonstrates the existence of entanglement of more than two spatially separated particles. Here we report the experimental observation of polarization entanglement of three spatially separated photons.

The original motivation to prepare three-particle entanglements stems from the observation by Greenberger, Horne and Zeilinger (GHZ) that three-particle entanglement leads to a conflict with local realism for non-statistical predictions of quantum mechanics [7]. This is in contrast to the case of Einstein, Podolsky and Rosen experiments with two entangled particles testing Bell’s inequalities, where the conflict only arises for statistical predictions [8]. We will experimentally address this issue in a forthcoming paper.

The incentive to produce GHZ states has been significantly increased by the advance of the field of quantum communication and quantum information processing. Entanglement between several particles is the most important feature of many such quantum communication and computation protocols [9,10].

We now describe the experimental arrangements for the observation of three-photon GHZ entanglement. The experimental techniques are similar to those that have been developed for our previous experiments on quantum teleportation [11] and entanglement swapping [12]. In fact, one of the main complications in the previous experiments, namely the creation of two pairs of photons by a single source, is here turned into a virtue.

The main idea, as was put forward in Ref. [4], is to transform two pairs of polarization entangled photons into three entangled photons and a fourth independent photon. Figure 1 is a schematic drawing of our experimental setup. Pairs of polarization entangled photons are generated by a short (approx. 200 fs) pulse of UV-light of wavelength $\lambda = 788$ nm from a mode-locked Ti-Sapphire laser, which passes through an optically nonlinear crystal (here Beta-Barium-Borate, BBO). The probability per pulse to create a single pair in the desired modes is rather low and of the order of a few $10^{-4}$. The pair creation is such that the following polarization entangled state is obtained [3].

$$\frac{1}{\sqrt{2}} (|H\rangle_a |V\rangle_b - |V\rangle_a |H\rangle_b).$$

(1)

This state indicates that there is a superposition of the possibility that the photon in arm $a$ is horizontally polarized and the one in arm $b$ vertically polarized $(|H\rangle_a |V\rangle_b)$, and the opposite possibility, i.e., $|V\rangle_a |H\rangle_b$. The minus sign indicates that there is a fixed phase difference of $\pi$ between the two possibilities. For our GHZ experiment this phase factor is actually allowed to have any value, as long as it is fixed for all pair creations.

The setup is such that arm $a$ continues towards a polarizing beam splitter, where $H$ photons are transmitted towards detector $T$ and $V$ photons are reflected, and arm $b$ continues towards a 50/50 polarization-independent beam splitter. From each beam splitter one output is directed to a second polarizing beam splitter. In between the two polarizing beam splitters there is a $\lambda/2$ retardation plate at an angle of 22.5° which rotates the vertical polarization of the photons reflected by the first polarizing beam splitter into a 45° polarization, i.e. a superposition of $|H\rangle$ and $|V\rangle$ with equal amplitudes. We use three more detectors, $D_1$, $D_2$, and $D_3$, in the remaining output arms. Narrow band-width interference filters are placed in front of the four detectors (\(\delta\lambda = 4.5\) nm for the detector T and $\delta\lambda = 3.6\) nm for the other three). Including filter losses, coupling into single-mode fibers, and the S-avalanche detector efficiency, the total collection and detection probability of a photon is about 10%.

Consider now the case that two pairs are generated by a single UV-pulse, and that the four photons are all detected, one by each detector T, $D_1$, $D_2$, and $D_3$. Our claim is that by the coincident detection of four photons and because of the brief duration of the UV pulse and
the narrowness of the filters, one can conclude that a three-photon GHZ state has been recorded by detectors $D_1$, $D_2$, and $D_3$. The reasoning is as follows. When a four-fold coincidence recording is obtained, one photon in path $a$ must have been horizontally polarized and detected by the trigger detector $T$. Its companion photon in path $b$ must then be vertically polarized, and it has 50% chance to be transmitted by the beamsplitter (see Figure 1) towards detector $D_2$ and 50% chance to be reflected by the beamsplitter towards the final polarizing beamsplitter where it will be reflected to $D_2$. Consider the first possibility, i.e., the companion of the photon detected at $T$ is detected by $D_3$ and necessarily carried polarization $V$. Then the counts at detectors $D_1$ and $D_2$ were due to a second pair, one photon travelling via path $a$ and the other one via path $b$. The photon travelling via path $a$ must necessarily be $V$ polarized in order to be reflected by the polarizing beamsplitter in path $a$; thus its companion, taking path $b$, must be $H$ polarized and after reflection at the beamsplitter in path $b$ (with a 50% probability) it will be transmitted by the final polarizing beamsplitter and arrive at detector $D_1$. The photon detected by $D_2$ therefore must be $H$ polarized since it came via path $a$ and had to transit the last polarizing beamsplitter. Note that this latter photon was $V$ polarized but after passing the $\lambda/2$ plate it became polarized at 45° which gave it 50% chance to arrive as an $H$ polarized photon at detector $D_2$. Thus we conclude that if the photon detected by $D_3$ is the companion of the $T$ photon, then the coincidence detection by $D_1$, $D_2$, and $D_3$ corresponds to the detection of the state

$$\ket{H}_1 \ket{H}_2 \ket{V}_3. \quad (2)$$

By a similar argument one can show that if the photon detected by $D_2$ is the companion of the $T$ photon, the coincidence detection by $D_1$, $D_2$, and $D_3$ corresponds to the detection of the state

$$\ket{V}_1 \ket{V}_2 \ket{H}_3. \quad (3)$$

In general the two possible states (2) and (3) corresponding to a four-fold coincidence recording will not form a coherent superposition, i.e. a GHZ state, because they could, in principle, be distinguishable. Besides possible lack of mode overlap at the detectors, the exact detection time of each photon can reveal which state is present. For example, state (2) is identified by noting that $T$ and $D_3$, or $D_1$ and $D_2$, fire nearly simultaneously. To erase this information it is necessary that the coherence time of the photons is substantially longer than the duration of the UV pulse (approx. 200 fs) [13]. We achieved this by detecting the photons behind narrow band-width filters which yield a coherence time of approx. 500 fs. Thus the possibility to distinguish between states (2) and (3) is no longer present, and, by a basic rule of quantum mechanics, the state detected by a coincidence recording of $D_1$, $D_2$, and $D_3$, conditioned on the trigger $T$, is the quantum superposition

$$\ket{H}_1 \ket{H}_2 \ket{V}_3.$$  

FIG. 1. Schematic drawing of the experimental setup for the demonstration of Greenberger-Horne-Zeilinger entanglement for spatially separated photons. Conditioned on the registration of one photon at the trigger detector $T$, the three photons registered at $D_1$, $D_2$, and $D_3$ exhibit the desired GHZ correlations.

$$\frac{1}{\sqrt{2}} (\ket{H}_1 \ket{H}_2 \ket{V}_3 + \ket{V}_1 \ket{V}_2 \ket{H}_3), \quad (4)$$

which is a GHZ state [14].

The plus sign in Eq. (4) follows from the following more formal derivation. Consider two down-conversions producing the product state

$$\frac{1}{2} (\ket{H}_a \ket{V}_b - \ket{V}_a \ket{H}_b) (\ket{H}_a^\prime \ket{V}_b^\prime - \ket{V}_a^\prime \ket{H}_b^\prime). \quad (5)$$

Initially we assume that the components $\ket{H}_{a,b}$ and $\ket{V}_{a,b}$ created in one down-conversion might be distinguishable from the components $\ket{H}_{a,b}'$ and $\ket{V}_{a,b}'$, created in the other one. The evolution of the individual components of state (5) through the apparatus towards the detectors $T$, $D_1$, $D_2$, and $D_3$ is given by

$$\ket{H}_a \rightarrow \ket{H}_T,$$  

$$\ket{V}_b \rightarrow \frac{1}{\sqrt{2}} (\ket{V}_2 + \ket{V}_3),$$  

$$\ket{V}_a \rightarrow \frac{1}{\sqrt{2}} (\ket{V}_1 + \ket{H}_2),$$  

$$\ket{H}_b \rightarrow \frac{1}{\sqrt{2}} (\ket{H}_1 + \ket{H}_3).$$  

Identical expressions hold for the primed components. Inserting these expressions into state (5) and restricting
ourselves to those terms where only one photon is found in each output we obtain

$$-\frac{1}{4\sqrt{2}} \left\{ |H\rangle_T \left( |V\rangle_1 |V\rangle_2 |H\rangle_3 + |H\rangle_1 |H\rangle_2 |V\rangle_3 \right) + |H\rangle_T \left( |V\rangle_1 |V\rangle_2 |H\rangle_3 + |H\rangle_1 |H\rangle_2 |V\rangle_3 \right) \right\}. \tag{10}$$

If now the experiment is performed such that the photon states from the two down-conversions are indistinguishable, we finally obtain the desired state (up to an overall minus sign)

$$\frac{1}{\sqrt{2}} |H\rangle_T \left( |H\rangle_1 |H\rangle_2 |V\rangle_3 + |V\rangle_1 |V\rangle_2 |H\rangle_3 \right). \tag{11}$$

Note that the total photon state produced by our setup, i.e., the state before detection, also contains terms in which, for example, two photons enter the same detector. In addition, the total state contains contributions from single down-conversions. The four-fold coincidence detection acts as a projection measurement onto the desired GHZ state (11) and filters out these undesirable terms. The efficiency for one UV pump pulse to yield such a four-fold coincidence detection is very low (of the order of $10^{-10}$). Fortunately, $7.6 \times 10^7$ UV-pulses are generated per second, which yields about one double pair creation and detection per 150 seconds, which is just enough to perform our experiments. Triple pair creations can be completely neglected since they can give rise to four-fold coincidence detection only about once every day.

To experimentally demonstrate that a GHZ state has been obtained by the method described above, we first verified that, conditioned on a photon detection by the trigger T, both the $H_1H_2V_3$ and the $V_1V_2H_3$ component are present and no others. This was done by comparing the count rates of the eight possible combinations of polarization measurements, $H_1H_2H_3$, $H_1H_2V_3$, $V_1V_2H_3$, $V_1V_2V_3$. The observed intensity ratio between the desired and undesired states was 12:1. Existence of the two terms as just demonstrated is a necessary but not yet sufficient condition for demonstrating GHZ entanglement. In fact, there could in principle be just a statistical mixture of those two states. Therefore, one has to prove that the two terms coherently superpose. This we did by a measurement of linear polarization of photon 1 along +45°, bisecting the H and V direction. Such a measurement projects photon 1 into the superposition

$$|+45^\circ\rangle_1 = \frac{1}{\sqrt{2}} (|H\rangle_1 + |V\rangle_1), \tag{12}$$

what implies that the state (11) is projected into

$$\frac{1}{\sqrt{2}} |H\rangle_T |+45^\circ\rangle_1 \left( |H\rangle_2 |V\rangle_3 + |V\rangle_2 |H\rangle_3 \right). \tag{13}$$

Thus photon 2 and 3 end up entangled as predicted under the notion of “entangled entanglement” [15]. We conclude that demonstrating the entanglement between photon 2 and 3 confirms the coherent superposition in state (11) and thus existence of the GHZ entanglement. In order to proceed to our experimental demonstration we represent the entangled state (2-3) in a linear basis rotated by 45°. The state then becomes

$$\frac{1}{\sqrt{2}} \left( |+45^\circ\rangle_2 |+45^\circ\rangle_3 - |-45^\circ\rangle_2 |-45^\circ\rangle_3 \right), \tag{14}$$

which implies that if photon 2 is found to be polarized along $-45^\circ$, photon 3 is also polarized along the same direction. We test this prediction in our experiment. The absence of the terms $|+45^\circ\rangle_2 |-45^\circ\rangle_3$ and $|-45^\circ\rangle_2 |+45^\circ\rangle_3$ is due to destructive interference and thus indicates the desired coherent superposition of the terms in the GHZ state (11). The experiment therefore consisted of measuring four-fold coincidences between the detector T, detector 1 behind a +45° polarizer, detector 2 behind a -45° polarizer, and measuring photon 3 behind either a +45° polarizer or a -45° polarizer. In the experiment, the difference of arrival time of the photons at the final polarizer, or more specifically, at the detectors D1 and D2, was varied.

The data points in Fig.2(a) are the experimental results obtained for the polarization analysis of the photon at D3, conditioned on the trigger and the detection of two photons polarized at 45° and -45° by the two detectors D1 and D2, respectively. The two curves show the four-fold coincidences for a polarizer oriented at $-45^\circ$ (squares) and $+45^\circ$ (circles) in front of detector D3 as function of the spatial delay in path a. From the two curves it follows that for zero delay the polarization of the photon at D3 is oriented along $-45^\circ$, in accordance

![FIG. 2](image-url)
with the quantum-mechanical predictions for the GHZ state. For non-zero delay, the photons travelling via path $a$ towards the second polarizing beamsplitter and those travelling via path $b$ polarizing distinguishable. Therefore increasing the delay gradually destroys the quantum superposition in the three-particle state.

Note that one can equally well conclude from the data that at zero delay, the photons at $D_1$ and $D_2$ have been projected onto a two-particle entangled state by the projection of the photon at $D_2$ onto $-45^\circ$. The two conclusions are only compatible for a genuine GHZ state. We note that the observed visibility was as high as 75%.

For an additional confirmation of state (11) we performed measurements conditioned on the detection of the photon at $D_1$ under 0° polarization (i.e. $V$ polarization). For the GHZ state $(1/\sqrt{2})(H_1H_2V_3 + V_1V_2H_3)$ this implies that the remaining two photons should be in the state $V_2H_3$ which cannot give rise to any correlation between these two photons in the $45^\circ$ detection basis. The experimental results of these measurements are presented in Fig.2(b). The data clearly indicate the absence of two-photon correlations and thereby confirm our claim of the observation of GHZ entanglement between three spatially separated photons.

Although the extension from two to three entangled particles might seem to be only a modest step forward, the implications are rather profound. First of all, GHZ entanglements allow for novel tests of quantum mechanics versus local realistic models. Secondly, three-particle GHZ states might find a direct application, for example, in third-man quantum cryptography. And thirdly, the method developed to obtain three-particle entanglement from a source of pairs of entangled particles can be extended to obtain entanglement between many more particles [16], which are at the basis of many quantum communication and computation protocols. Most applications of GHZ states imply that the three particles have to be detected. Therefore, even as our setup only produces GHZ entanglement upon the condition that the three photons and the trigger photon are actually detected, our scheme can readily be used for many applications. The detection plays the double role of projecting onto the GHZ state and of performing a specific measurement on this state. Finally, we note that our experiment, together with our earlier realization of quantum teleportation [11] and entanglement switching [12] provides necessary to tools to implement a number of novel entanglement distribution and network ideas as recently proposed [17,18].

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Rigorously speaking, this ensurance technique is perfect, hence produces a pure GHZ state, only in the limit of infinitesimal pulse duration and infinitesimal filter bandwidth, but detailed calculations [M.A. Horne, Fortschr. Phys. 46, 6 (1998)] reveal that our pulse and filter values are sufficient to create a clearly observable entanglement, as confirmed by our experimental data.