A transversity bound and higher order effects*+

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Next-to-leading order (NLO) effects on Soffer's transversity bound are studied. It is shown that defining the NLO corrections to the transversity via Drell-Yan lepton-pair production, but to the unpolarized and longitudinally polarized distributions via deep inelastic scattering, results in a substantial suppression of the bound for all $q = u, d$ and sea. Moreover assuming that the bound is saturated at $Q_o^2 = 4 \text{ GeV}^2$ and evolving the NLO distributions to $Q^2 > Q_o^2$ via 2-loop split functions, it is shown that the bound is not violated.

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1 INTRODUCTION

The state of a quark q in a nucleon is specified by three distributions :

- (i) The unpolarized $F_q(x, Q)$ related with the structure function $f_1(x, Q)$ through the well-known expression.
- (ii) The longitudinally polarized or helicity $\Delta_L F_q(x, Q)$ related with $g_1(x, Q)$ of the EMC experiment through a similar expression and
- (iii) The transversely polarized or transversity $\Delta_T F_q(x, Q)$ related with a third structure function $h_1(x, Q)$.

All three of them are important for understanding the structure of the nucleon.

Under a chirality transformation, f_1 and g_1 are even, but h_1 is odd. Then, it turns out that f_1 and g_1 are of leading twist in deep inelastic scattering (DIS), and F_q and $\Delta_L F_q$ are conveniently determined in DIS. However, h_1 is of leading twist in Drell-Yan (DY) lepton pair production, and DY production by transversely polarized hadrons is the best way to determine $\Delta_T F_q$. Unfortunately on the latter, no information from

experiment is available so far [1] and this severely hampers theoretical predictions. Thus, the inequality

$$2|h_1(x, Q)| \leq f_1(x, Q) + g_1(x, Q) \quad (1)$$

proposed by Soffer [2] has been very welcome. Its status in QCD is not very clear [3], but it is generally taken to imply

$$2|\Delta_T F_q(x, Q_o)| \leq F_q(x, Q_o) + \Delta_L F_q(x, Q_o) \quad (2)$$

at some $Q_o^2 = O(1 \text{ GeV}^2)$ and for each q and $\bar{q}$ species. Subsequently we define the leading order bound :

$$\Delta_T F_{q, \max}^{(o)}(x, Q) \equiv \frac{1}{2} \left(F_q(x, Q) + \Delta_L F_q(x, Q) \right) \quad (3)$$

With this definition, two questions arise :

- (A) What is the effect of next-to-leading order corrections(NLOC) on $\Delta_T F_{q, \max}^{(o)}$?
- (B) Suppose we evolve all F_q , $\Delta_L F_q$ and $\Delta_T F_q$ from Q_o^2 to some $Q^2 > Q_o^2$. Is the bound preserved ?

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2 EFFECT OF HIGHER ORDER CORRECTIONS

At some Q^2 perturbation theory gives $([\Delta_L]F)$ stands for either F and $\Delta_L F$:

$$[\Delta_L]\bar{F}_q(x, Q) = [\Delta_L]F_q(x, Q) + \alpha_s(Q) \int_x^1 \frac{dx'}{x'} [\Delta_L]C_q\left(\frac{x}{x'}\right) [\Delta_L]F_q(x') + \alpha_s(Q) \int_x^1 \frac{dx'}{x'} [\Delta_L]C_g\left(\frac{x}{x'}\right) [\Delta_L]F_g(x') \quad (4)$$

where $[\Delta_L]C_q(x)$, $[\Delta_L]C_g(x)$ the NLOC and $[\Delta_L]F_g(x)$ the gluon distribution. For the transversities ($\Delta_T F_g = 0$):

$$\Delta_T \bar{F}_q(x) = \Delta_T F_q(x) + \alpha_s(Q) \int_x^1 \frac{dx'}{x'} \Delta_T C_q\left(\frac{x}{x'}\right) \Delta_T F_q(x') \quad (5)$$

Now, given $[\Delta_L]C_q$, $[\Delta_L]C_g$ we determine $[\Delta_L]\bar{F}_q$. We write

$$\Delta_T \bar{F}_{q,max}(x, Q) \equiv \frac{1}{2} \left\{ \bar{F}_q(x, Q) + \Delta_L \bar{F}_q(x, Q) \right\}; \quad (6)$$

this is not our NLO bound, but as it will be seen, it serves a useful purpose. Now, insert $\Delta_T \bar{F}_{q,max}(x, Q)$ in the l.s. side of Eq.(5); with the correction $\Delta_T C_q$ given this is an integral eq. for $\Delta_T F_q$. At some Q^2 , $\Delta_T \bar{F}_{q,max}$ can be parametrized by the form

$$\Delta_T \bar{F}_{q,max}(x) = \bar{C} x^{\bar{a}} (1-x)^{\bar{b}} \quad (7)$$

Then the integral eq. can be solved by Mellin transform and the convolution theorem. The resulting $\Delta_T F_q(x, Q)$ ($\equiv \Delta_T F_{q,max}(x, Q)$)

is our NLO bound to be compared with $\Delta_T F_{q,max}^{(o)}(x, Q)$.

We determined our NLOC via two different procedures :

- (a) For all $[\Delta_{L,T}]F_q$ the NLOC are determined via DY.
- (b) The NLOC for $\Delta_T F_q$ are determined via DY, but for F_q and $\Delta_L F_q$ via DIS ; this appears to be a more natural way [3].

The procedure (a) was used in [4]. In the figure (upper part) we reproduce the results at $Q_o^2 = 4 \text{ GeV}^2$ for $q = u$ quark, which is the most characteristic case. Note that all F stand for $\Delta_T F_{u,max}$. We note that $\bar{F}$ is significantly greater than the original $F^{(o)}$, but that the final F differs little from $F^{(o)}$. Clearly, the effect of HOC is not very significant.

Now we apply procedure (b). In the $\overline{MS}$ scheme the NLOC for the unpolarized distributions and the helicities are [5,6] :

$$C_q(x) = \Delta_L C_q(x) = \frac{C_F}{2\pi} \left\{ -\left(\frac{9}{2} + \frac{\pi^2}{3}\right) \delta(1-x) + x \left[(1+x^2) \left(\frac{\ln(1-x)}{(1-x)} \right)_+ - \frac{3}{2} \frac{1}{(1-x)_+} - \frac{1+x^2}{1-x} \ln x + 3 + 2x \right] \right\} \quad (8)$$

where $C_F = 4/3$ and $1/(1-x)_+$, $(\ln(1-x)/(1-x))_+$ well-known distributions. Also

$$C_g = \frac{x}{2\pi} \left\{ \frac{x^2 + (1-x)^2}{2} \ln \frac{1-x}{x} + 3x(1-x) \right\} \quad (9)$$

and

$$\Delta_L C_g = \frac{x}{2\pi} \left\{ \left(x - \frac{1}{2}\right) \ln \frac{1-x}{x} + 3(1-x) \right\} \quad (10)$$

The NLOC $\Delta_T C_q(x)$, defined via DY, are given in [4] but in dimensional reduction. However, the distributions $F_q(x)$, $\Delta_L F_q$ to be used below, the corrections (8)-(10) and the (2-loop) evolu-

tion of $\Delta_T F_q(x, Q^2)$ [7-9] are determined in the $\overline{MS}$ (or $\overline{MS}$ -equivalent) scheme. Then by adding proper conversion terms [10] we find in $\overline{MS}$

$$\Delta_T C_q(x) = \frac{C_F}{2\pi} \left\{ \left(\frac{2\pi^2}{3} - 8 \right) \delta(1-x) + 8x^2 \left(\frac{\ln(1-x)}{1-x} \right)_+ - 4x^2 \frac{\ln x}{1-x} + 2x(1-x) \right\} \quad (11)$$

In the subsequent calculations we use (2-loop) running coupling $\alpha_s(Q)$ with $\Lambda = 0.2 \text{ GeV}$ and 4 flavours. for F_q the distributions of CTEQ4M [11] and for $\Delta_L F_q$ the set A of Ref. [12]. The renormalization and factorization scales are taken equal ($=Q^2$), and for DIS : $Q^2 = -(\text{virtual photon's } 4 - \text{momentum})^2$, whereas for DY : $Q^2 = (l^+ l^- \text{ invariant mass})^2$.

The figure (lower part) shows our results for $q = u$ again at $Q_o^2 = 4 \text{ GeV}^2$. Now $\overline{F}$ does not much differ from $F^{(o)}$, but the final F is significantly smaller. The effect of HOC is very significant. Similar are our results for $q = d$ and $q = \text{sea}$.

To explain why the procedure (b) affects the bound much more than (a), first notice that in DY, relative to LO, the NLO terms are known to introduce a *large* correction. This easily follows from Eq.(10): As least for x not too small, the NLOC are dominated by the contributions of terms involving distributions. There are three such terms : $(2\pi^2/3)\delta(1-x)$, $-8\delta(1-x)$ and $8x^2(\ln(1-x)/(1-x))_+$, and their contributions are comparable and large in size. The contribution of the third term roughly cancels that of the second, so the first remains as large NLOC. However, in DIS the NLO terms are known to introduce a *small* correction. This also follows from Eqs (8)-(10): First, $[\Delta_L]C_j$ involve no distributions and give rather small NLOC. Second, $[\Delta_L]C_q$ do involve distributions, but the

contributions of them roughly cancel.

Now, in both procedures (a),(b) the effect of NLOC is seen by first comparing $\overline{F}$ and $F^{(o)}$. In (a), due to the large DY NLOC, $\overline{F}$ and $F^{(o)}$ differ much. In (b), due to the small DIS NLOC, $\overline{F}$ and $F^{(o)}$ differ little. Then compare the final bound F with $\overline{F}$ (Eq.(5)): In both (a),(b), $\Delta_T C_q$ is determined via DY, so it gives a large contribution ; hence F is suppressed relative to $\overline{F}$, so in (b) F is significantly smaller than $F^{(o)}$, i.e. the bound is much suppressed by NLOC.

3 PRESERVATION OF THE BOUND UPON EVOLUTION

We finally turn to question (B) regarding the evolution to $Q^2 > Q_o^2$. First, in LO, evolving $\Delta_T F_q$ via 1-loop split functions, it was shown in a simple way that the bound is preserved [13]. Evolving $\Delta_T F_q$ via 2-loop split functions complicates matters, but in certain usual factorization schemes including $\overline{MS}$ and for the valence transversities, the same was shown in a numerical way [14].

Now, with procedure (b), take Eq.(5) with $\Delta_T \overline{F}_q = \Delta_T \overline{F}_{q,max}$ and solve it for $\Delta_T F_q$ first at $Q^2 = Q_o^2$ and then at some $Q_j^2 > Q_o^2$, to determine $\Delta_T F_q(x, Q_o)$ and $\Delta_T F_q(x, Q_j)$. Then evolve $\Delta_T F_q(x, Q_o)$ to $Q^2 = Q_j^2$ and compare the result with $\Delta_T F_q(x, Q_j)$. This was done for $Q_j^2 = 100$ and 10^4 GeV^2 . It was found that the evolved $\Delta_T F_q(x, Q_o)$ was smaller than $\Delta_T F_q(x, Q_j)$ for all $q = u, d$, and sea, i.e. that the presence of NLOC does not violate the bound.

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