COMMENT ON VISCOS STABILITY OF RELATIVISTIC KEPLERIAN ACCRETION DISKS

Rafal Moderski and Bożena Czerny
Nicolaus Copernicus Astronomical Center, 00-716 Warsaw, Bartycka 18, POLAND

Received ________________; accepted ________________
Recently Ghosh (1998) reported a new regime of instability in Keplerian accretion disks which is caused by relativistic effects. This instability appears in the gas pressure dominated region when all relativistic corrections to the disk structure equations are taken into account. We show that he uses the stability criterion in completely wrong way leading to inappropriate conclusions. We perform a standard stability analysis to show that no unstable region can be found when the relativistic disk is gas pressure dominated.

Subject headings: accretion, accretion disks, stability – relativity
1. Introduction

Stability analysis of Shakura-Sunyaev $\alpha$ disk (Shakura & Sunyaev 1973) was performed by many authors (Lightman & Eardley 1974; Lightman 1974; Shakura & Sunyaev 1976; Piran 1978) and yield the well known conclusions: the disk is stable for gas pressure dominated (GPD) region and becomes viscously and thermally unstable for radiation pressure dominated (RPD) region. While the general relativity generalization of the Shakura & Sunyaev model were available (Novikow & Thorne 1973) the stability studies were presented only for the Newtonian case.

Recently Ghosh (1998) reported a new regime of instability for $\alpha$ disks when general relativity effects are taken into account. He found that even in GPD disks there exist a region not far from the inner edge of the disk which is viscously unstable. This might have an enormous influence on the models of active galactic nuclei and galactic low-mass X-ray binaries.

In this paper we repeat a stability analysis studied by Lightman & Eardley (1974) but in a relativistic case. We obtain the same results as for the Newtonian case i.e., disk is viscously stable for gas pressure dominated region, but becomes viscously unstable in the region where radiation pressure dominates. Thus we do not confirm the results obtained by Ghosh (1998). We discuss an error made by Ghosh (1998) leading to wrong conclusions.

2. Disk structure

Surface density evolution is described by the equation (Ghosh 1998)

\[ \frac{A^{\frac{1}{2}} D^{\frac{1}{2}}}{\partial t} \frac{\partial \Sigma}{\partial t} = D^\frac{1}{2} \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{D^{-\frac{1}{2}}}{(\partial \ell/\partial r)} \frac{\partial}{\partial r} \left( B C^{-1} \ell r^2 W \right) \right], \]  

(1)
where \( W \) is the vertically integrated viscous stress, \( \ell \) is the specific angular momentum, and \( A, B, C, D \) are relativistic corrections (Novikov & Thorne 1973)

\[
A \equiv 1 + \frac{A^2}{\tilde{r}^2} + \frac{2A^2}{\tilde{r}^3}, \quad B \equiv 1 + \frac{A}{\tilde{r}^{3/2}}, \quad C \equiv 1 - \frac{3}{\tilde{r}} + \frac{2A}{\tilde{r}^{3/2}}, \quad D \equiv 1 - \frac{2}{\tilde{r}} + \frac{A^2}{\tilde{r}^2}. \tag{2}
\]

Here \( A \equiv cJ/GM^2 \) is the dimensionless spin of the central accreting object, and \( \tilde{r} \equiv r c^2/GM \) is the dimensionless distance.

Equations of disk structure under the assumption of thermal equilibrium yield following expressions for the relation between the surface density of the disk and the viscous stress

\[
\Sigma_{RPD} = \left( \frac{16c^2}{9\alpha \kappa_{es}} \right) W^{-1} CKD^{-2}, \tag{3}
\]

\[
\Sigma_{GPD} = \left( \frac{m_p}{k\alpha} \right)^{4/5} \left( \frac{bc}{18\kappa_{es}} \right)^{1/5} \left( \frac{r^3}{GM} \right)^{1/10} W^{3/5} C^{1/5} D^{-1/5}, \tag{4}
\]

where \( K \equiv 1 - \frac{4A}{\tilde{r}^{3/2}} + \frac{2A^2}{\tilde{r}^2} \) is relativistic correction introduced by Riffert & Herold (1995); \( \Sigma_{GPD} \) and \( \Sigma_{RPD} \) are surface densities in the gas pressure and radiation pressure dominated regimes respectively.

3. Viscous stability analysis

Newtonian version of Equations (1) and (3) was used by Lightman & Eardley (1974; see also Lightman 1974) to study the stability of the Keplerian disk. The assumption of thermal equilibrium was proved to be justified by Shakura & Sunyaev (1976) because in the limit of wavelengths of the radial perturbation much longer than the disk thickness the viscous instability is clearly decoupled from the thermal instability. As was correctly noticed by Ghosh (1998), the basic stability criterion is the sign of the partial derivative, \( \partial W/\partial \Sigma \), and remains the same for both relativistic and non relativistic disks.

The point is that in order to determine the stability of the disk we should compute this partial derivative at a given radius directly from Equations (3) and (4). It corresponds
to the linear perturbation of the surface density distribution in Equation (1) under the assumption of thermal equilibrium leading to known relation \( W(\Sigma) \) at a given radius. In that case the perturbation \( \delta W \) of \( W \) is expressed through

\[
\delta W = (\partial W/\partial \Sigma)_r \delta \Sigma,
\]

and in the next step \( \delta \Sigma \) is decomposed into linear waves according to standard local analysis. Spatial derivatives of unperturbed stress and surface density are neglected in this process.

Thus for radiation dominated disk we have

\[
W \propto (\alpha \Sigma)^{-1} \mathcal{K} \mathcal{D}^{-2}
\]

From (6) we can calculate viscous stress derivative

\[
(\partial W/\partial \Sigma)_{RPD} \propto -\alpha^{-1} \Sigma^{-2} \mathcal{K} \mathcal{D}^{-2} < 0
\]

which immediately shows that the disk is viscously unstable in the whole radiation pressure dominated region. This is a relativistic generalization of the well known result obtained first by Lightman & Eardley (1974) exactly in the same way.

For gas pressure dominated disk from (4) we have

\[
(\partial W/\partial \Sigma)_{GPD} \propto \Sigma^{2/3} r^{-11/6} \mathcal{C}^{-1/3} \mathcal{D}^{1/3} > 0
\]

which shows that gas pressure dominated disk is stable. No evidence is found for any unstable region, in opposite to the claim of Ghosh (1998).

The conclusion of Ghosh (1998) was based on the computation of entirely different partial derivative, namely \( (\partial W/\partial \Sigma)_M \), or equivalently, the expression \( (\partial W/\partial r)/(\partial \Sigma/\partial r) \) which has no straightforward relation to \( (\partial W/\partial \Sigma)_r \).
To show this clearly we first replot the Figure 1 of Ghosh (1998) showing the radial dependence of the stress and the surface density in a stationary solution for a single value of accretion rate. The loop-like character of this plot simply reflects the fact that the stress and the surface density are zero at the marginally stable orbit (upper branch) and they again tend to zero for radius approaching infinity (lower branch). In this Figure the region where \( \frac{\partial W}{\partial r} / \left( \frac{\partial \Sigma}{\partial r} \right) < 0 \) corresponds to radii between \( \tilde{r} \simeq 14 \) and \( \tilde{r} \simeq 22 \).

Since Ghosh (1998) claimed that the negative part of the slope indicate the unstable region we show the correct stability plot for the two radii from this range. A single such plot, as explained above, show the dependence of the stress vs. surface density for a fixed radius but variable accretion rate. Upper and lower part of a curve is well approximated by Equation (3) and Equation (4), correspondingly, and more general approach to gas/radiation pressure contribution matches smoothly those two asymptotic solutions giving a single curve. The slop of this curve determines the disk viscous stability. The accretion rate adopted in Figure 1 is marked in Figure 2 with crosses. It is well within a stable region, as claimed in all papers prior to Ghosh (1998).

To confirm our analysis we perform numerical simulations of surface density evolution according to Equation (1) for both radiation pressure and gas pressure dominated disks. The results are presented on Figure 3 and 4 respectively. We can see that for radiation pressure dominated disk the initial small perturbation grows leading to total disruption of the disk, while for gas pressure dominated disk initial perturbation is smoothed out to stationary configuration.
4. Conclusions

We study the viscous stability of relativistic, Keplerian accretion disk in the thermal equilibrium. We confirm the results previously obtained in the Newtonian case i.e., the disk is stable in gas pressure dominated regime and becomes viscously unstable in radiation pressure dominated regime. We show that instability recently reported by Ghosh (1998) was caused by improper use of the stability criterion for accretion disk, and does not appear if the correct approach is applied. We confirm our results by performing numerical simulations of surface density evolution which completely agree with our analytical analysis.

This work was supported by Polish KBN grants 2P03D00410 and 2P03D00415. RM also acknowledge support from the Foundation for Polish Science.
REFERENCES


This manuscript was prepared with the AAS \LaTeX\ macros v4.0.
FIGURE CAPTIONS

Fig. 1.— Integrated viscous stress $W$ versus surface density $\Sigma$ obtained in the same way like Ghosh (1998). The accretion rate is fixed ($\dot{m} = 0.01$) and the quantity used to parameterize the plot along the curve is the distance from the central object. This relation shows the “unstable region” between $\tilde{r} \sim 14$ (maximum of $W$) and $\tilde{r} \sim 22$ (maximum of $\Sigma$).

Fig. 2.— Proper relation between integrated viscous stress and surface density usually used for stability analysis. This time, for a given curve, the radius is fixed and accretion rate is used to parameterize the plot. Two curves are shown: for $\tilde{r} = 15$ and $\tilde{r} = 21$. Accretion rate from the Fig. 1 is marked by big crosses. There is no evidence for instability in these points.

Fig. 3.— Evolution of the surface density of the radiation pressure dominated disk. Small initial perturbation grows and finally disrupt the disk.

Fig. 4.— Same as Figure 3 but for gas pressure dominated disk. Initial perturbation is placed at $\tilde{r} = 18$ (inside the “unstable region” postulated by Ghosh (1998)). Perturbation is smoothed out showing no evidence for instability.