PHOTOFLUID INSTABILITIES OF HOT STELLAR ENVELOPES

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October 19, 1998

Abstract

Beginning from a relatively simple set of dynamical equations for a fluid permeated by a radiative field strong enough to produce significant forces, we find the structure of plane-parallel equilibria and study their stability to small acoustic disturbances. In doing this, we neglect viscous effects and complications of nongreyness. We find that acoustic instabilities occur over a wide range of conditions below the Eddington limit. This result is in line with findings reported twenty years ago but it contradicts some more recent reports of the absence of instabilities. We briefly attempt to identify the causes of the discrepancies and then close with a discussion of the possible astrophysical interest of such instabilities.

PACS: 47.70.Mc, 97.10Ex
Keywords: radiation gas dynamics, stellar atmospheres, stellar activity and pulsation

1 Introduction

Evidence of high levels of fluid dynamical activity in hot stellar atmospheres has been available since O. Struve detected large line widths that he attributed to macroturbulence. In fact, the actual nature of the motions is still not certain. Given the many possible causes of vigorous motion, this is not surprising, and it remains unclear whether the activity is driven by rotation, pulsation, radiation, or all of these. Huang and Struve (1960) favored the notion that the large partial pressure of radiation in hot stars plays a key rôle in this problem, but they never made it clear how. Indeed, in the context of early cosmology, radiation pressure is thought of as a stabilizing influence.

In this discussion we wish to isolate the effects of the radiative forces and to discuss the possible consequences they may have for the fluid dynamics of a stratified layer. It goes without saying that aspects of this topic should be of interest for stellar interiors and hot disks as well as stellar atmospheres.

A concrete image of the kind of fluid motions that radiative forces might produce is to be found in the dynamics of fluidized beds (Davidson and Harrison, 1963). Fluidization occurs when a fluid is forced to flow upward through a bed of particles. When the upward drag on each particle balances its weight, the particles are levitated and they form a second fluid. The bed is then said to be fluidized and, once this happens, it is in a state analogous to that of a stellar atmosphere at the Eddington limit, when radiative levitation balances gravity (Prendergast and Spiegel, 1973).

In fluidized beds, the drag per particle is a sensitive function of the density of particles. Hence, the fluidized state can be sustained over a wide gamut of conditions ranging from the lethargic case of quicksand to the kind of vigorous bubbling that may also occur in a hot stellar atmosphere close to the Eddington limit. The bubbling state arises when the typical mass density of a single particle in the bed greatly exceeds the density of the driving fluid. In that case, voids in the particle distribution form, rise to the top of the bed and collapse, throwing particles upward, as in a boiling liquid. This phenomenon has interested chemical engineers for some time and, if the analogy to the astrophysical situation holds, there is much interest in it for the astrophysicist as well.

While it is not clear exactly how fluidization bubbles form, it is generally supposed that the formation process is driven by instability. In the case of the fluidized bed, there are known instabilities resulting from the dependence of the drag per particle on the density of particles. This mechanism is thus a mechanical analogue of the $\kappa$-mechanism of stellar pulsation.
Under suitable conditions, when such mechanisms operate, we can expect a Hopf bifurcation that, in the stellar case, causes pulsation.

The hottest stars are mostly ionized and so they are not highly susceptible to instability by the $\kappa$-mechanism that operates in many cooler stars. Nevertheless, very hot stars do pulsate. That may be sufficient grounds for suspecting that bubbling occurs in their atmospheres, since pulsation can produce parametric instability of convective modes in a star (Poyet and Spiegel, 1979). In very hot stars, pulsation may be driven from the stellar cores or be the result of instability in the outer layers (Umurhan, 1998). In the present work we investigate the possibility of radiatively driven instabilities in hot stellar atmospheres, or rather, in hot slabs stratified under gravity and radiative forces.

The stability problem of hot stellar atmospheres has been studied for a few decades with mixed results. Over twenty years ago, many participants at a meeting in Nice (Cayrel, R. and Steinberg, M. 1976) concluded that there were instabilities in very hot atmospheres. At that time, the main issue under discussion was whether the instabilities were convective or absolute. However, subsequent studies offered the conclusion that radiatively driven instability could occur only under very special conditions if at all. Marzek (1977) found that instability occurred only just below the Eddington limit. Marzek worked directly with the transfer equation by numerical means and thus had more detail in his system than most previous investigators. Recently, Asplund (1998) claimed that there is no instability until the Eddington limit is surpassed. In his treatment of the transfer theory, Asplund omitted the same terms that many early workers dropped in the closure of the transfer equations and so had less detail than some previous workers. So the different conclusions of the older and newer work is not just a question of more or less detail in the basic equations.

The reason for the change in the prevailing results is not clear. It may just be that workers who had reproduced the earlier results simply saw no point in announcing this, so they have not come forward. The results of Marzek are also unpublished. On the other hand, the observed evidence of great activity in the hottest stars does suggest some form of instability. Perhaps, there are other mechanisms at work. In this vein, Arons (1992) has included magnetic effects to promote instability. There are also no doubt rotational instabilities and parametric instabilities that may be present. However, the issue of the radiative instabilities has become clouded and it seems a sufficiently important astrophysical issue that the problem should be reconsidered. That is the purpose of this paper.

In the next section we present the equations we shall use with some discussion of their provenance. Then we turn to the equilibria they allow and linearize the full equations about them. We do find unstable acoustic waves, and make an attempt to understand their origin before concluding with a brief discussion of their possible implications.

2 The Photofluid Equations

For this study we use the continuum description of both the matter and the radiation. For each, we have a stress tensor and the vanishing of the divergence of the sum of these provides the equations of the problem, though they must be supplemented by equations of state for the material and radiative fluids and by formulae for scattering and absorption coefficients.

We express the equations in an inertial frame that we call the basic frame. In this case, the equations of motion are (Hsieh and Spiegel, 1979; Simon 1963):

$$\frac{\partial p}{\partial t} + \nabla \cdot (\rho u) = 0$$ (2.1)

$$\rho \left[ \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = -\nabla p - \rho g \hat{z} + \rho \left( \frac{(\kappa + \sigma)}{c} \right) \left( F - \frac{4}{3} E u \right)$$ (2.2)

$$\frac{\partial E}{\partial t} + \nabla \cdot F = \rho c (S - E) + \frac{(\kappa - \sigma)}{c} u \cdot F$$ (2.3)

$$\frac{\partial F}{\partial t} + \frac{c^2}{3} \nabla E = -\rho (\kappa + \sigma) c (F - \frac{4}{3} E u) + \rho c (S - E) u - \nabla \cdot (u F + Fu) + \frac{2}{3} \nabla (u \cdot F)$$ (2.4)

$$\frac{\partial p}{\partial t} + (u \cdot \nabla) p - \frac{c^2}{3} \left[ \frac{\partial p}{\partial t} + (u \cdot \nabla) p \right] = -\gamma - 1 \left[ \rho c (S - E) + 2 \rho \kappa - u \cdot F \right]$$ (2.5)

$$S = a T^4$$ (2.6)

$$p = R \rho T$$ (2.7)
In these equations, all quantities are expressed in the basic frame. The fluid variables $u, \rho, p$ and $T$ are the velocity, the mass density, pressure and temperature, respectively. The main radiative variables, $F$ and $E$ are the radiative flux, and energy density, both integrated over frequency while $R, a$ and $\gamma$ are the gas constant, the radiation constant and the ratio of specific heats. The Thomson cross section is $\sigma$, $\kappa$ is the mean absorption coefficient, $c_s$ is the speed of sound, and $c$ is the speed of light.

We adopt the plane-parallel geometry of an atmosphere with constant gravity; the unit vector $\hat{z}$ points in the vertical direction. Since the radiation is relativistic, we have included here some $O(v/c)$ corrections to the radiative quantities. For example, the radiative flux as seen in a frame locally moving with the matter is $F = \frac{4}{3}Eu$. Such corrections are not quantitatively important in the present discussion, as we shall see.

In continuum physics one writes constitutive relations expressing the stress in terms of other basic fluid properties. Here we consider an ideal fluid in which the dissipation is caused by interaction with a coexisting radiation fluid. For the latter, a constitutive relation, or closure approximation, was derived from the equation of transfer by taking its first two moments. That system was then closed by the Eddington relation between the radiative pressure tensor and the radiative energy density, $P_{ij} = \frac{4}{3}E\delta_{ij}$ where $\delta_{ij}$ is the Kronecker delta. The resulting system was then expanded in $u/c$, where $c$ is the speed of light, and truncated at order $u/c$.

A number of simplifications have been made in deriving the basic equations given here. We assumed that the energy exchanged between the fluid and the radiation field may be qualitatively accounted for by absorption and re-emission. Thomson scattering is assumed with the qualitative notion that Compton effect can be represented as part of the absorption term. The details of such processes have been blurred here because we have replaced absorption coefficients by a single mean absorption coefficient. Such replacement has been a standard astrophysical practice on which much discussion has centered. The best means to use depends on which term in which equation one is focused on and we have simply assumed that they are all the same. The problem is even more delicate when the medium is in motion, for then the means take on tensor character. We have assumed that these tensors are diagonal and that all their nonzero elements are equal.

We are working in the Eddington approximation, which represents the radiative pressure as diagonal. It therefore does not include viscous stresses and we have not added any. When those are included they are likely to promote stability, unless they are anisotropic. The effects of bulk viscosity have also not been explored. All these issues await discussion in future work, but we feel the present level of complexity makes a good starting point for revisiting the stability problem.

Finally, we need to worry about boundary conditions. The ones we use shall be mentioned in connection with the linear stability problem below. These will be solved in a slab of finite thickness extracted from the equilibrium structure, to which we turn next.

### 3 An Equilibrium State

The question of instability naturally involves a statement of the state that is unstable. Finding such states can in itself be a complicated matter, especially in cases where the Eddington limit has been exceeded. Studies such as that of Asplund in which the Eddington limit is exceeded over the whole layer are in fact difficult to assess since it is not clear if such local conditions can be matched to a realistic stellar model that has a proper spherical shape both within and without. In the books of Eddington and of Chandrasekar on stellar structure one finds some spherical models where the Eddington limit is locally exceeded in the deep interior, but this reversal of the local effective gravity is possible only if the opacity is suitably dependent on physical conditions (Underhill, 1949). Such situations do not arise in an atmosphere dominated by Thomson scattering.

We are here studying the plane parallel case and the only way in which to have a portion of it exceed the Eddington limit in a physically reasonable way is to have a suitably variable opacity. This is not easily achieved in a very hot atmosphere where scattering dominates so we shall not consider super-Eddington conditions here. Indeed, we are here mainly interested in instabilities in more typical stellar atmospheric conditions.

From equation (2.5) we find that in a steady state with no motion, we must have $E = S = aT^4$. Then we see that $F = F\hat{z}$ with constant $F$. For a state that is horizontally homogeneous and depends only on the vertical coordinate, $z$, with the flux in the $\hat{z}$-direction, the basic equations are the following. The hydrostatic equation of the fluid is

$$\frac{dp}{dz} = g_s \rho \quad (3.1)$$
where $p$ is the gas pressure and

$$g_* = g - g_n \quad \text{with} \quad g_n = \frac{\sigma + \kappa}{c} F ;$$  (3.2)

$g_*$ is the acceleration of gravity corrected for photolevitation. The analogous relation for the radiative pressure, $\frac{1}{3}E$ is

$$\frac{dE}{dz} = -3g_n \rho .$$  (3.3)

With these equations we also use the equation of state (2.7).

This is a standard problem in the theory of atmospheres and by analogy with what is done there we introduce an optical coordinate,

$$\tau = \int_0^z \rho (\sigma + \kappa) ds',$$  (3.4)

where the origin of $z$ is at some convenient level in the atmosphere where we also locate the origin of $\tau$. We see then that

$$E = \hat{E} - 3\frac{F}{c} \tau ,$$  (3.5)

where the integration constant, $\hat{E}$, is the value of $E$ at $\tau = 0$. This result is a standard of the Eddington approximation of radiative transfer in a stellar atmosphere.

From equation (3.1) and the equation of state we readily find the integral

$$p = \hat{p} - gq + \frac{F}{c} \tau ,$$  (3.6)

where $\hat{p}$ is the pressure at the origin of $\tau$ and of $z$ and

$$q = \int_0^z \rho dz'$$  (3.7)

is a column density. Thus,

$$\mathcal{P} \equiv p + \frac{1}{3}E = \hat{P} - gq$$  (3.8)

which indicates that the total pressure supports the weight of the atmosphere.

At this point, it becomes complicated to discuss the general case and it is best to separate the case of the hot atmosphere with $\sigma \gg \kappa$ from the opposite case of the cool atmosphere, or even that of the earth’s atmosphere. In the former case, since $\sigma$ is constant, we can write a closed expression for the properties. In the latter case, if we may approximate $\kappa$ as a product of powers of density and temperature we can obtain similar results by using matched expansions. In the cool case, we may omit the radiative forces. Nevertheless, in both cases, the atmosphere splits into two parts, a lower polytropic layer and an upper isothermal layer. We illustrate with an idealized case that we shall concentrate on here, with $\sigma + \kappa$ taken constant.

For our illustrative static atmosphere, we have $\tau = (\kappa + \sigma)q$ and so (3.6) and (3.8) combine into

$$p = \hat{p} + \frac{g_*}{3g_n} (E - \hat{E}) .$$  (3.9)

As $z \to \infty$, the matter thins out and we require that the gas pressure should vanish in that limit. Nevertheless, radiation pours out of the atmosphere, so the radiation pressure need not vanish. If we use a tilde to denote evaluation at $z = \infty$, (3.9) becomes

$$\tilde{E} = \hat{E} - 3g_n \hat{p} .$$  (3.10)

We obtain

$$p = \frac{g_*}{3g_n} (E - \tilde{E}) .$$  (3.11)
Then, if we write \( \tilde{T} = a \bar{T}^4 \) and \( \Theta = T/\bar{T} \), (3.3) becomes

\[
\frac{4 \bar{R} \tilde{T}}{g_c} \frac{d \Theta}{dz} = \Theta^4 - \frac{1}{\Theta^4}.
\]  
(3.12)

This is readily solved and we find (Spiegel, 1977)

\[
-\frac{g_c z}{4RT} = \Theta - \frac{1}{2} \tan^{-1} \Theta - \frac{1}{2} \coth^{-1} (\Theta).
\]  
(3.13)

The temperature profile for this atmosphere is linear for \( z \) very negative and constant for \( z \) very positive. So we have two rather distinct regions: a polytrope below and an isothermal atmosphere above. When we set \( \bar{T} = 0 \), we find that \( T \) is linear and we have simply a polytropic atmosphere whose upper boundary is at \( z = 0 \). To complete the solution, we note that (3.11) gives us \( p \) in terms of \( E \), that is, \( a \bar{T}^4 \) and that the ideal gas law is then used to compute \( \rho \).

It is interesting that composite structures like this one occur in cool star stellar atmospheres as well as in the earth’s atmosphere. In 1899 Teisserenc de Bort reported his discovery of (what was then called) the isothermal region in the high atmosphere of the earth. This layer is now known as the stratosphere and the early attempts to understand it did involve radiative transfer as well as other complications that were thought essential (Humphreys, 1964). It is interesting that the present theory gives rise to a composite structure resembling that of the troposphere and stratosphere and seems to give a rough description of the combination of the photosphere and chromosphere of the sun as well. In our simplified version, the lower layer is limited to a polytropic exponent of \( 4/3 \), but this is readily generalized and, in any case, is quite appropriate to our present interest in hot stars.

We need to stress that a plane-parallel model are to be used over sufficiently confined range of depths, even though the solution just mentioned extends over an infinite range of depths. We shall study the stability of only a finite slab cut out of this solution in the next section.

### 4 Stability Theory

#### 4.1 Linear Equations

To describe small perturbations to the static solution just described, we indicate the value of a variable in equilibrium by the subscript naught. The linearized equations are separable in space and time and horizontal and vertical coordinates so that a typical variable, such as density, has the form

\[
\rho(x, y, z, t) = \rho_0(z) + \rho_1(z) \exp(ikx - i\omega t)
\]  
(4.1)

where the perturbation eigenfunction has to be determined along with the eigenvalue \( \omega \).

When we put the forms (4.1) for all the variables into the basic equations and neglect quadratic and higher terms we obtain a set of equations for the perturbation quantities. It is easier to read the content of this set if we suppress the subscript one from the perturbation amplitudes. When we do this, the equations are

\[
\frac{i \omega \rho}{\rho_0} = iku + \frac{\partial}{\partial z} w + w \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0
\]  
(4.2)

\[
-i \omega \rho_0 w = -\frac{\partial}{\partial z} p - g_c \rho + \rho_0 \frac{(\kappa + \sigma)}{c} \left(F_z - \frac{4}{3} E_0 w\right)
\]  
(4.3)

\[
-i \omega \rho_0 u = -ik p + \rho_0 \frac{(\kappa + \sigma)}{c} \left(F_x - \frac{4}{3} E_0 u\right)
\]  
(4.4)

\[
-i \omega E + ik F_x + \frac{\partial}{\partial z} F_z = \rho_0 \kappa c (S - E) + \rho_0 \frac{(\kappa - \sigma)}{c} F_0 w
\]  
(4.5)

\[
-i \omega F_x + ik \frac{c^2}{3} E = -\rho_0 (\kappa + \sigma) c \left(F_x - \frac{4}{3} E_0 u\right) - F_0 \frac{\partial}{\partial z} u + \frac{2}{3} ik F_0 w
\]  
(4.6)
\[-i\omega F_z + \frac{c^2}{3} \frac{\partial}{\partial z} E = -\rho_0 (\kappa + \sigma) c \left( F_z - \frac{4}{3} E_0 w \right) - \rho(\kappa + \sigma) c F_0 - ik F_0 u - \frac{4}{3} F_0 \frac{\partial}{\partial z} w \] (4.7)

\[-i\omega p - \rho_0 g_* w + c_s^2 \rho_0 (iku + \frac{\partial}{\partial z} w) = -\gamma(1) \rho_0 \kappa c (S - E) - 2(\gamma - 1) \frac{\rho_0 k F_0}{c} w \] (4.8)

\[S = 4 E_0 \left( \frac{p}{p_0} - \frac{\rho}{\rho_0} \right) \] (4.9)

where \(F_z\) is the \(z\)-component of the perturbed flux, and so on.

### 4.2 Two-point boundary–value eigenvalue problem

In our solutions of the linear equations, we have found that the terms coming from frame changes have very little effect. For example, the last two terms of equation (2.4) come from making the Eddington approximation in the frame co-moving with the fluid and then Lorentz-transforming back into the star frame. We find that when we solve the equations without those terms and then evaluate those from the solutions so obtained, they are indeed quite small. On the basis of such consistency checks, we omit them. The full set of reduced equations is exhibited in the next section.

In the problem simplified in this way, there are four first-order equations in \(z\) for four dependent variables. The ones we work with are the vertical component of the velocity field, \(w\), the gas pressure, \(p\), the vertical component of the radiation flux, \(F_z\), and the radiative energy density, \(E\). Thus we have the following set of first order equations:

\[
\frac{\partial}{\partial z} w = \left( \frac{N^2}{g_*} - \frac{g_*}{c_s^2} \right) w - i\omega \frac{\rho}{\rho_0} \quad (4.10)
\]

\[
\frac{\partial}{\partial z} p = \left[ i\omega - \frac{4(\kappa + \sigma) E_0}{3c} \right] \rho_0 w + \rho_0(\kappa + \sigma) c F_z - g_* \rho \quad (4.11)
\]

\[
\frac{\partial}{\partial z} F_z = -\left( \rho_0 \kappa c - i\omega \right) E + \frac{\rho_0 (\kappa - \sigma) F_0}{c} w + 4 \frac{\kappa E_0 c}{c_s^2} p - ik F_z - 4 \kappa E_0 c p \quad (4.12)
\]

\[
\frac{\partial}{\partial z} E = -\frac{3}{c^2} [\rho_0 (\kappa + \sigma) c - i\omega] F_z + 4 \frac{\rho_0 (\kappa + \sigma) E_0}{c} w - 3 \frac{(\kappa + \sigma) F_0}{c} \rho \quad (4.13)
\]

where \(g_*\) is the effective gravity and \(N\) is the buoyancy frequency given by

\[
\frac{N^2}{g_*} = -\left( \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 - \frac{g_*}{c_s^2} \right) \quad (4.14)
\]

Furthermore, the density, horizontal velocity, and horizontal radiative flux perturbations satisfy

\[
\rho = \frac{1}{c_s^2} p - \left[ i\omega - \frac{4(\gamma - 1) \kappa c E_0}{c_s^2} \right] \left[ 2(\gamma - 1) \rho_0 k F_0 w + \left[ i\omega - \frac{4(\gamma - 1) \kappa c E_0}{c_s^2} \right] \left[ \frac{k}{\omega \rho_0} \left( 1 - i \frac{\omega}{\rho_0 (\kappa + \sigma) c} \right) p + \frac{k}{3 \omega \rho_0} E \right] \right] \quad (4.15)
\]

\[
u = \left( 1 + \frac{4}{3} \frac{\rho_0 c^2}{\omega} - \frac{i \omega}{\rho_0 (\kappa + \sigma) c} \right) \left[ k \frac{\omega \rho_0}{\rho_0 (\kappa + \sigma) c} \right] \left[ \frac{k}{3 \omega \rho_0} \left( 1 - i \frac{\omega}{\rho_0 (\kappa + \sigma) c} \right) p + \frac{k}{3 \omega \rho_0} E \right] \quad (4.16)
\]

\[
F_x = \left( 1 + \frac{4}{3} \frac{\rho_0 c^2}{\omega} - \frac{i \omega}{\rho_0 (\kappa + \sigma) c} \right) \left[ \frac{4 k E_0}{3 \omega \rho_0} p + \frac{k E_0}{9 \omega \rho_0} \left( 1 - i \frac{\omega c}{4 (\kappa + \sigma) E_0} \right) E \right] \quad (4.17)
\]

We shall solve these equations in a slab of finite vertical extent in the domain \(z_0 \leq z \leq z_1\). The choices of the lower and upper boundaries have a significant effect on the stability results since we may have a nearly pure polytrope, an almost isothermal atmosphere, or a composite of the two. Though the degree of instability does change in response to variations
in such choices, the instabilities do seem to occur over a range of choices. In all cases we truncate the equilibrium so that the background density is finite everywhere in the computational domain. This prescription minimizes the influence of singularities in the equations.

We assume that there is no mass flow through the boundaries and so set

\[ w(z_b) = w(z_t) = 0 \]  

\( (4.18) \)

We also adopt standard Eddington boundary conditions on the boundaries (Hsieh, 1977), namely

\[ E(z_b) + 2F_z(z_b) = 0 \quad (bottom) \]  
\[ (4.19) \]
\[ E(z_t) - 2F_z(z_t) = 0 \quad (top) \]  
\[ (4.20) \]

to fix the incident radiation at the lower boundary and to ensure that no radiation comes from the upper boundary.

4.3 Numerical results

We have solved the equations numerically in a variety of parameter ranges. The reliability of the program we used has been checked by independent calculations. In brief, the main result of the study is that we find instabilities in a wide range of conditions. We give here a brief sampling of the results in a parameter range that is not extreme from the standpoints of stiffness and other numerical considerations.

For a given horizontal wave number, \( k \), the equations and boundary conditions (4.10) - (4.20) form a two-point boundary value eigenvalue problem with the (complex) frequency of oscillation \( \omega \) as the eigenvalue. We seek solutions in a sequence of horizontal wavenumbers to trace particular branches of modes. In the case without radiative effects, this procedure correctly recovers the adiabatic dispersion relations that are known analytically. However, in the present case with radiative driving, we have only numerical results to offer so far. To obtain these, we have used a Newton-Raphson-Kantorovich relaxation scheme to find and track the various branches of radiative fluid waves. Here we describe the unstable photo-acoustic modes, which are the most likely to produce significant observable effects.

We find instability even for the rather mild case of \( \alpha = \gamma_\ast / g = 0.9 \), and for a wide range of physical parameters as we raise the level of radiative levitation by lowering \( \alpha \). The horizontal wave number of the fastest growing mode in each case depends sensitively on the truncation of the basic state. We find a \( k = 0 \) instability for basic states that are mostly polytropic; however, in general, the inclusion of the isothermal region reduces the growth rate of the \( k = 0 \) instability, and may even move the most unstable modes to finite wave numbers. Here we illustrate the kind of results obtained with the case \( \alpha = 0.7 \).

In Figs. 1-3 we show the eigenvalues as a function of horizontal wave number \( k \) for \( \alpha = 0.7 \). In each case the upper panel is the real part of \( \omega \), the frequency, and the lower panel shows (minus) the imaginary part, or growth rate. Fig. 1 is for a polytrope, Fig. 2 is from the approximately isothermal region of the equilibrium only, and Fig. 3 is for a composite containing the transition region at \( z = 0 \). The time of growth of unstable modes are tens or hundreds of oscillation periods, depending on the details of the structure. In fact, this represents reasonably strong instability by the standards of pulsation theory.

The mode with \( k = 0 \) corresponds to radial oscillation. It is typically the most unstable one in the purely polytropic case. As we go to the composite case, the wave number of maximum growth goes to a finite value. Even if the main instability were radial, it would probably trigger higher wavenumbers through secondary instability if its amplitude became large enough. We show the eigenfunctions corresponding to the \( k = 0 \) case of Fig. 3 in Fig. 4; the panels on the right are the real parts and those on the left the imaginary parts. The eigenfunctions do not show appreciable dependence on \( k \), so we may spare the reader further details of these.

5 Discussion

Having computed the stability calculation for a variety of parameters, we then evaluated the terms individually and, in this way, we learned which terms are important for the stability. These results should be useful elsewhere in developing approximation techniques for such problems. Here we show the equations with these terms omitted. They are negligible in the problems studied and probably remain so for all nonrelativistic cases. The reduced equations are
In Fig. 5 we show the eigenvalues from Fig. 3 with the corresponding ones computed with the reduced equation set.

In fact, for the modes studied here, the terms $\partial_t E$ and $\partial_t F$ are also negligible. Omitting these terms is somewhat analogous to neglecting the displacement current in hydromagnetics: we are giving up here the acoustic modes of the photon fluid when we throw out those terms. And, when we do discard those terms in this reduced set of equations, we obtain the set of equations studied by Asplund (1998), who finds instability only in the case of negative $g_s$. Perhaps the absence of instability in situations like those we have studied here comes from Asplund’s use of a local approximation.

Instabilities of acoustic modes are found in models of cooler stellar atmospheres with very similar descriptions to the one given in this section. Such studies have $\alpha = 1$ and they typically replace the description of the radiative terms by the diffusion limit or the optically thin limit, in which Newton’s law of cooling may be used. We believe that the instabilities we see at small horizontal wavenumber are descendants of those thermoacoustic instabilities (Umurhan, 1998) with possibly an admixture of some form of $\kappa$-mechanism (Unno et al. 1979).

It is not clear why the treatment of Marzek does not show at least these instabilities inherited from cooler conditions. It may be that this is a matter of underlying structures. The cool-star studies are largely confined to polytropic layers and we have seen that the full atmosphere typically has a composite structure. On the other hand, we have run calculations in layers confined to $z > 0$, that is with the polytropic part cut out, and we still find instabilities. The fact that these were not seen (or at least not reported by) Marzek is therefore disquieting. He did work with the transfer equation and so his results contain the influence of radiative viscosity that we have neglected. On the other hand, at large horizontal wavenumbers, when the modes become optically thin, these effects should not be large. This discrepancy remains unresolved.

These results show why we felt that it is appropriate to reopen the stability discussion here. We claim that hot stellar atmospheres are subject to instabilities in the form of growing waves. Another contributor to this volume, N. Shaviv, has also offered this general conclusion, though as yet we have not made detailed comparisons with this work. We have here confined our calculations to compact domains, thus finding the absolute instabilities, but we suspect that there are also strong drift instabilities. These are harder to study by numerical means, but we are now undertaking some analytic work on that issue.

If it is agreed that radiative instabilities do occur in the atmospheres of the hottest stars, it is of interest to look ahead to the possible nonlinear developments one may expect from such instabilities. Cassinelli (1985) has argued that hot stars have spots on the grounds that they exhibit the phenomena that go with them in the solar case, such as coronas and other high excitation features. Since one does not expect the conditions for ordinary convection in hot stellar atmospheres, another mechanism is needed if this suggestion is correct.

The acoustic instabilities we have discussed here are likely to produce strong density inhomogeneities with perhaps the kind of bubbling that one sees in fluidized beds (Prendergast and Spiegel, 1973). Since hot stars are typically rapid rotators, we may expect the usual symbiosis of vigorous stellar fluid dynamics and stellar rotation that is believed to produce dynamo action. Cassinelli’s conjecture seems to us to have a strong chance to be correct. We believe that photofluidodynamic instabilities are likely to be of interest in the study of hot stars and disks. While the need for further stability calculations remains, we feel that numerical simulation in the nonlinear regime may now be a fruitful pursuit. We hope to have results on such calculations with photon bubbles and radiative vortices (phortices) in time for Giora’s hundred and twentieth birthday.

L.T. acknowledges support from an NSF postdoctoral fellowship. We were participants in the Geophysical Fluid Dynamics summer school at Woods Hole Oceanographic Institution during the completion of this work.
References

Cayrel, R. and Steinberg, M., eds. 1976 Physique des Mouvements dans les Atmospheres Stellaires, Colloques Internationaux du C.N.R.S., No. 250; (see therein the papers of Castor, Hearn and Spiegel).
Unno, W., Osaki, Y., Ando, H & Shibahashi, H. 1979 Nonradial Oscillations of Stars (Univ. of Tokyo Press).
Figure Captions

Figure 1. Dispersion relation of the fundamental acoustic mode for $\alpha = 0.7$. The top panel shows the acoustic frequency and the bottom panel is the instability growth rate. This is the case of the polytrope atmosphere.

Figure 2. Dispersion relation of the fundamental acoustic mode for $\alpha = 0.7$. The top panel shows the acoustic frequency and the bottom panel is the instability growth rate. This is the case of the isothermal atmosphere.

Figure 3. Dispersion relation of the fundamental acoustic mode for $\alpha = 0.7$. The top panel shows the acoustic frequency and the bottom panel is the instability growth rate. This is the case of the composite atmosphere, where the transition from the polytrope to the isothermal atmosphere occurs at $z = 0$.

Figure 4. Eigenfunctions of the fundamental acoustic mode for $\alpha = 0.7$. The left and right panels correspond to real and imaginary parts. This is the case of the composite atmosphere, where the transition from the polytrope to the isothermal atmosphere occurs at $z = 0$.

Figure 5. Comparison of dispersion relations between the full equations and the reduced equations. Solid lines are the full calculation and the dashed lines are the reduced one. This is for the composite atmosphere at $\alpha = 0.7$.