Abstract

Chiral symmetry is known to be decisive for an understanding of the low energy sector of strong interactions. It is thus important for a model of relativistic heavy ion collisions to incorporate the dynamical breaking and restoration of chiral symmetry. Thus we study an expansion scenario for a quark-meson plasma using the Nambu-Jona-Lasinio (NJL) model in its three flavor version. The equations of motion for light and strange quarks as well as for pions, kaons and etas are solved using a QMD type algorithm, which is based on a parametrization of the Wigner function. The scattering processes incorporated into this calculation are of the types $qq \leftrightarrow q\bar{q}$, $q\bar{q} \leftrightarrow q\bar{q}$, $q\bar{q} \leftrightarrow MM$ and $M \leftrightarrow q\bar{q}$.

In relativistic heavy ion collisions, a very hot and dense system of strongly interacting matter is created. At these high temperatures and densities, the state of matter changes drastically. The two main effects which are predicted from quantum chromodynamics (QCD) are (i) deconfinement and (ii) chiral symmetry restoration. Lattice simulations indicate, that these two effects are not independent of each other, but occur at the same time [1]. Although experiments searching for this new state of matter have been performed at the Cern SPS and will be performed at RHIC and LHC, the theoretical description still leaves many open questions. Some of these are (i) the question, if a local thermal and/or chemical equilibrium will be established, (ii) the influence of the in-medium changes of hadronic properties and (iii) the description of the hadronization. The treatment of these problems is by no means simple, since a satisfactory, simple model for the description of the phase transition to a quark-gluon plasma (QGP) does not yet exist. Since the QGP, if created, is a transient state, a realistic model for its evolution should be able to handle nonequilibrium effects. A nonequilibrium theory of nonabelian gauge theories such as QCD, however, cannot be provided using present day knowledge, since these theories even in thermal equilibrium are not yet sufficiently understood. Lattice gauge theories, on the other hand, have been successfully applied in order to extract the thermal behaviour of strongly interacting matter. They work, however, only in equilibrium situations. Another practical approach which is frequently used is hydrodynamics. These models, however, suffer from the fact that they work in the limit of infinitely many collisions per time interval, which might not be true in the first stages and surely is not true in the late stages of the collision.

The ansatz presented here thus the modelization of the expansion of a hot system using an effective Lagrangian. The model interaction we use is the three flavor Nambu–Jona-Lasinio (NJL) model [2], defined by

\[ \mathcal{L} = \sum_{f=u,d,s} \bar{\psi}_f (i \slashed{D} - m_0) \psi_f + G \sum_{a=0}^8 \left[ \left( \bar{\psi} \gamma_a \lambda_i \psi \right)^2 + \left( \bar{\psi} i \gamma_5 \lambda_i \psi \right)^2 \right] + K \left[ \det \bar{\psi} (1 + \gamma_5) \psi + \det \bar{\psi} (1 - \gamma_5) \psi \right] \]  

(1)

In this model, the exchange interaction has been contracted to give a pointlike interaction in flavor space. This approximation works as long as the effective mass of the gluon is high. Recent QCD calculations support this assumption even at high temperatures [3]. The most important feature of the Lagrangian (1) is that
it preserves the chiral symmetry of QCD, i.e. in the limit $m_{qf} \to 0$ it is invariant under transformations of the form

$$\psi \to \exp (-i\theta_a \lambda a \gamma_5) \psi$$ \hspace{1cm} (2)

It is a well established fact, that this symmetry is the essential ingredient for the description of the low lying hadronic states which are produced copiously in a heavy ion collision [4].

The equilibrium properties of the NJL model in thermal equilibrium have been studied in great detail elsewhere [2,5–8], so that only those properties will be mentioned here, which are important for the understanding of the present article. For simplicity, first the case $m_{qf} = 0$ will be detailed. At temperature $T = 0$, chiral symmetry is spontaneously broken, which leads to a finite effective quark mass. As a consequence of the Goldstone theorem, $N_f^2 - 1$ massless modes appear as quark-antiquark bound states. These states are identified with the $\pi$, $K$ and $\eta$ mesons. At a certain finite temperature, however, chiral symmetry gets restored and one has again massless quarks, whereas the mesons become massive. In this phase, they do no longer exist as bound states but rather as resonances. If one introduces a finite current quark mass, $m_{qf} \neq 0$, this picture gets slightly changed in that chiral symmetry is no longer an exact, but rather an approximate symmetry. Thus mesons have a small but finite mass at $T = 0$, as they have in nature, whereas the quark mass does not go to zero at large temperatures, but stays finite and for $T \to \infty$ goes to the current quark mass.

The quark mass spectrum for this case is shown in Fig. 1. The light quarks $u$ and $d$ are taken to be degenerate and will be denoted by the generic index $q$ in the following. The current quark masses used in Fig. 1 are $m_{uq} = 5.5 \text{MeV}$ and $m_{oq} = 140 \text{MeV}$. Due to chiral symmetry breaking, one has at $T = 0$ an effective mass of $m_q = 368 \text{MeV}$ and $m_s = 550 \text{MeV}$. In the temperature region $T \approx 200 \text{MeV}$, these masses drop and the effective mass of the light quarks becomes low. For the strange quarks, however, the situation is different. Due to the high current quark mass, the effective mass stays relatively high. At $T = 350 \text{MeV}$, which is far beyond any temperature to be realistically expected in present days heavy ion experiments, one still has $m_s = 300 \text{MeV}$, which is about twice as high as the current quark mass. The pseudoscalar mesonic mass spectrum using the same parameters is shown in Fig. 2. At $T = 0$, the mesons have their physical masses of $m_\pi = 135 \text{MeV}$,
$m_K = 498$ MeV and $m_\eta = 515$ MeV. With rising temperatures, these values stay more or less constant, until to a certain temperature, where their masses become equal to the masses of their constituents, i.e. $m_\pi = 2m_q$, $m_K = m_q + m_s$ or $m_\eta = 2m_q$, respectively. This happens when the $m_\pi$ and $m_\eta$ lines of Fig. 2 cross the $2m_q$ line, or the $m_K$ line crosses the $m_q + m_s$ line, respectively, as is indicated by the arrows. At these temperatures, the mesons become unstable via a Mott transition [6]. At higher temperatures, they are unstable with respect to the decay $M \to q\bar{q}$ and obtain a finite width. The qualitative form of the mass spectrum of Fig. 2 has been confirmed by lattice calculations [1].

A finite temperature study of this model will surely not lead to an event generator, since the model still contains free quark states, which are not observed in nature. It will, however, be able to study qualitative features of the plasma expansion, such as expansion time scales, the approach to equilibrium and thus the applicability of hydrodynamics, the production mechanism of hadrons etc. While the simulation program presently is limited to zero baryochemical potential, future extensions will remove this limitation and also allow for the study of strangeness distillation and DCC formation.

The treatment of the NJL model in nonequilibrium starts from the observation, that both quark and meson degrees of freedom can be simultaneously described by an equation of the Boltzmann type [9],

$$\left( \partial_t + \vec{\partial}_p E \vec{\partial}_x - \vec{\partial}_x E \vec{\partial}_p \right) n(t, \vec{x}, \vec{p}) = I_{\text{coll}} \left[ n(t, \vec{x}, \vec{p}) \right] ,$$

where $I_{\text{coll}}[n]$ is a collision integral. In principle, one is thus able to describe a transition from a pure quark regime to a hadronic regime, where quarks are converted to hadrons via collision processes. The solution method we choose for Eq. (3) is an algorithm of the QMD type [10], i.e. we parametrize the particle distribution functions by

$$n(t, \vec{x}, \vec{p}) = \sum_i \exp \left( -\frac{(\vec{x} - \vec{x}_i(t))^2}{2w^2} \right) \exp \left( -\frac{w^2}{2}(\vec{p} - \vec{p}_i(t))^2 \right) .$$

The center points of the distribution move on the characteristics of Eq. (3), i.e.

$$\dot{\vec{x}}_i(t) = \vec{p}_i(t)/E \quad \quad \dot{\vec{p}}_i(t) = -\vec{\partial}_x E + \text{collision contributions} .$$

Equation (5) has to be solved together with the gap equation
\[
m_i = m_{qi} - \frac{GN_c}{\pi^2} m_i A_i + \frac{KN_c^2}{8\pi^4} m_j m_ka_k, \quad i \neq j \neq k \neq i \quad (6a)
\]

\[
A_i = -8\pi^2 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{E} \left(1 - \frac{n_i + n_j}{2N_c}\right), \quad (6b)
\]

where the indices \(i, j\) and \(k\) run over all three quark flavors. For the computation of mesonic properties we take a shortcut by defining an effective temperature via \(m_q(\vec{x}, t) = m_{q}^{\text{eq}}(T_{\text{eff}}(\vec{x}, t))\), where \(m_{q}^{\text{eq}}(T)\) is the equilibrium form of the temperature dependence of \(m_q\) as shown in Fig. 1. The mesonic properties can then be obtained using the equilibrium expressions, which are functions of \(T_{\text{eff}}, m_q\) and \(m_s\). The collision processes entering Eq. (5) are (i) quark quark scattering \(qq \leftrightarrow q\bar{q}, q\bar{q} \leftrightarrow q\bar{q}\) and \(q\bar{q} \leftrightarrow q\bar{q}\) [7], (ii) hadronization \(q\bar{q} \leftrightarrow MM\) [8], and (iii) meson decay \(M \rightarrow q\bar{q}\) [9]. The latter process is only possible in the early phases of the expansion, when the effective temperature is higher than the Mott temperature.

The initial conditions chosen presently do not contain strange quarks, whereas light quarks are distributed thermally within a sphere of a given radius. This kind of initial conditions has the immediate consequence, that the strange quark mass is even higher than in thermal equilibrium. The reason for this can be seen by writing Eq. (6) explicitly for strange quarks. One recognizes that the second term on the right hand side, which is proportional to \(G\), only couples to the strange quark condensate. Since initially no strange quarks are present, this term does not receive medium corrections and thus makes the effective quark mass higher than in thermal equilibrium. Medium corrections do arise from the third term of Eq. (6), which is proportional to the product of the up and down quark condensate. This contribution is, however, not sufficient to produce a large mass drop.

Quantitatively, this can be seen from Fig. 3, which shows the quark masses and the effective temperatures as a function of \(r\) at various times during the initial phase of the expansion. At \(t = 0\), the light quark mass in the center of the system is low, according to the high particle density here. At larger radii, the mass goes up due to the gaussian form of the particle distribution. Note that the quark masses are only known at those points where a particle is present, thus the curves stop at the edge of the fireball. The effective temperature in the centre amounts to approximately 250 MeV. At this temperature in equilibrium, one would expect a strange quark mass of 380 MeV according to Fig. 1. In reality, however, one has a strange quark mass of approximately 450 MeV in the centre due to nonequilibrium effects. At later
times, the system expands and thus the quark mass curve becomes flatter, while its width grows. Accordingly, the effective temperature drops. In the final state, the quark masses will tend towards the vacuum value, which means that the mean field part of the interaction ceases [11].

Mesons are created during the evolution by collisions of quarks and antiquarks. Most of all mesons created are pions, which are most frequently produced by the collision of two light quarks. Kaons, on the other hand, are produced most easily by the collision of a light with a strange quark or two strange quarks, while the hadronization cross section for the creation of a kaon pair from a light quark antiquark pair is rather low [8]. Eta mesons are most frequently created collisions of two light quarks, forming a pion and an eta. The time dependence of the multiplicities is shown in Fig. 4. It can be seen here, that the production of mesons starts immediately after the beginning of the expansion. The production rate is maximal at \( t = 0 \), when the particle density is high, giving thus rise to a large number of collisions. At later times the density drops and thus the number of collisions per time decreases. This in turn leads to a flattening of the multiplicity curves.

Figure 5 shows the particle density, averaged over all solid angles, at time \( t = 10 \text{ fm/c} \). It is clearly visible that the meson density is maximal at the same places where also the quark density is high. This confirms the picture gained from Fig. 4, i.e. that mesons are created within the bulk of the fireball. More insight can be gained from Fig. 6, which shows the distribution of the effective temperatures at the creation points of the mesons for pions, kaons and etas respectively. All three distributions agree more or less with each other, up to the effects of lower statistics for the kaon and the eta. It can be seen that mesons are created most likely at temperatures directly below the Mott temperature, and within a temperature range of 50 MeV. This plot should be compared to the mean hadronization times for light and strange quarks in equilibrium, as have been calculated in [8]. It has been shown there, that these hadronization times have a minimum in the temperature range \( 150 \text{ MeV} < T < 200 \text{ MeV} \), which agrees nicely with Fig. 6.

Figure 7 shows the density along the coordinate axes for \( t = 0 \) and \( t = 30 \text{ fm/c} \). It can be seen here, that, although one starts with a system, which is to a good approximation spherically symmetric, one ends up with a final state which shows large fluctuations of the density with respect to the direction. This behaviour contradicts
hydrodynamics, where an initially symmetric system stays symmetric.

To conclude, it has been demonstrated how a chirally symmetric quark-meson plasma containing strangeness behaves out of equilibrium and how mesons are produced. This investigation has been performed at zero baryochemical potential. Future investigations will remove this limitation and thus be able to study the mechanism of strangeness distillation. Also the study of DCC formation is planned.

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REFERENCES


FIGURES

FIG. 1. Quark masses at finite temperature in equilibrium. Solid line: light quarks, dashed line: strange quarks.

FIG. 2. Meson masses as a function of temperature. Also shown are $2m_q$ and $m_q + m_s$. The Mott transitions of the pion, the kaon and the eta are marked by the arrows.

FIG. 3. Effective quark masses (left) and temperatures (right) as a function of $r$ for $t = 0$, 2 and 4 fm/$c$. In the left panel, the upper curves are for strange quarks and the lower ones for light quarks, respectively.

FIG. 4. Time dependence of the particle multiplicities.

FIG. 5. Angular averaged particle density at $t = 10$ fm/$c$.

FIG. 6. Histogram of the creation temperatures of pions (solid line), kaons (dashed line) and etas (dot-dashed line).

FIG. 7. Particle density along the coordinate axes. Upper panel: $t = 0$, lower panel: $t = 30$ fm/$c$. 

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