Hot Gas in Clusters of Galaxies: the Punctuated Equilibria Model

A. Cavaliere
Astrofisica, Dipartimento Fisica, II Università di Roma,
via Ricerca Scientifica 1, 00133 Roma, Italy

N. Menci
Osservatorio Astronomico di Roma, via Osservatorio, 00040 Monteporzio, Italy

and

P. Tozzi
Astrofisica, Dipartimento Fisica, II Università di Roma,
via Ricerca Scientifica 1, 00133 Roma, Italy

Present address: Department of Physics and Astronomy, The Johns Hopkins University,
Baltimore, MD 21218, USA.

ABSTRACT

We develop our model of “punctuated equilibria” for the hot intra-cluster gas emitting powerful X-rays. The model considers the gravitational potential wells set by the dark matter as they evolve by hierarchical clustering and engulf outer gas; it assumes that the gas re-adjusts to a new hydrostatic equilibrium after each merging event. Before merging the gas is heated at the virial temperature when bound in subclusters; at early $z$ it is preheated by supernova activity following star formation.

In detail, we compute analytically the following steps: the dynamic histories of dark matter halos with their merging events; the associated infall of gas into a halo, with compressions and shocks establishing the conditions at the cluster boundary; the updated disposition of the gas in the potential well matching such conditions; the statistical convolution of observable quantities over the merging histories.

For the individual objects from groups to clusters, the model yields profiles of density and surface brightness with no free parameters; in particular, the so-called $\beta$ parameter is itself an outcome of the model, and the polytropic index $\gamma$ is internally constrained to a narrow range. We obtain declining temperature profiles, and profiles for the density and for the surface brightness shallower in groups compared with clusters; our model groups also contain a lower baryonic fraction on average, but with a scatter considerably larger.

We present various key quantities over the whole range from groups to clusters. In particular, we predict in different cosmologies the statistical correlation $L - T$ of luminosity with temperature; similarly, we derive the correlation $R_X - T$ for the size of the X-ray emitting region. The intrinsic scatter in both correlations is also predicted.

key words: galaxies: clusters: general – intergalactic medium – X-rays: galaxies
1. INTRODUCTION

Groups and clusters of galaxies constitute large, nearly virialized condensations. Within their virial radii $R$ ranging from $1/2$ to about $2$ Mpc the density contrasts attain or exceed $\delta \rho / \rho_u \sim 2 \times 10^2$ relative to the background, and the corresponding masses $M$ range from some $10^{13}$ to $10^{15} M_\odot$, mainly in dark matter (DM).

These structures contain a large baryonic fraction $f \simeq 0.1$ in the form of a hot intra-cluster plasma (ICP) at temperatures $kT \sim GM m_H/10R \sim 0.5 - 10$ keV ($m_H$ is the proton mass) with particle densities up to $n \sim 10^{-3}$ cm$^{-3}$. The ICP emits by thin thermal bremsstrahlung X-ray luminosities $L \propto n^2 T^{1/2} R^3$ ranging from $10^{42}$ to $10^{45}$ erg/s.

Recent observations of individual objects resolved in energy and angle with the ASCA and with the SAX satellites indicate radial temperature profiles declining outwards (Markevitch, Sarazin & Henriksen 1997; Molendi, private communication; see also Hughes, Gorenstein & Fabricant 1988). In several cases the thermal structure is complicated by hot spots (Honda et al. 1997; Markevitch et al. 1998). As for the statistical aspects, a steep correlation close to the overall form $L \propto T^3$ is known to hold for local clusters, but with a substantial scatter (Edge & Stewart 1991; Mushotzky 1994). Recently the observations have sampled higher redshifts out to $z \sim 0.5$ (Tsuru et al. 1996; Mushotzky & Scharf 1997) finding little significant evolution. Low-temperature, local systems have been also sampled, finding there indications of a slope steeper yet (Ponman et al. 1996). For $kT > 5$ keV a flattening toward $L \propto T^2$ has been detected by Allen & Fabian (1997).

In the near future the AXAF mission will substantially improve the space-resolved spectroscopy of many individual clusters, and shortly after the mission XMM will greatly enlarge the statistics of the $L - T$ correlation. Corresponding upgrades are called for in the theoretical understanding of the complex astrophysics concerning both the DM and the ICP over the full range from groups to clusters.

The force approach uses numerical computations for both the DM and the ICP, striving for wide dynamic range and complete hydrodynamics. The N-body simulations have shown, in accord with the hierarchical clustering picture (see Peebles 1993), the evolution of the DM halos to occur largely through a sequence of merging and accretion events which involve generally smaller partners down to nearly diffuse matter (see, e.g., Tormen, Bouchet & White 1997), and are correlated with the surrounding large scale structures (Colberg et al. 1998).

As for the ICP, pioneering work by Schindler & Müller (1993) taken up by Roettiger, Stone & Mushotzky (1998) used 3D Eulerian codes with adaptive mesh and advanced shock-capturing methods to study how the large merging events of the DM halos affect the ICP component. The outcomes show how such events produce anisotropic shocks and non-uniform compressions, resulting in a complex thermal structure lasting a few Gyrs.

At the other extreme, a sequence of radial, highly resolved Eulerian computations (progressing
from Perrenod 1980 to Takizawa & Mineshige 1998a) have shown that isotropic accretion of smooth gas also causes a strong and slowly expanding shock, which remains close to the (growing) virial radius for some dynamical times.

Most recently, state-of-the-art N-body codes coupled with advanced hydro (Bryan & Norman 1998, Gheller et al. 1998) have been run on supercomputers, aiming at resolutions below 100 kpc in rich clusters as necessary for deriving reliable luminosities. But such simulations are hard-pressed in implementing at the same time the full physics of the ICP. In fact, preheating at temperatures $\sim 0.5$ keV is expected from stellar formation and evolution to supernovae (see Renzini 1997, and references therein); this is particularly relevant for the shallower potentials of groups where $T$ is close to 0.5 keV. Inclusion of the preheating in the numerical work is technically taxing, as it involves cooling, star formation and energy feedbacks resolved down to subgalactic scales; but in its absence the simulations produce a correlation of the form $L \propto T^2$ at all temperatures, at variance with the data. Suginohara & Ostriker (1998) stress how delicate may become at high resolutions the balance of cooling and feedbacks, and how difficult becomes reproducing ICP cores as observed. Thus for now and for some time to come it will be hard to combine into a realistic numerical picture wide dynamic range from galaxies to large scale structures, stellar preheating and large statistics.

The state of the numerical approach and the challenge from the data motivates us to present here an analytic model which includes, though in a simplified form, the physical processes outlined above. We describe the cluster history as a sequence of punctuated equilibria (PE). That is to say, we envisage such history as a sequence of hierarchical merging episodes of the DM halos which we compute analytically (with its variance) in the framework of the standard hierarchical clustering, specifically using the so-called “extended Press & Schechter theory” (Bond et al. 1991; Bower 1991; Lacey & Cole 1993). We stress that such episodes cause in the gas shocks of various strengths depending on the mass ratio of the merging subclusters, ranging from nearly adiabatic compressions for comparable clumps up to shocks with high Mach numbers in the accretion of loose gas. Our point is that the most effective such shocks and compressions overlap to provide the boundary conditions for the new hydrostatic equilibrium to which the ICP is assumed to re-adjust.

The PE model as presented here takes up our previous work (Cavaliere, Menci & Tozzi 1997, 1998; hereafter CMT97, CMT98), but differs in that the histories of the DM halos are now computed analytically rather than based on Monte Carlo simulations. This goes beyond the technical aspect since it allows us to explore efficiently the dependences of the density, temperature and luminosity on the parameters of the clusters and on the cosmology. In the same vein, the present approach allows us to quantify the connection between slope and scatter of the $L - T$ correlation and the cosmological scenario. In addition, the parameters of the ICP thermal state are now fixed or bounded in terms of constraints internal to the model, and new predictions are presented.

In Sect. 2 we describe the PE model and our computational steps: in Sect. 2.1 we recall the
statistical formalism for hierarchically merging DM halos; in Sect. 2.2, we derive from the mass ratios involved in each merging episode the boundary conditions for the plasma equilibrium; in Sect. 2.3 we compute from such boundary conditions the ICP equilibrium; in Sect. 2.4 we derive the statistics of $L$ and of the size $R_X$ using the formalism of Sect. 2.1. In Sect. 3 we give, and compare with the observations, the model results for the profiles $n(r)$ and $T(r)$, for the surface brightness $\Sigma(r)$, for the relations $M - T$ and $f - T$, and for the correlations $R_X - T$ and $L - T$. The final Sect. 4 is devoted to discussion and conclusions.

2. THE PUNCTUATED EQUILIBRIA MODEL

The X-ray bolometric luminosity of a cluster is given in its basic dependences by

$$L \propto \int_0^{r_2} n^2(r) T^{1/2}(r) d^3r.$$  \(1\)

Here $T(r)$ is temperature in the plasma and $r_2$ is the cluster boundary, that we take to be close to the virial radius $R \propto M^{1/3} \rho^{-1/3}$, where $\rho(z) \propto (1 + z)^3$ is the DM density in the cluster, proportional to the average cosmic DM density $\rho_u(z)$ at formation.

It will be convenient to separate the internal profiles $n(r)$ and $T(r)$ from their boundary conditions at $r_2$. As to the latter, the infalling gas is expected to become supersonic near $r_2$ (see, e.g., Perrenod 1980; Takizawa & Mineshige 1997) so that a shock front will form there. The conservations across the shock of mass, energy and stresses yield the Rankine-Hugoniot conditions, i.e., the temperature and density jumps from the outer values $T_1$ and $n_1$ to $T_2$ and $n_2$ just interior to $r_2$ (spelled out in Sect. 2.2). Then the luminosity may be rewritten in the form

$$L \propto r_2^3 n_2^2 T_2^{1/2} \int_0^1 d^3x \left[ \frac{n(x)}{n_2} \right]^2 \left[ \frac{T(x)}{T_2} \right]^{1/2},$$  \(2\)

where $x \equiv r/r_2$.

Note that the values $n_2$ and $T_2$ at the boundary are not uniquely determined by the cluster mass $M$; rather, they are related to the outer values $n_1$ and $T_1$ by the named shock conditions. In turn, $n_1$ is fixed by $n_1 \propto f_u \rho_u/m_H$, in terms of the universal baryonic fraction $f_u$; whereas $T_1$ is determined only statistically, through the diverse merging histories ending up in the mass $M$. Specifically, as explained in detail in Sects. 2.1 and 2.2, in each merging episode $T_1$ takes on the values appropriate to the other merging partner, constituted by subclumps or even by smooth gas. In sum, a given dark mass $M$ admits a set of ICP equilibrium states characterized by different boundary conditions, each corresponding to a different realization of the dynamical merging history. It is the convolution over such set which provides the average values of $L$ and $R_X$, and their scatter.

So the development of our PE model proceeds along the following steps: i) we first give the statistics of the current DM halo of mass $M$ and of the merging clumps $\Delta M$;
ii) we compute the shock strength relating at the boundary the inner values \( T_2 \) and \( n_2 \) to the exterior ones \( T_1 \) and \( n_1 \), as a function of \( M \), and \( \Delta M \);  
iii) from such boundary values, we compute the internal profiles \( T(x)/T_2 \) and \( n(x)/n_2 \) for the post-merging hydrostatic equilibrium, involving the cluster potential and hence \( M \);  
iv) we convolve the results of steps ii) and iii) with the statistics i).

Below we describe these steps in turn.

### 2.1. Histories of the Dark Matter Halos

Here we recall the basic merging probabilities provided by the “extended Press & Schechter theory”, see Lacey & Cole (1993; 1994). This is based on the dark halos formed by hierarchical merging of smaller structures.

The halo mass distribution at the cosmic time \( t \) is given by the standard Press & Schechter (1974) formula

\[
N(m, t) = \sqrt{\frac{2}{\pi}} \frac{\delta_c(t) \rho_c}{M_o^2} \left| \frac{d \ln \sigma}{d \ln m} \right| \frac{m^{-2}}{\sigma(m)} \exp \left\{ -\frac{[\delta_c(t) - \delta_{co}]^2}{2 \sigma^2(m)} \right\},
\]

where the masses \( m \equiv M/M_o \) are normalized to the current value \( M_o = 0.6 \times 10^{15} \Omega_0 h^{-1} M_\odot \) (i.e., to the mass enclosed within a sphere of \( 8 h^{-1} \) Mpc), and \( \delta_c(t) = \delta_{co} D(t) \) is a critical threshold for the collapse and virialization of the primordial density perturbations. The local value \( \delta_{co} \) depends weakly on the cosmological parameters, while the growth factor \( D(t) \) sensitively depends on them.

The mass variance \( \sigma(m) \) is computed in terms of the perturbation spectrum; for definiteness, we use the CDM spectra given and discussed by White, Viana & Liddle (1996). For \( \Omega = 1 \) we adopt the primordial “tilted” index \( n_p = 0.8 \); for \( \Omega < 1 \) we adopt \( n_p = 1 \). The associated normalizations are taken from the COBE/DMR data (Gorski et al. 1998), and expressed in terms of the amplitude \( \sigma_8 \) at the relevant scale of \( 8 h^{-1} \) Mpc (see Bunn & White 1997).

Corresponding to eq. (3), the probability distribution that a given mass \( m \) at time \( t_o \) has a progenitor of mass \( m' \) at time \( t_1 < t_o \) reads

\[
\frac{df}{dm'}(m', t_1 | m, t_o) = \frac{\delta_c(t_1) - \delta_{co}}{(2\pi)^{1/2}(\sigma^2 - \sigma')^{3/2}} \frac{m}{m'} \left| \frac{d \ln \sigma}{d \ln m} \right| \frac{m'}{m} \exp \left\{ -\frac{[\delta_c(t_1) - \delta_{co}]^2}{2 \sigma^2 - \sigma'} \right\},
\]

where \( \sigma' \) is the mass variance at the scale \( m' \). On the other hand, at a given time \( t \) a progenitor \( m' \) increases its mass by a merging event with a clump of mass \( \Delta m \) (producing a cluster with mass \( m = m' + \Delta m \)), with the probability distribution per unit time given by

\[
\frac{d^2 p(m' \to m' + \Delta m)}{d \Delta m \, dt} = \left( \frac{2}{\pi} \right)^{1/2} \left| \frac{d \ln (\delta_c)}{dt} \right| \left| \frac{d \ln \sigma}{d m} \right| (m' + \Delta m) \frac{\delta_c(t)}{\sigma(m' + \Delta m)} \times
\]

\[
\times \frac{1}{\left[ 1 - \sigma^2(m' + \Delta m)/\sigma^2(m') \right]^{3/2}} \exp \left\{ -\frac{\delta_c^2(t)}{2} \left[ \frac{1}{\sigma^2(m' + \Delta m)} - \frac{1}{\sigma^2(m')} \right] \right\}.
\]
We have compared the analytical probabilities above with the results of the Monte Carlo code developed by P. Tozzi to simulate the hierarchical merging history of halos, based on the excursion set approach of Bond et al. (1991). A realization from the Monte Carlo simulations is shown in fig. 1a as an illustration of the basic process of DM halo growth. To show the agreement of the two approaches for a relevant quantity, we plot in fig. 1b the probability distribution of progenitors of mass \( m' \) at different \( z \) that end up in a given mass \( m \) at \( z = 0 \), computed from the Monte Carlo and from the eqs. above. In fig. 1c we show as a function of \( m \) the fraction of objects which, during the last 2 Gyrs, accreted mass in events involving comparable clumps (specifically, those with mass ratio 1/2.5), and in events involving very unequal clumps (with mass ratio 1/10); during that interval, more than 60 % of the clusters with \( M \geq 10^{15} M_\odot \) will have merged with clumps smaller than \( M/10 \).

### 2.2. Boundary Conditions

The pre-shock temperature in a merging event is that of the infalling gas. If the latter is contained in a sufficiently deep potential well, \( T_1 \) is the virial temperature \( T_{1v} \propto \Delta m/r \) of the secondary merging partner; on using \( r \propto (\Delta m/\rho)^{1/3} \) this writes

\[
 kT_{1v} = 4.5 (\Delta m)^{2/3} (\rho/\rho_0)^{1/3} \text{ keV},
\]

where the numerical coefficient is taken from Hjorth, Oukbir & van Kampen (1998). Where necessary, the \( z \)-dependence of \( \rho/\rho_0 = (1+z)^3 \) is converted to \( t \)-dependence following the standard FRW cosmologies.

But an independent lower bound \( kT_{1s} \approx 0.5 \text{ keV} \) is provided by preheating of diffuse external gas, due to feedback energy inputs following star formation and evolution all the way to supernovae (David et al. 1995; Renzini 1997). We recall that preheating temperatures in excess of 0.1 keV are believed to constitute essential complements to the hierarchical clustering picture to prevent the “cooling catastrophe” from occurring, see White & Rees (1978); Blanchard, Valls Gabaud & Mamon (1992). In point of fact, Henriksen & White (1996) find from X-rays evidence for diffuse gas at 0.5 – 1 keV in the outer regions of a number of clusters. So in the following the actual value of \( T_1 \) will be

\[
 T_1 = \max [T_{1v}, T_{1s}].
\]

Given \( T_1 \), the boundary conditions for the ICP in the cluster is set by the strength of the shocks separating the inner from the infalling gas. We report here from CMT98 the explicit expression of the post-shock temperature \( T_2 \) for three degrees of freedom and for a nearly hydrostatic post-shock condition with \( v_2 << v_1 \), assuming the shock velocity to match the growth rate of the virial radius \( R(t) \):

\[
 kT_2 = \frac{\mu m_H v_1^2}{3} \left[ \frac{(1 + \sqrt{1 + \epsilon})^2}{4} + \frac{7}{10} \epsilon - \frac{3}{20} \frac{\epsilon^2}{(1 + \sqrt{1 + \epsilon})^2} \right].
\]
Here $\epsilon \equiv 15kT_1/4\mu m_H v_1^2$ and $\mu$ is the average molecular weight; the inflow velocity $v_1$ is set by the potential drop across the region of nearly free fall, to read $v_1 \simeq \sqrt{-\phi_2/m_H}$ in terms of the potential $\phi_2$ at $r_2$. For a “cold inflow” with $\epsilon << 1$ the shock is strong, and the expression simplifies to $kT_2 \approx \mu m_H v_1^2/3 + 3kT_1/2$. Instead, for $\epsilon \gtrsim 1$ the shock is weak, and $T_2 \simeq T_1$ is recovered as expected. Note that $T_2$ depends through both $T_1$ and $v_1^2$ on the mass $\Delta m$ of the clump being accreted.

From $T_2$ and $T_1$, the density jump at the boundary $n_2/n_1$ is found to read (see CMT97)

$$
\frac{n_2}{n_1} = 2 \left(1 - \frac{T_1}{T_2}\right) + \left[4 \left(1 - \frac{T_1}{T_2}\right)^2 + \frac{T_1}{T_2}\right]^{1/2}.
$$

(9)

It is seen that the density jump takes on the limiting value $n_2/n_1 = 4$ for very strong shocks, while the adiabatic approximation $n_2/n_1 \approx 1 + 3(T_2 - T_1)/2T_1$ is recovered for weak shocks.

### 2.3. Hydrostatic Equilibrium

We adopt the polytropic temperature description $T(x)/T_2 = [n(x)/n_2]^{-1}$, with the index $\gamma$ in the range $1 \leq \gamma \leq 5/3$ to begin with. In terms of the virial temperature $T_v$ (see Sarazin 1988), eq. (2) writes $L \propto r_2^3 (n_1/\rho)^2 (n_2/n_1)^{2T_v/\rho^2 (T_2/T_v)^1/2} [n(r)/n_2]^{2+(\gamma-1)/2}$, where the bar denotes the integration over the emitting volume $r^3 \leq r_2^3$, and $\rho$ is the average DM density in the cluster, proportional to $\rho_u$ and so to $n_1$ (see under eq. 2). The radius $r_2$ may be rewritten in terms of the temperature $T_v \propto m/r_2 \propto \rho r_2^2$. We finally obtain

$$
L \propto \left(\frac{n_2}{n_1}\right)^2 T_v^2 \rho^{1/2} \left[\frac{T_2}{T_v}\right]^{1/2} \frac{n(r)/n_2}{2+(\gamma-1)/2}.
$$

(10)

The underlying assumption is that after a merging event the cluster re-adjusts to a hydrostatic equilibrium with boundary conditions $n_2$, $T_2$ corresponding to its dynamical history (see eq. 6-9). Actually, this requires sound propagation times shorter than the dynamical timescale taken anyway by the DM to relax to its own steady configuration; the condition is seen to be satisfied, though marginally, for all merging events except for the rare ones involving comparable clumps. As we discuss in detail in the concluding §4, the observations and the hydrodynamical N-body simulations concur in supporting not only the hydrostatic equilibrium approximation for the relevant merging events, but also its parametrization with a polytropic equation of state.

The ratio $n(x)/n_2$ is obtained from the hydrostatic equilibrium $dP/m_H n \, dr = -GM(<r)/r^2$ through the polytropic pressure $P(r) = kT_2 n_2 [n(r)/n_2]^{\gamma}$. This yields (see Cavaliere & Fusco Femiano 1978; Sarazin 1988, and bibliography therein) the profiles

$$
\frac{n(r)}{n_2} = \left[\frac{T(r)}{T_2}\right]^{1/(\gamma-1)} = \left\{1 + \frac{\gamma - 1}{\beta} \left[\phi_2 - \phi(r)\right]\right\}^{1/(\gamma-1)},
$$

(11)
where \( \tilde{\phi} \equiv \phi/\mu m_H \sigma_2^2 \) is the potential normalized to the associated one-dimensional DM velocity dispersion at \( r_2 \). The ICP disposition in eq. (11) relative to the DM depends on the parameter

\[
\beta = \mu m_H \sigma_2^2 / kT_2 ,
\]

and is further modulated by the second parameter \( \gamma \), to yield as the latter increases flatter profiles \( n(r) \) and steeper \( T(r) \). In our PE \( \beta \) is by \( T_2 \) given by eq. (8), and considering the statistics of \( T_2 \) we obtain the results discussed in §3.1; the other parameter \( \gamma \) will be bounded as also discussed there.

We shall focus on the “universal” forms of \( \phi(r) \) and \( \sigma(r) \) given by Navarro, Frenk & White (1997). When relevant, we will discuss also results for the (simplified) King potential (see Sarazin 1988; see also Adami et al. 1998) where the DM itself has a core, and somewhat fatter ICP cores obtain. We shall also discuss the steeper cusp found by Moore et al. (1997) in highly resolved, CDM simulations; correspondingly, we still obtain a core-like ICP distribution, albeit slightly slimmer. Actually, in hydrostatic equilibrium a DM cusp flatter than \( \rho(r) \propto r^{-2} \) – corresponding to a gravitational force flatter than \( r^{-1} \) – implies at the centre a finite ICP density \( n_c \) but a high derivative, which however at observable resolutions is flattened by a modest increase of \( \gamma \).

### 2.4. Statistics

Our purpose is to compute the average value of \( L \) and its dispersion, associated with a given cluster mass \( m \). We re-iterate from Sect. 2 that the diverse merging histories ending up in such a mass give rise to a set of equilibrium states characterized by different values of \( T_2, n_2 \). These are related by eqs. (8) and (9) to the values of \( T_1 \) associated with the the clump of mass \( \Delta m \) incoming onto a cluster progenitor. So to meet our purpose we must sum over the shocks produced at a time \( t' < t \) in all possible progenitors \( m' \) (weighting with their number) by the accreted clumps \( \Delta m \) (weighting with their merging rate); finally, we integrate over times \( t' \) from an effective lower limit \( t - \Delta t \).

The average \( L \) is then given by

\[
\langle L \rangle = Q \int_{t-\Delta t}^{t} dt' \int_{0}^{m} dm' \int_{0}^{m'_{\text{min}}} dm \int_{m_{\text{min}}}^{m'} df(m', t', m, t) \frac{df(m' \to m' + \Delta m)}{d\Delta m \, dt'} L ; \tag{13}
\]

and the variance is given by

\[
\langle \Delta L^2 \rangle = Q \int_{t-\Delta t}^{t} dt' \int_{0}^{m} dm' \int_{0}^{m'_{\text{min}}} dm \int_{m_{\text{min}}}^{m'} d\Delta m \frac{df(m', t', m, t)}{d\Delta m \, dt'} \left( \frac{d^2 p(m' \to m' + \Delta m)}{d\Delta m \, dt'} \right) (L - \langle L \rangle)^2 . \tag{14}
\]

Higher moments – if needed in case of non-Poissonian statistics – are given by similar expressions; the full distribution of \( L \) requires aimed computations or simulations, as noted by CMT98. In the integrals, the luminosity \( L [T_2(m'), T_1(\Delta m')] \) depends on \( m' \) and \( \Delta m' \) through the boundary
conditions discussed in Sect. 2.2. The compounded probability distribution in eqs. (13) and (14) has been normalized to 1 (we do not write down the normalization factor $Q$ for the sake of simplicity). The effective lower limit for the integration over masses is set as follows.

The merging events relevant to $\langle L \rangle$ and to $\langle \Delta L^2 \rangle$ after eqs. (13) and (14) are those lasting enough as to overlap with similar ones. Since $\Delta t \propto \Delta m/v_1 \rho r^2$, the above condition results in an effective lower limit for the masses entering eq. (13) and (14); physically, lumps with masses smaller than such limit yield a small mass flux and so produce shocks which dissipate before new clumps income. We pinned down the minimum $\Delta m$ numerically, by looking at the saturation of $\langle L \rangle$ for increasing values of $\Delta t$. This occurs at $\Delta t \approx 0.7 t_d$ which corresponds to a lower limit about $m/20$ for $\Delta m$ on using $r^2 \propto \Delta m^{2/3}$.

Note that the above procedure acts like an effective mass weight. Heuristically, this may be seen with the $\Delta m$ and $t$ integrations interchanged; then the lower limit contains $\Delta t(\Delta m)$ which must be convolved with the distribution of $\Delta m$. But for $\Delta t > 0.7 t_d$ the resulting average saturates; so we have written the $t$-integration as the outmost integral, having adopted the lower limit for $\Delta m$ said above.

Thus very small accreted lumps do not affect our average $\langle L \rangle$, due to the small associated mass accretion rate. On the other hand, merging events involving comparable partners (though contributing $\sim 1/2$ of the total mass) affect the overall $\langle L \rangle$ only marginally; in fact, such events not only are few ($< 10\%$), but also they involve lumps with temperatures $T_1$ comparable to $T_2$, and so produce a compression factor $n_2/n_1 \approx 1$ (eq. 9). The major contribution to $\langle L \rangle$ is by far ($\sim 90\%$) provided by intermediate merging lumps, which yield an integrated contribution $\sim 1/2$ to the mass, but dominate the number of events and produce large compressions $(n_2/n_1)^2 \propto L$. For such events, the isotropic hydrostatic equilibrium for the ICP is physically motivated and robust (see §2.3 and §4), substantiating our step-by-step rendition of the hydrodynamics.

3. RESULTS

Here we present various results from the PE model, and compare them with observations. To this aim, we shall express our results in terms of the observed emission weighted temperature, which we denote simply by $T$. Moreover, the contribution of relevant emission lines to $L$ (from updates of Raymond & Smith 1977) has been added to the bremsstrahlung emission underlying the simple scaling in eq. 10.

3.1. Profiles

Our reference cluster will have a mass $m$, and DM potential $\phi(r)$ as said in Sect. 2.3. The density and temperature profiles are given by eq. (11), and are to match the shock boundary
conditions at the position $r_2 \simeq R$. The key quantity is the parameter $\beta$ defined by eq. (12); its average value and scatter are predicted by the PE model (using convolutions analogous to eqs. 13-14) to be as shown in fig. 2. Note that $\beta(T)$ grows slowly with the temperature; we obtain values ranging from about $\beta = 0.5$ at the group scale to $\beta \approx 0.9$ for rich clusters, see fig. 2. These values imply that in our model the ICP profiles are smoother and more extended than the DM’s, an effect becoming more pronounced in downscaling from clusters to groups.

In fig. 3 we show the baryonic fraction $f_2$ (integrated out to the shock) as a function of $T$ and for different values of $\gamma$. The polytropic index $\gamma \geq 1$ describes the equation of state for the ICP. An upper bound to it arises if the overall thermal energy of the ICP is not to exceed its gravitational energy, with only minor contributions from other energy sources, like radiosource heating or energy transfer from DM to ICP, as discussed in Sect. 4. The thermal and the gravitational energy are computed using the profiles in eq. (11), and their ratio is given in fig. 4 to show that the upper bound $\gamma \leq 1.3$ holds.

In fig. 5 we show temperature profiles $T(r)$ for different values of $\gamma$, in terms of the normalized coordinate $x = r/R$. It turns out that observations by Markevitch et al. (1997) are consistent with the $T(r)$ predicted when $\gamma = 1.2 \pm 0.1$, in our allowed range. Hereafter we shall focus on $\gamma = 1.2$.

In fig. 6 we show the density profiles $n(r)$ for two local clusters with different temperatures; the associated surface brightness $\Sigma(r)$ is shown in fig. 7 along with representative data. It can be seen that groups have flatter $\Sigma(r)$ than rich clusters, an outcome persisting when the King or the potentials by Moore et al. 1997 are used.

### 3.2. Correlations

We show first in fig. 8 the $M - T$ relation in view of its important role. Note that, given the mass function, our flattening at low temperatures translates into a steepening of the corresponding temperature function; such an effect has been also noted by Balogh, Babul & Patton (1998).

The $L - T$ correlation is given by the double convolution (13), and likewise for $\Delta L$ after eq. (14). The results are shown in fig. 9 for a tilted CDM spectrum of perturbations in the critical universe. For the reasons discussed in CMT98, the normalization has been best-fitted on the data.

As stressed in our previous works (CMT97, CMT98), the correlation we predict and show in fig. 9 is not a single power-law; it starts as $L \propto T^2$ for very rich clusters with high $T$, but bends down with decreasing $T$, due to the threshold effect of the preheating temperature $kT_1 \approx 0.5$ keV. Note that our correction to 0.3 solar metallicity tends to increase, if anything, the luminosities and to flatten the slope at low $T$.

A convenient fitting formula for the predicted $L - T$ correlation (precise to better than 10%
for $T > 2T_1$) is as follows:

$$L = a_L T^{2+\alpha_L} (\rho/\rho_o)^{1/2} \tag{15}$$

\[a_L \propto 3.8 \Omega^{-0.3}_o (1+z)^{0.22/\Omega_o} + (1 - \Omega_o) e^{-0.7(1+z)}\]

\[\alpha_L = 1.12 (1+z)^{-0.2} e^{-0.25 (T-T_1)/\Omega_o^{0.5} (1+z)^{0.5}} ,\]

where the luminosity is expressed in units of $10^{44}$ erg/s and the temperature in keV. The $z$-dependence of $a_L$ results from the interplay of the following effects: i) the evolution of the Navarro et al. (1997) potential; ii) the abundance of clusters with given $T$; iii) the evolution of the progenitor probability distributions in eqs. (4) and (5). Such effects are small, and moreover they balance out very nearly, leaving the basic $z$-dependence $[\rho(z)/\rho_o]^{1/2}$. In turn, the latter dependence goes as $(1+z)^{0.5-1}$, considering (CMT98) the evolution of the overdensities in the large scale structures – filamentary or sheet-like – hosts to the clusters.

At temperatures substantially larger than the threshold $kT_1 \simeq 0.5$ keV the intrinsic, dynamic dispersion grows with $T$, but the relative $\Delta L/L$ stays nearly constant around 25% $(2\sigma)$, as shown by fig. 9. We have deliberately chosen to keep this figure simple and not to include the conceivable spread of $T_1$ already represented by CMT98 in their fig. 2; the effect of such a spread is to widen the dispersion below 1 keV adding another, large component to the intrinsic dynamic variance.

As our analytical approach allows us to span a wide range of cosmologies/cosmogonies, we show in fig. 10a the dependence of $\langle L \rangle$ on $\Omega_o$ at two temperatures and at the current epoch. It is seen that $\langle L \rangle$ increases with $\Omega_o$ increasing; this is because the underlying strength of the current shocks grows on average as the merging rate (moderately) increases on approaching the critical cosmology, see Lacey & Cole (1993). A similar behaviour is followed by the corresponding dispersion $\Delta L$, see fig. 10b.

Similarly, we derive a correlation with $T$ of the effective size of the X-ray emission. If (following Evrard & Mohr 1997) we define $R_X$ in terms of the isophote corresponding $1.9 \times 10^{-3}$ ct/s arcmin$^2$ in the ROSAT band (consistent with our normalization for $L$), we find the $\langle R_X \rangle - T$ correlation shown in fig. 11. We find $R_X \propto T$ in the range of clusters, with a steepening at the group scale and a flattening at large temperatures. A corresponding fitting formula is as follows:

$$R_X = a_R T^{0.5+\alpha_R} (\rho/\rho_o)^{-1/2} \tag{16}$$

\[a_R = 0.6 \Omega^{-0.6}_o (1+z)^{-1.5+0.3(1-\Omega_o)/(1+z)}\]

\[\alpha_R = 0.5 \Omega^{-0.6}_o (1+z)^{0.6} e^{-0.37 (T-T_1)/\Omega_o^{0.2} (1+z)^{1.4}} ,\]

where $R_X$ is expressed in Mpc and $T$ in keV.

4. CONCLUSIONS AND DISCUSSION

This paper is based on hierarchical clustering; group and cluster formation is envisaged in terms of DM potential wells evolving hierarchically, and engulfing outer baryons by accretion of...
smooth gas or by merging with other clumps. We consider the diffuse baryonic component to increase as the deepening wells overcome the external gas energy provided by preheating stars or by virialized subclumps. After a merging episode, the ICP in the wells falls back to a new, approximate hydrostatic equilibrium (hence the name “punctuated equilibria”).

We have modeled this complex astrophysics using an analytic approach based on the standard hierarchical clustering and comprising three main steps: the hydrostatic equilibrium for the ICP is computed for a given boundary condition; the latter is derived from the effects on the ICP of the dynamical evolution of the DM halos; the intrinsic stochastic character of such evolution is accounted for on convolving with the statistics of the DM merging histories. All such aspects are treated in a fashion which is necessarily simplified; however, the resulting model – whose parameters are all internally or physically constrained – proves to be gratifyingly efficient in explaining and predicting a variety of observations. We shall discuss such aspects in turn.

As for the ICP equilibrium, we note that the condition $R/c_s < t_d$ (sound crossing time shorter than the dynamical time) is weakly satisfied. However, this is enough to ensure ICP equilibrium during the intervals (at least 2/3 of the total time, see Tormen et al. 1997) when the DM halos themselves are in approximate dynamical equilibrium. In point of fact, ever since Jones & Forman (1984) to the recent Cirimele, Nesci & Trevese (1997), it has been recognized that hydrostatic equilibrium provides for many clusters a fitting description of the averaged profiles of surface brightness in X-rays (except for the central region when cooling flows set in). Even the high-resolution observations provided by ASCA (Markevitch 1998) and those being derived from SAX data can be accounted for in terms of average profiles. Conspicuous hot spots do occur, but only in a minority of sources, and then in correlation with other signs of ongoing major dynamical events, as discussed next.

This body of evidence supports the case that the gas stays close to hydrostatic equilibrium, or falls back to it after a short transient from the merging event, except for the rare major episodes involving a partner of comparable mass. The limits to the above picture may be defined with the help of aimed hydrodynamical simulations. A quantitative account of how much and how long cluster collisions displace the ICP out of equilibrium can be found, e.g., in the N-body experiment of Roettiger et al. (1998) for the case of a merging event with a mass ratio of 1/2.5. Even for such ratio (already a rare event in the hierarchical clustering picture) the simulation shows that some 2 Gyr after the event hot spots and space variations of the luminosity are reduced to under 20%.

So a sequence of hydrostatic equilibria of the ICP is physically motivated for all merging events except for those involving comparable clumps (a mass ratio larger than $\sim 1/4$). However these sum up to less than 10% in the number; in addition, these events yield a shock compression factor $n_2/n_1 \approx 1$ (see eq. 9), with an overall contribution to $\langle L \rangle$ less than 10%. This is actually the precision level of our model.

Note that these considerations also support the use of the polytropic relation $T \sim n^{\gamma - 1}$. In fact, when equilibrium holds, a macroscopic $\gamma = d\ln p / d\ln n$ may be always defined in principle to
describe the ICP state; the question concerning whether this is constant on scales $\gtrsim 0.1$ Mpc can be probed with observations. In fact, as we discuss later on in this Section, observed temperature and surface brightness profiles agree well with those predicted by the polytropic equation of state with $\gamma \approx 1.2$.

The hydrostatic equilibrium is described by a first order differential equation (see §2.3) requiring one condition at the boundary. Physically, this is provided by the place where a shock converts most of the kinetic and gravitational energy of the inflowing colder gas into thermal energy, as it must occur for the accreted ICP to be contained in the well. The boundary condition may be referred to in terms of the stress balance $P_2 = P_1 + n_1 m_H v_1^2$, one conservation law contributing to eqs. (8) and (9). If it were somehow possible to shut off the r.h.s. completely, the intrachannel medium would expand (the more so the closer is its state to isothermal, and the shallower is the potential at the shock position; the density would decrease everywhere including the centre and $L \propto n^2$ would quench considerably over a sound crossing time. However, we shall argue next that the dynamic stress acts steadily.

In closer detail, the shock jump conditions are set in terms of $T_2/T_1$, basically the height of the current potential well (see eq. 8) compared with the thermal energy of the infalling gas. The latter is initially due to stellar preheating (of nuclear origin); then it is increased to the virial value (of gravitational origin) when the accreted gas is bound in DM subclumps. So the preheating sets an effective threshold $kT_1 \sim 0.5$ keV to gas inclusion, which breaks the self-similar correlation $L \propto T^2$ not only in its vicinity but also up to a few keV. In our model, this occurs through the specific dependence of $n_2/n_1$ on $T_2/T_1$ at the cluster boundary holding for spherical shocks, strong or weak. This is a fair representation for the conditions prevailing when the cluster growth occurs by nearly isotropic accretion of smooth gas or of many small clumps, as shown by a sequence of spherical hydrodynamical simulations up to the recent one by Takizawa & Mineshige (1997). Surely, this representation looks as a rather crude approximation to the aftermaths (lasting up to 2 Gyr) of major merging events, as those simulated in detail by Roettiger et al. (1998). But in point of fact, our average quantities, their scatter and the profiles agree with the data over the whole range of $T$ as we stress next, while an explanation in point of principle is offered thereafter.

For example, the equilibrium parameter $\beta(T)$ is set by the boundary conditions in terms of shock strengths to values which increase from about 0.5 at the group scale to about 0.9 for rich clusters, see fig. 2. Correspondingly, the surface brightness profiles in groups – beyond the generally larger observational noise – ought to be flatter than in rich clusters (see figs. 6, 7). In fact, similar values are obtained from fits to the brightness $\Sigma(r)$ observed in groups and in rich clusters ever since Kriss, Cioffi & Canizares 1983 and Jones & Forman 1984. More recently, a similar trend has been found from spectroscopic measurements of $\beta$ by Edge & Stewart (1991), by Henriksen et al. (1996), and by Girardi et al. (1998). We add that – as another straightforward consequence of the threshold $T_1$ – our model groups differ from rich clusters also for their lower, average baryonic fraction (see fig. 3) in accord with the average values inferred by Reichart et al. (1998) from observations, however noisy.
The other parameter of the ICP equilibrium, namely the polytropic index $\gamma$, is constrained to the rather narrow range $1 \leq \gamma < 1.3$. The upper bound holds if the ICP thermal energy content is not to exceed its gravitational energy in the DM potential (see figs. 4); it may be extended to 1.4 only if kinetic energy transfer from residual clumps in DM to ICP contributes more than 20%, the limit set by Kravtsov & Klypin (1998). Values of $\gamma$ around 1.2 imply the entropy distribution $S(r) \propto \log[T(r)/n^{\gamma-1}(r)]$ to have a neat central minimum, in accord with the notion of dominant entropy deposition by shocks in the outer regions (see David, Jones & Forman 1996; Bower 1997) against the central contribution deposited by supernovae. In fact, specific calculations based on entropy production at the shock show that $\gamma \approx 1.2$ holds with little variations from clusters to groups (Tozzi & Norman 1999, in preparation). Values of $\gamma \approx 1.2$ turn out to yield temperature gradients (see fig. 5) consistent not only with the results from advanced simulations of rich clusters (Bryan & Norman 1998), but also with aimed recent observations by Markevitch et al. (1997) (see also Fusco Femiano & Hughes 1994). Preliminary data from SAX (S. Molendi, private communication) indicate in some cases a somewhat flatter central gradient, but still consistent with the range of $\gamma$ given above. That such gradients cannot be realistically traced back to imperfect thermalization of the electrons has been argued by Ettori & Fabian (1998) (for a discussion see also Takizawa & Mineshige 1998b). Note that the arguments may be reversed, opening an interesting perspective to gauge the baryon thermal history and its link with the DM dynamics, see fig. 6; in fact, very steep DM cusps would require larger values of $\gamma$ to fit the core-like shape of $\Sigma(r)$, but this in turn would produce steep profiles of $T(r)$ and imply additional central inputs of energy and entropy, leading to a strongly bent $L-T$ relation.

As for the statistical aspects governing average correlations and their scatter, these are derived from convolution of the boundary conditions over the merging histories. We have already predicted and discussed the $L-T$ correlation in our previous papers (CMT97; CMT98); here we only note, and illustrate in fig. 9, the points discussed by Markevitch (1998), Allen & Fabian (1998), namely, that once the effect of large cooling flows is removed or accounted for, the average correlation is flattened to a slope around 2.5 and the scatter is reduced down to 13 % ($1\sigma$), both in good agreement with the intrinsic, dynamic scatter from the model.

Here instead we expand on the $R_X-T$ correlation, see fig. 11. That our model predicts $R_X \propto T$ in the range of rich clusters with a steepening at the group temperatures, is due both to the non selfsimilar form of the shock strength as a function of $T$, and to the shape of $\beta(T)$ discussed above. The results agree with the data, whilst all the self-similar computations (including the simulations without preheating discussed by Evrard & Mohr 1997) yield $R_X \propto T^{0.5-0.7}$. The dispersion we find from the average over the merging histories also compares well with the existing data.

To understand such overall effectiveness of the model one has to consider two features of the hierarchical clustering: i) the main contributions to the growth of cluster-sized DM halos is given by the many lesser, closely isotropic events (see fig. 1), which are described well by the model; ii) such events contribute the most to statistics like the $\langle L \rangle - T$ correlation, as can be seen on
examining the convolutions with the complete merging histories represented by eqs. (13) and (14). The argument reduced to the bones goes as follows: the average $\langle L \rangle \propto \langle (n_2/n_1)^2 \rangle$ may be expressed by summing the values $(n_2/n_1)^2$ after a merging episode, weighted with the probability $\Pi$ of a given mass ratio (see fig. 1c), and with the mass increment $\Delta M/M$. Considering a cluster with $M \approx 10^{15} M_\odot$, the rare events ($\Pi \approx 0.15$) corresponding to $\Delta M/M \approx 0.5$ yield $(n_2/n_1)^2 \approx 1; \text{ instead, events with } \Delta M/M \lesssim 0.1 \text{ have } \Pi \gtrsim 0.6 \text{ and yield } (n_2/n_1)^2 \gtrsim 6 \text{ (from eq. 9 averaged over the merging histories)}, \text{ whose product makes an overwhelming contribution to the average.}$

In summary, our picture envisages the combined effect over the effective time $\Delta t$ of all shocks which overlap. Barring the lumps too small to overlap and yield an appreciable mass accretion rate, and those too warm and too few to yield relevant compressions, our physical picture is focused onto a nearly continuous accretion of intermediate and colder lumps overlapping within a sound crossing time. From these, the ICM feels a nearly steady external pressure with only minor fluctuations. Such pressure exerted at the cluster boundary sets the internal density via the connection between boundary and centre provided by hydrostatic equilibrium, and so determines the steady $\langle L \rangle$, possibly varying on cosmological time scales.

Finally, we stress the efficiency of the present analytical approach in making predictions for a wide range of cosmologies/cosmogonies. These are easily spanned in terms of the the convolutions (13) and (14) over the merging histories to yield the dependence of the amplitude, shape and scatter of the predicted correlations $L - T, R_X - T$ on the cosmology as given in fig. 10 and fitted with eqs. 15 and 16. Such histories are dominated by those merging events between very unequal clumps (with $T_1 \ll T$) which occur close to the observation time (Lacey & Cole 1993), as shown in fig. 1b. Though the merging rate does depend on $\Omega_\text{o}$, the sum over time of the merging events is weakly dependent on it, and so do the average luminosity and the dispersion (see fig. 10).

The advances attained by the PE model are as follows. The free parameters (the central density $n_c$, $\beta$ and $\gamma$) of the previous hydrostatic models are now computed or constrained. The boundary conditions that yield $n_c$ and $\beta$ are derived from an approximate rendition of the hydrodynamics, and are related to the DM dynamics. The stochastic character of the latter implies variance in the merging histories even at given $T$, and this is enhanced by the $n^2$ dependence of the emission to yield the intrinsic scatter expected in $L$. The resulting, narrowly constrained model predicts temperatures declining outwards, and – in scaling down from rich clusters to groups – smaller $\beta$ and shallower brightness profiles, decreasing baryonic content on average, and the $L - T$ relation bending down strongly on approaching the preheating threshold $kT_1 \sim 0.5$ keV.

Actually, any reasonable spread in such threshold as discussed by CMT98 implies for groups an increased luminosity dispersion $\Delta L/L > 25\% \text{ (2}\sigma)$ along with a considerable scatter $\Delta f/f \approx \Delta T_1/T_1$ in the baryonic fraction, apart from the larger uncertainties affecting the group observations. We have deliberately chosen to keep the model simple and to implement here neither such spread nor the $z$-dependence of $T_1$ which is expected for $z \gtrsim 1$, corresponding to the star formation rates at such early $z$. We plan to expand on such issues while the high-$z$ data are
drawing near.

Acknowledgements: we are indebted to M. De Simone and D. Trevese for communicating their data prior to publication, to F. Governato for discussing with us his high-resolution N-body simulations, and to S. Molendi for several informative discussions concerning the temperature profiles from SAX. Thanks are due to our referee C. Lacey for pointing a number of errors and omissions in the MS, and for having stimulated us to clarify the exposition of several important points. Grants from ASI and from MURST are acknowledged.
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FIGURE CAPTIONS

Fig. 1.— Top panel: a Monte Carlo realization illustrates the merging history of a DM halo with final mass $10^{15} M_\odot$. Middle panel: for the same halo it is shown the probability distribution of progenitors with mass $M'$ at the redshifts $z = 0.03$ and $z = 0.5$ that end up in a mass $M$ at $z = 0$. The circles ($z = 0.03$) and the triangles ($z = 0.5$) represent the results from the Monte Carlo simulations, while the lines show the analytical results from eq. 4. Bottom panel: the fraction of merging events during the last 2 Gyrs involving a mass $M$ and a partner with mass $3 \times 10^{13} M_\odot < M' < M/10$ (solid line); the lower limit arises from requiring a virial temperature $T_v > 0.5$ keV for the partner (see §2.2). The dashed line is the corresponding fraction for events with $M' > M/2.5$. Tilted, COBE-normalized spectrum of perturbations, as given in White et al. (1996) in the critical universe with $H_0=50$ km/s Mpc.

Fig. 2.— The predicted dependence of $\beta$ on the (emission-weighted) temperature $T$, see eq. 12 and 8. DM potential as given by Navarro et al (1997), computed for the cosmological parameters and the perturbation power spectrum used in fig. 1. The shaded region indicates the 2–$\sigma$ scatter due to the merging histories.

Fig. 3.— The predicted ratio of the baryonic fraction $f_2$ at the cluster boundary to the external value $f_u$ (White et al. 1993; White & Fabian 1995) as a function of the temperature $T$ for $\gamma = 1$ (solid line), $\gamma = 1.1$ (dashed) and $\gamma = 1.3$ (dotted).

Fig. 4.— The ratio of the thermal ICP energy $E_{ther} = 3 \int d^3r n(r) T(r)$ to the ICP gravitational energy $E_{grav} = G m_H \int d^3r n(r) M(<r)/r$ is shown as a function of $\gamma$ for the DM potential of fig. 2 (solid line), and for the King form (dashed line) with core radius $R/10$. The profiles $n(r)$ and $T(r)$ are provided by eq. (11).

Fig. 5.— Temperature profiles for the model cluster of $10^{15} M_\odot$ at $z = 0$ in polytropic equilibrium; $\gamma = 1$ (solid line), 1.2 (dashed) and 1.66 (dotted), see eq. (11). DM potential as in fig. 2. The profile is smoothed out with a filter width of 100 kpc. The dashed area taken from Markevitch et al. (1997) summarizes the observations of 30 clusters.

Fig. 6.— Upper panel: The predicted ICP density profile $n(r)$ for a rich clusters with $kT = 8$ keV (upper line), and for a poor cluster with $kT = 2$ keV (lower line). Radii are normalized to the virial radius $R$. The profiles are computed using the DM potential as in fig. 2, and for the ICP the polytropic index $\gamma = 1.2$. Bottom panel: to illustrate the variations produced by the use of the King potential as in fig. 4, with $\gamma = 1.1$ (dotted line); and of the Moore et al. 1997 potential, with $\gamma = 1.3$ (dashed line).

Fig. 7.— The surface brightness profile $\Sigma(r)$ from the PE model is compared with the data for two oppositely extreme clusters. Top panel: A539, rather sparse and with low T, at $z = 0.026$ (David et al. 1996, De Simone private communication). Bottom panel: A2390, relaxed and hot, at
$z = 0.23$ (B"ohringer et al. 1998). No attempt has been made at excluding emissions from cooling flows.

Fig. 8.— The predicted mass-temperature relation for $T_1 = 0.8$, 0.5, 0.3, 0 keV, from top to bottom.

Fig. 9.— The average L-T correlation with its 2σ dispersion (shaded region) is shown for the PE model with the tilted CDM cosmogony of fig. 1. The value $H_0 = 50$ km/s Mpc is assumed for the Hubble constant. The luminosities are corrected to 0.3 solar metallicity using the standard Raymond Smith code. Group data from Ponman et al. (1996, solid squares); cluster data from Markevitch (1998, open triangles). Here $T_1 = 0.5$ keV with no dispersion; the effect of the latter is shown by fig. 2 of CMT98.

Fig. 10.— Left panel: the dependence of the average luminosity $L \propto \langle (n_2/n_1)^2 \rangle$ on $\Omega_o$.
Right panel: the dependence of the dispersion $\Delta L$ on $\Omega_o$. In both panels CDM cosmogonies are assumed, see Liddle et al. (1996).

Fig. 11.— The correlation of the radius $R_X$ (see text) with $T$ (emission weighted) is plotted for $\Omega = 1$. The 2σ dispersion is shown by the shaded region; the cosmological parameters and the perturbation power spectrum are as in fig. 2. The data are from Mohr & Evrard (1997).