Standard Model and CP Violations in Neutral K- and B-meson Systems

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Abstract

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In this paper we have discussed CP violations in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ systems in the framework of the Standard Model. To start with, using only the experimental values of the moduli of the elements in the first two rows of the CKM matrix and its unitarity, we have sought to fix the values of the complex phase angle δ , where $V_{ub} = |V_{ub}|e^{-i\delta}$. We find, in the PDG representation, that $\delta = \delta_1 = 70^\circ \pm 6^\circ$ or $\delta = \delta_2 = \pi - \delta_1$. Next, we have utilized the empirical information on the CP violation parameter $|\epsilon_K|$ in neutral Kaon decays, the $B^0 - \bar{B}^0$ mixing parameter x_d , the ratio $|V_{ub} / V_{cb}|$ and the theoretical results from the box diagrams for $|\epsilon_K|$ and x_d in order to evaluate the range of values for the angle δ . The result is $32^\circ \leq \delta \leq 125^\circ$, which includes both δ_1 and δ_2 obtained earlier. This analysis also allows us to obtain the possible values of β for a given angle δ , where β is defined by $V_{td} = |V_{td}|e^{-i\beta}$. Thus we are able to predict the possible values of the CP-asymmetry parameter a in the neutral B-meson decays to CP-self conjugate states. Our results will be useful as the parameter a is expected to be measured in the near future.

1 Introduction

A great deal of work has been done, within the framework of the Standard Model (SM), to explain CP violation in neutral Kaon decays (Donoghue *et al* 1986, Nir 1992, Peccei 1993, Buras 1997, Parodi *et al* 1998). In this model CP violation is attributed to a complex phase angle δ in the CKM matrix (Kobayashi and Maskawa 1973). Though there are different parametrizations of the CKM matrix, the unitarity of the matrix allows us to construct a universal quantity J which is two times the area of a unitarity triangle (Jarlskog 1989) and, of course, the areas of all six unitarity triangles are equal. In the Wolfenstein parametrization (Wolfenstein 1988), where the matrix is expressed in terms of four parameters, namely, A , λ , σ and δ , the quantity J is given by

$$J = A^2 \lambda^6 \sigma \sin \delta = \lambda |V_{cb}| |V_{ub}| \sin \delta, \quad (1.1)$$

where

$$s_{12} \equiv \lambda, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta} \equiv A\lambda^3\sigma e^{-i\delta} = A\lambda^3(\rho - i\eta).$$

Out of the nine elements of the CKM matrix, the moduli of the elements of the first two rows except $|V_{ub}|$ are known experimentally with reasonable accuracy (PDG 1996, Buras 1997, Parodi *et al* 1998). The quantity $|V_{ub}|$ has a large uncertainty. The moduli of the three elements of the last row may be obtained by using the unitarity relation of the CKM matrix. The ranges of possible values for the moduli of all elements are given in PDG 1996. However, the numbers in the third row and the third column are required to be readjusted to some extent in the light of the recent information on the experimental values of $|V_{cb}|$ (Buras 1997, Parodi *et al* 1998).

In our work, by using the moduli of the elements of the CKM matrix in the Källén function (Jarlskog 1989), the areas of six unitarity triangles have been evaluated. Then comparing them with the expression for J (Eq. 1.1), estimates for $|\sin \delta|$ have been made. Here we notice that in order to get δ from Eq. (1.1) we need the values of λ , $|V_{cb}|$ and $|V_{ub}|$ as inputs. Although λ and $|V_{cb}|$ are well known experimentally, $|V_{ub}|$ is not. Nevertheless, the value of $|\sin \delta|$ is found to be almost independent of $|V_{ub}|$

provided the same value of $|V_{ub}|$ is employed in the numerical computations of J , by using the Källén function, as well as in Eq. (1.1) which is needed to obtain $|\sin\delta|$ from J . This statement is true about the four unitarity triangles which involve $|V_{ub}|$. For the remaining two unitarity triangles, the average value of $|V_{ub}|$ is used in order to compute $|\sin\delta|$ from Eq. (1.1). Thus, in the PDG parametrization of the CKM matrix, the estimated value of $|\sin\delta|$, obtained from the unitarity triangles and experimental information about the matrix elements in the first two rows, is $0.899 \leq |\sin\delta| \leq 0.970$ and the corresponding values for δ with $0 \leq \delta \leq \pi$ are $\delta = \delta_1 = 70^\circ \pm 6^\circ$ and $\delta = \pi - \delta_1$.

The admissible values of δ can also be obtained from other sources. Let us first consider the CP violation parameter $|\varepsilon_K|$ for $K^0 - \bar{K}^0$ system which is rather well known empirically (PDG 1996). Using the box diagrams for $K^0 - \bar{K}^0$ mixing in the SM, we can obtain a formula for $|\varepsilon_K|$ involving $\sigma \sin\delta$ (η) and $\sigma \cos\delta$ (ρ) in addition to other parameters which are reasonably well known (Inami and Lim 1981, Gilman and Wise 1983, Buras *et al* 1984, Herrlich and Nierste 1996). Now, by using the experimental value of $|\varepsilon_K|$, bounds on the values of ρ and η can be obtained.

The neutral B-meson mixing parameter x_d is now known experimentally with sufficient accuracy (PDG 1996, Buras 1997). From box diagrams of $B_d^0 - \bar{B}_d^0$ mixing a theoretical expression for x_d can be found (Inami and Lim 1981, Buras *et al* 1984). This is proportional to $|V_{td}|^2$ which depends on ρ and η as well as on A and λ . By using the current values of the quantities which are known empirically and theoretical estimates for the remaining parameters (Buras *et al* 1990, Parodi *et al* 1998), another constraint can be imposed on the possible values of ρ and η , and therefore, on δ and σ .

Finally, there is an empirical bound on the admissible values of σ and this is,

$$0.268 \leq \sigma \leq 0.456, \quad (1.2)$$

corresponding to (PDG 1996, Buras 1997, Parodi *et al* 1998)

$$0.06 \leq \frac{|V_{ub}|}{|V_{cb}|} \leq 0.10. \quad (1.3)$$

Implications of all the constraints coming from three different sources are shown graphically in Fig. (3.1). The shaded region in the diagram indicates the possible values of (ρ, η) or (δ, σ) . We observe that both values of δ , namely δ_1 and δ_2 , obtained from the unitarity triangles of the CKM matrix are compatible with the range of values of δ shown in Fig. (3.1). However, figure (3.2) seems to indicate that the probable value of δ is approximately 62° which favours the solution $\delta_1 = 70^\circ \pm 6^\circ$ obtained earlier.

The admissible values of ρ and η shown in Figure (3.1) allow us to predict the asymmetry in the decays of B^0 and $\bar{B}^0$ to a CP-self conjugate final state provided one weak amplitude dominates the decays. Theoretical expectations from our analysis are shown in Fig. (4.2). It remains to be seen whether these predictions will be borne out, should CP violation in B systems be observed and measured in future.

2 The universal quantity J and $|\sin \delta|$

The unitarity of the CKM matrix implies that there are six triangles of equal area (Jarlskog 1989) in a complex plane if the phase angle δ is different from 0 or π . These triangles are geometrical representations of the orthogonality of any two rows or any two columns of the matrix. The area Δ of each unitarity triangle and hence J ($=2\Delta$) can be evaluated by using the moduli of the CKM matrix elements and the Källén function. Then using Eq.(1.1), that is, $|\sin \delta| = J/\lambda|V_{cb}| |V_{ub}|$, the value of δ can be determined.

To proceed, first we consider the two of the unitarity triangles which involve $|V_{ub}|$ but not $|V_{td}|$. These are the sb and uc triangles. Of the CKM matrix elements in these two triangles, the moduli of all the elements are fairly well known except $|V_{ub}|$ which has

a large range, namely, $0.0024 \leq |V_{ub}| \leq 0.0040$. This has been obtained from Eq. (1.3) and the experimental value of $|V_{cb}|$ (Parodi *et al* 1998) which is $(40.0 \pm 1.6) \times 10^{-3}$. Therefore, for both sb and uc triangles $|\sin \delta|$ has been evaluated twice corresponding to the lower and upper values of $|V_{ub}|$ and the average values for the other matrix elements which we take as:

$$|V_{ud}| = 0.9751, \quad |V_{us}| = \lambda = 0.2215, \quad |V_{cd}| = 0.2213, \quad |V_{cs}| = 0.9743, \\ |V_{ts}| = 0.0392 \text{ and } |V_{tb}| = 0.9992.$$

For the uc triangle we have obtained $|\sin \delta| = 0.957$ with $|V_{ub}| = 0.0024$ and $|\sin \delta| = 0.968$ with $|V_{ub}| = 0.0040$. Assuming that $0 < \delta < \pi$, the values of δ are, respectively, $\delta = \delta_1 = 73.1^\circ$ or $\delta = \delta_2 = \pi - \delta_1$, and $\delta = \delta_1 = 75.5^\circ$ or $\delta = \delta_2 = \pi - \delta_1$. The values of J are 2.035×10^{-5} and 3.431×10^{-5} for $|V_{ub}| = 0.0024$ and $|V_{ub}| = 0.0040$ respectively. Similarly, for the sb triangle, $|\sin \delta| = 0.910$ and $|\sin \delta| = 0.953$ corresponding to the lower and upper values of $|V_{ub}|$, respectively. The respective values of δ are $\delta = \delta_1 = 65.5^\circ$ and $\delta = \delta_1 = 72.3^\circ$. For each δ_1 there is of course another possible value of δ , i.e., $\delta_2 = \pi - \delta_1$. We notice that $|\sin \delta|$ is nearly independent of the variations of $|V_{ub}|$ within the range of values allowed by experiment. This result is expected since, according to Eq. (1.1), J is proportional to $|V_{ub}|$ and the same value of $|V_{ub}|$ is used in the evaluation of the Källén function as well as in Eq. (1.1). Furthermore, this result indicates that the values of the experimentally measured elements of the CKM matrix are consistent with unitarity. Thus it is possible to ascertain a value of δ which is not affected so much by the large uncertainty in $|V_{ub}|$.

Before calculating J for the remaining four triangles all of which involve $|V_{td}|$, we have tried to minimize the uncertainty in the value of this element. The modulus of V_{td} as given in PDG (1996) is

$$|V_{td}| = 0.004 \text{ to } 0.014, \quad (2.1)$$

and the more recent result according to Buras (1997) is

$$|V_{td}| = 0.0069 \text{ to } 0.0113. \quad (2.1a)$$

In the Wolfenstein parametrization, retaining terms up to λ^5 , V_{td} can be written as

$$V_{td} = A\lambda^3 \left\{ 1 - \left(1 - \frac{\lambda^2}{2}\right) \sigma e^{i\delta} \right\} = \lambda |V_{cb}| \left\{ 1 - \left(1 - \frac{\lambda^2}{2}\right) \sigma e^{i\delta} \right\}. \quad (2.2)$$

Using this equation we get a bound on $|V_{td}|$:

$$|V_{td}| = 0.0084 \text{ to } 0.0108, \quad (2.3)$$

where we have used the average experimental values of λ , $|V_{cb}|$, and have allowed σ to range over its admissible empirical values as given in Eq. (1.2). For δ we have used the average values of δ_1 as well as δ_2 obtained from the sb and the uc triangles. This bound on $|V_{td}|$ has a somewhat smaller spread than that in Eqs. (2.1), (2.1a) and is of course within the range of values suggested in these equations.

Next, in order to examine whether Eq. (2.3) is consistent with the unitarity of the CKM matrix, consider the db and ut unitarity triangles both of which involve $|V_{td}|$ and $|V_{ub}|$ apart from other elements. To find $|\sin \delta|$ from these two triangles we proceed similarly as we did in cases of sb and uc triangles, that is, for each triangle we find four values of δ corresponding to the four different combinations of the lower and the upper values of $|V_{ub}|$ and $|V_{td}|$. For the db triangle the extreme values of δ are: $\delta_1 = 64.0^\circ$ and $\delta_1 = 76.0^\circ$. Similarly for the ut triangle, the extreme values are $\delta_1 = 67.3^\circ$ and $\delta_1 = 76.9^\circ$. For each δ_1 there is of course the supplementary angle $\pi - \delta_1$. For these two triangles the upper bound on $|V_{td}|$ has to be adjusted to the value 0.010 so that all four calculations for each triangle give consistent results for δ_1 . Thus we have

$$|V_{td}| = 0.0084 \text{ to } 0.0100. \quad (2.3a)$$

Here we find that the central values of δ obtained from the db and the ut unitarity triangles are close to each other and are nearly the same as those obtained earlier from the sb and the uc triangles.

Finally, we consider the remaining two triangles which involve $|V_{td}|$ but not $|V_{ub}|$. These are the ds and ct triangles. Here a real value of J does not exist for all allowed values of $|V_{td}|$ as given in Eq. (2.3a). Therefore we use values of $|V_{td}|$ within its admissible range so that δ_1 obtained from these two triangles are comparable with δ_1 obtained from the previous four triangles. More explicitly, we choose $|V_{td}| = 0.0100$ for the ds triangle and $|V_{td}| = 0.0086$ for the ct triangle. Although for these two triangles we do not need $|V_{ub}|$ to evaluate the Källén function, we need $|V_{ub}|$ to evaluate $|\sin \delta|$ in the second step by using Eq. (1.1). We find that all admissible values of $|V_{ub}|$ do not yield a real δ . So we use the mid-value of $|V_{ub}|$ to calculate $|\sin \delta|$. Thus we obtain $\delta = \delta_1 = 70.0^\circ$ from the ds triangle and, $\delta = \delta_1 = 70.8^\circ$ from the ct triangle. For each δ_1 , $\pi - \delta_1$ is also a possible value of δ . Our calculations with the ds and ct triangles indicate that the unitarity of the CKM matrix requires better values for $|V_{ub}|$ and $|V_{td}|$ with narrower ranges of uncertainties.

Combining all these results obtained from the six unitarity triangles we get in the PDG representation (PDG 1996) the following values of $|\sin \delta|$ and δ :

$$0.899 \leq |\sin \delta| \leq 0.970, \quad (2.4)$$

$$\delta_1 = (70 \pm 6)^\circ, \quad \delta_2 = \pi - \delta_1. \quad (2.5)$$

We note here that in our calculations with the six unitarity triangles we have found J to vary within the range

$$1.91 \times 10^{-5} \leq J \leq 3.45 \times 10^{-5}. \quad (2.6)$$

3 Estimates of δ from other sources

In the previous section the values of the phase angle δ are obtained by using the unitarity of the CKM matrix and the currently available experimental information on the moduli of the elements of the matrix. However, possible values of ρ ($\sigma \cos \delta$) and η ($\sigma \sin \delta$) can also be ascertained by considering simultaneously the empirical bounds on σ , the experimental values of the CP violation parameter $|\epsilon_K|$ in the neutral Kaon decays, and the mixing parameter x_d in the $B_d^0 - \bar{B}_d^0$ system. Now we consider these cases in some details.

3.1 Experimental bounds on σ

Among the four parameters A , λ , σ and δ of the CKM matrix in the Wolfenstein representation, A and λ are known experimentally with sufficient accuracy. These are

$$\lambda = 0.2215 \pm 0.0025, \quad (3.1.1)$$

$$A = 0.815 \pm 0.050. \quad (3.1.2)$$

But there is a large uncertainty in the value of σ . The current information on the value of σ is obtained from the ratio of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$. In the Wolfenstein parametrization, retaining terms up to λ^6 , this ratio can be expressed as

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{|A\lambda^3 \sigma e^{-i\delta}|}{|A\lambda^2|} = \lambda \sigma, \quad (3.1.3)$$

and the accepted experimental value of this ratio is 0.08 ± 0.02 as quoted in Eq.(1.3). Hence combining the Eqs. (3.1.1), (3.1.3) and (1.3), the bound on σ is

$$\sigma = 0.362 \pm 0.094, \quad (3.1.4)$$

i.e.,

$$\bar{\sigma} = 0.353 \pm 0.092, \quad (3.1.5)$$

where $\bar{\sigma} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$, with $\bar{\rho} = (1 - \frac{\lambda^2}{2})\rho$ and $\bar{\eta} = (1 - \frac{\lambda^2}{2})\eta$. Eq. (3.1.5) can be represented by two circles σ_1 and σ_2 in the $\bar{\rho} - \bar{\eta}$ plane with the origin at (0, 0). This has been shown in Fig. 3.1.

3.2 CP violation parameter $|\varepsilon_K|$ for the $K^0 - \bar{K}^0$ system

The CP violation parameter $|\varepsilon_K|$ for the neutral Kaon complex is rather well known and the most recent result is (PDG 1996)

$$|\tilde{\varepsilon}| \equiv |\varepsilon_K| = |\eta_{+-}| = (2.285 \pm 0.019) \times 10^{-3}, \quad (3.2.1)$$

where

$$|\eta_{+-}| = \frac{|\text{Amp}(K_L \rightarrow \pi^+ \pi^-)|}{|\text{Amp}(K_S \rightarrow \pi^+ \pi^-)|}.$$

Now, the theoretical expression for $|\varepsilon_K|$ in the SM can be derived from the box diagrams of $K^0 - \bar{K}^0$ mixing and is given by (Inami and Lim 1981, Buras et al 1984, Peccei 1993)

$$\begin{aligned} |\varepsilon_K| = & \frac{G_F^2 f_K^2 m_K}{6\sqrt{2}\pi^2 \Delta m_K} B_K A^2 \lambda^6 \bar{\eta} \left[m_c^2 \left\{ -\eta_1 (1 - \lambda^2/2) + \eta_3 \left(\ln(m_t^2/m_c^2) + f_3(y_t) \right) \right\} + \right. \\ & \left. m_t^2 \eta_2 f_2(y_t) A^2 \lambda^4 \left\{ (1 - \bar{\rho}) - (\bar{\rho}^2 + \bar{\eta}^2) \lambda^2 \right\} \right], \end{aligned} \quad (3.2.2)$$

where the coefficients η_i are QCD short distance corrections to the box diagrams. Including the next-to-leading order corrections these are given by (Herrlich and Nierste 1996, Buras 1997)

$$\eta_1 \cong 1.38 \pm 0.20; \quad \eta_2 \cong 0.57 \pm 0.01; \quad \eta_3 \cong 0.47 \pm 0.04. \quad (3.2.3)$$

The kinematical functions $f_2(y_t)$ and $f_3(y_t)$ are (Inami and Lim 1981, Peccei 1993)

$$f_2(y_t) = 1 - \frac{3y_t(1+y_t)}{4(1-y_t)^2} \left[1 + \frac{2y_t}{1-y_t^2} \ln y_t \right], \quad (3.2.4)$$

$$f_3(y_t) = -\frac{3y_t}{4(1-y_t)} \left[1 + \frac{y_t}{1-y_t} \ln y_t \right], \quad (3.2.5)$$

where $y_t = \frac{m_t^2}{m_W^2}$. With $m_t = 168$ GeV and $m_W = 80.33$ GeV, one obtains

$$f_2(y_t) = 0.554, \quad f_3(y_t) = -0.888. \quad (3.2.6)$$

The values of A and λ are given in Eqs. (3.1.1) and (3.1.2), and the current experimental value of Δm_K is

$$\Delta m_K = (3.491 \pm 0.009) \times 10^{-15} \text{ GeV}, \quad (3.2.7)$$

while B_K is estimated to be (Parodi *et al* 1998)

$$B_K = 0.90 \pm 0.09.$$

The value of the Kaon decay constant f_K is taken as $f_K \cong 160$ MeV. Substituting the values of all parameters except ρ and η in Eq. (3.2.2), we obtain

$$1 = 0.876_{-0.310}^{+0.408} \bar{\eta} \left[1 + 3.01_{-0.73}^{+1.11} \left\{ 1 - \bar{\rho} - (\bar{\rho}^2 + \bar{\eta}^2) \lambda^2 \right\} \right] \quad (3.2.8)$$

This equation represents two curves in the $\bar{\rho} - \bar{\eta}$ plane, labelled ε_1 and ε_2 in Fig. (3.1).

3.3 The $B_d^0 - \bar{B}_d^0$ mixing parameter x_d

There is another parameter, the $B_d^0 - \bar{B}_d^0$ mixing parameter x_d , which is known empirically and that imposes some constraints on the values of ρ and η . The parameter x_d is defined as

$$x_d = \Delta m_{B_d} \tau_{B_d}, \quad (3.3.1)$$

where Δm_{B_d} and τ_{B_d} are the mass difference and average life time of the two mass eigenstates B_H and B_L . The current experimental value of Δm_{B_d} as quoted by Parodi *et al* (1998) is

$$\Delta m_{B_d} = (0.472 \pm 0.018) \text{ps}^{-1}. \quad (3.3.2)$$

The parameter Δm_{B_d} can also be evaluated from the box diagrams of neutral B meson mixing and the explicit expression is

$$\Delta m_{B_d} = \frac{G_F^2 m_{B_d}}{6\pi^2} \left[B_{B_d} f_{B_d}^2 \eta_B \right] m_t^2 f_2(y_t) |V_{td}|^2. \quad (3.3.3)$$

In the Wolfenstein parametrization,

$$\Delta m_{B_d} = \frac{G_F^2 m_{B_d}}{6\pi^2} [B_{B_d} f_{B_d}^2 \eta_B] m_t^2 f_2(y_t) A^2 \lambda^6 [(1-\bar{\rho})^2 + \bar{\eta}^2]. \quad (3.3.4)$$

The quantities η_B and $B_{B_d} f_{B_d}^2$ are not known empirically, and the theoretical estimates (Buras *et al* 1990, Parodi *et al* 1998) are

$$\eta_B = 0.55 \pm 0.01 \quad (3.3.5)$$

and

$$f_{B_d} \sqrt{B_{B_d}} = (220 \pm 40) \text{ MeV}. \quad (3.3.6)$$

Now, using the values of all these quantities in Eq. (3.3.4), we obtain

$$[(1-\bar{\rho})^2 + \bar{\eta}^2]^{1/2} = 0.879^{+0.291}_{-0.197}. \quad (3.3.7)$$

These are shown in Fig. (3.1) as two concentric circles x_1 and x_2 in the $\bar{\rho} - \bar{\eta}$ plane with (1,0) as the centre, and with radii 0.683 and 1.170.

The shaded region in Fig. 3.1 corresponds to the admissible values of $\bar{\rho}$ and $\bar{\eta}$. This puts a bound on δ which is

$$32^\circ \leq \delta \leq 125^\circ. \quad (3.3.8)$$

However, as the shaded region includes both the maximum and minimum values of σ given in Eq. (3.1.4), the bounds on σ cannot be further improved from this analysis. The bound on δ which we have found in this section of our work is quite comparable with the results obtained recently by Buras (1997) which is $40^\circ \leq \delta \leq 135^\circ$. We note here that the range of δ is very sensitive to the input value of the factor $f_{B_d} \sqrt{B_{B_d}} \eta_B$ which is not known precisely.

Taking the central values in Eqs. (3.1.5), (3.2.8) and (3.3.7) three curves are drawn in Fig. (3.2). These curves do not intersect at a single point, but there is a small region of confluence in $\bar{\rho} - \bar{\eta}$ plane, corresponding to a value of $\delta \approx 62^\circ$ which favours the solution $\delta = \delta_1$ obtained in the previous section. Parodi *et al* (1998) have very recently considered the probability distribution for the angle δ and found that δ equals $(64 \pm 12)^\circ$.

4 CP violation in neutral B meson decays

As we have already noted, the CP violation parameters in neutral K meson decays are quite small -- of the order of 10^{-3} . In contrast to this, the CP violation effects in the decays of neutral B meson may be large, since in such decays all three generations of quarks may be directly involved. Here we will study CP violation in neutral B meson decays to CP-self conjugate final states f . The relevant asymmetry parameter can be defined as

$$a(t) = \frac{\Gamma(B^{\text{phys}}(t) \rightarrow f) - \Gamma(\bar{B}^{\text{phys}}(t) \rightarrow f)}{\Gamma(B^{\text{phys}}(t) \rightarrow f) + \Gamma(\bar{B}^{\text{phys}}(t) \rightarrow f)} \quad (4.1)$$

where $B^{\text{phys}}(t)$ and $\bar{B}^{\text{phys}}(t)$ are the states at time t which evolved from a pure B^0 and a pure $\bar{B}^0$ respectively at $t = 0$. Assuming that only one weak amplitude dominates this ratio, the asymmetry parameter can be shown to be (Peccei 1993)

$$a(t) = \mp \sin(\varphi_B + \varphi_D) \sin \Delta m_B t, \quad (4.2)$$

where the upper (lower) sign corresponds to decays to CP even (odd) eigenstate f . The quantity φ_B in Eq. (4.2) is given by

$$\varphi_B \equiv \begin{cases} -2\beta & \text{for neutral } B_d \text{ decay} \\ 0 & \text{for neutral } B_s \text{ decay} \end{cases} \quad (4.3)$$

if the box diagrams for $B^0 - \bar{B}^0$ mixing is assumed to be dominated by the top-quark, and $-\beta$ is the phase of the CKM matrix element V_{td} , that is,

$$V_{td} = |V_{td}| e^{-i\beta}. \quad (4.4)$$

The decay phase φ_D in Eq. (4.2) is defined through the following equation

$$\eta_f e^{i\varphi_D} = \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)}. \quad (4.5)$$

We have assumed that only one weak amplitude dominates, then corresponding to CP even or odd eigenstate f ,

$$\eta_f = \pm 1$$

and the angle φ_D is purely a weak CP violating phase.

From the dominant decay channels of neutral B meson and retaining only terms up to the order of λ^3 in the CKM matrix, we find

$$\varphi_D \cong \begin{cases} -2\delta & \text{for } b \rightarrow u \text{ transitions} \\ 0 & \text{for } b \rightarrow c \text{ transitions} \end{cases} \quad (4.6)$$

where the angle δ is already defined in Eqs. (1.1, 3.1.3).

The CP violation parameter in neutral B meson system, for decays to CP-self conjugate state f , is denoted by α_f which is defined as

$$\alpha_f = \mp \sin(\varphi_B + \varphi_D) \quad (4.7)$$

and the time-integrated asymmetry parameter is given by

$$a = \mp \sin(\varphi_B + \varphi_D) \frac{x}{1+x^2}, \quad x = \frac{\Delta m_B}{\Gamma_B}. \quad (4.8)$$

The asymmetry parameter can now be calculated using Eq. (4.3), (4.6) and (4.8) for both neutral B_d and B_s decays involving either $b \rightarrow u$ or $b \rightarrow c$ transitions.

It is interesting to note that the angles β and δ appearing in the asymmetry parameters of the various neutral B decays to CP conjugate states, are just two of the angles of the db unitarity triangle. This triangle with sides rescaled by $A\lambda^3$ is shown in Fig. (4.1). We have neglected the terms $O(\lambda^7)$ in the parametrization of the sides of the triangle.

Now, from Fig. (3.1) we can obtain the admissible values of δ and sets of allowed values for β corresponding to individual values of δ . The results are displayed in Fig. (4.2). The values of the asymmetry parameters for the relevant decays of neutral B mesons can be read off from the figure.

5 Results and conclusions

In this work we have tried, first of all, to find the value of the CKM phase δ using only the unitarity and the moduli of the elements of the CKM matrix. The moduli of six of the elements in the first two rows are known experimentally, and the remaining three moduli are determined from the unitarity of the matrix. The notable point is that,

in spite of the large uncertainty in the value of $|V_{ub}|$, δ can be fixed within a narrow range. In the PDG parametrization of the CKM matrix, we have obtained $\delta = \delta_1 = 70^\circ \pm 6^\circ$ and $\delta = \delta_2 = \pi - \delta_1$, if δ is assumed to lie between zero and π .

Then we have proceeded to find the possible values of δ and β simultaneously from the experimental values of σ , $|\varepsilon_K|$ and x_d . Both the phase angles δ and β are needed to predict CP violation in the $B^0 - \bar{B}^0$ system. The empirical values of σ , $|\varepsilon_K|$ and x_d put some constraints on the values of δ and β . The results are shown in the Fig. (3.1) where the shaded region gives the admissible values of δ and β . The allowed values of δ are $32^\circ \leq \delta \leq 125^\circ$, which include both δ_1 and δ_2 .

Having obtained the possible values of δ and β from Fig. (3.1), we have been able to predict CP violation asymmetries in the decays of neutral B-mesons to CP-self conjugate final states with the assumption that a single weak amplitude dominates these decays. The time-integrated asymmetry parameter for $B_d^0 - \bar{B}_d^0$ system in decays to CP even final states involving $b \rightarrow u$ and $b \rightarrow c$ transitions are expected to lie within the ranges -0.476 to 0.470 and 0.170 to 0.375 , respectively (Fig. 4.2). The asymmetries in decays to CP odd eigenstates are just the negative of the asymmetries mentioned above.

References:

- Buras A J, Slominsky W and Steger H 1984 *Nucl. Phys.* **B237** 529
- Buras A J, Jamin M and Weisz P H 1990 *Nucl. Phys.* **B347** 491
- Buras A J in *Proceedings of ICHEP96, 28th International Conference of High Energy Physics*, Warsaw, Poland ed Ajduk Z and Wroblewski A K (World Scientific, Singapore 1997) p 243
- Donoghue J F, Golowich E and Holstein B 1986 *Phys. Rep.* **131** 319
- Gilman F and Wise M 1983 *Phys. Rev.* **D27** 1128
- Herrlich S and Nierste U 1996 *Nucl. Phys.* **B476** 27
- Inami T and Lim C S 1981 *Prog. Theor. Phys.* **65** 1297
- Jarlskog C in *CP Violation*, ed Jarlskog C (World Scientific, Singapore 1989) p 3
- Kobayashi M and Maskawa T 1973 *Prog. Theor. Phys.* **49** 652
- Nir Y in *Perspectives in the Standard Model*, ed Ellis R K *et al* (World Scientific, Singapore 1992) p 339
- Parodi F, Roudeau P and Stocchi A , “Constraints on the parameters of the V_{CKM} matrix at the end of 1997”, hep-ph 9802289
- PDG 1996 *Phys. Rev.* **D54** 1
- Peccei R D in *Particle Physics and Cosmology at the Interface*, ed Pati J, Ghose P and Maharana J (World Scientific, Singapore 1993) p 79
- Wolfenstein L 1988 *Phys. Rev. Lett.* **51** 1945

Figure Captions.

Fig.3.1 Displayed are the constraints on $\bar{\rho}$ and $\bar{\eta}$ obtained from the experimental values of (a) σ (curves σ_1 and σ_2), (b) $|\epsilon_K|$ (curves ϵ_1 and ϵ_2), and (c) x_d (curves x_1 and x_2). The shaded region shows the allowed values of $\bar{\rho}$ and $\bar{\eta}$.

Fig.3.2 Three curves in the $\bar{\rho} - \bar{\eta}$ plane corresponding to σ , $|\epsilon_K|$ and x_d for the central values in Eqs. (3.1.5), (3.2.8) and (3.3.7).

Fig.4.1 The db unitarity triangle with sides rescaled by $A\lambda^3$.

Fig.4.2 The allowed values of $\sin 2\beta$ and $\sin(2\beta + 2\delta)$ obtained from Fig. 3.1.

Fig. 3.1

Fig. 3.2

Fig. 4.1

Fig. 4.2