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Study of three-dimensional laser cooling method based on resonant linear coupling

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ABSTRACT

Three-dimensional (3D) laser cooling method of fast stored ion beams based on linear coupling mechanism is explored. We extensively study the two approaches proposed in the previous publications, i.e. the dispersive coupling scheme and the coupling-cavity scheme, confirming how much one can improve the transverse cooling efficiency. A possible design of a coupling cavity is presented. We employ the tracking code "SAD" and the molecular dynamics code "SOLID" to carry out reliable numerical experiments where realistic lattice structures of storage rings are taken into account. Through systematic simulations, it is demonstrated that the resonant coupling remarkably enhances transverse cooling rates for any initial beams, making it feasible to reach the equilibrium temperature far below the current achievable level. We further emphasize the essential importance of avoiding the Mathieu instability. Discussion is also made on the minimum cooling power required for beam crystallization as well as on the interpretation of the past experimental results in the TSR and ASTRID rings.

I. INTRODUCTION

To our current knowledge, Doppler laser cooling, among various cooling techniques, is most promising in achieving the highest possible phase-space density of ion beams [1]. Limiting temperature attainable with this relatively new technique is even below milli-Kelvin, sufficient to realize the theoretically-predicted phase transition of fast stored beams [2]. At present, there exist only two storage rings which are fully equipped with a laser cooler, i.e. the TSR ring in Germany [3] and the ASTRID ring in Denmark [4]. Nevertheless, beam temperature much lower than ever recorded with conventional techniques has already been obtained, at least, in the longitudinal degree of freedom. The high potential demon-

strated independently by the TSR and ASTRID group has now let us reasonably anticipate that one may be able to produce crystalline beams in the near future.

In contrast with experimental success in reducing longitudinal temperature to an ultra-cold range, laser cooling of fast circulating beams has been shown much less effective in transverse directions. Although natural Coulomb interactions between individual particles can be a source of heat exchange, the rate of the heat transfer is quite low. The transverse temperature appears to be saturated far above the longitudinal final temperature as far as the past experimental results are concerned [5]. As this difficulty has not been overcome for the last decade in spite of continuous experimental effort, it is now thought that one may need to develop some novel approach in order to extend the powerful laser cooling force to the transverse degrees of freedom.

In the previous papers [6,7], we proposed, for the first time, the idea of using artificial dynamical coupling to indirectly provide a strong transverse dissipative force. As the coupling source, we took advantage of the linear potential induced either by a special *coupling cavity* operating in TM_{210} mode [6] or by *momentum dispersion* in a regular RF cavity [7]. It was theoretically proved that the transverse cooling rates can be considerably enhanced under the resonance conditions

$$\nu_x - \nu_s \approx k, \quad \nu_y - \nu_s \approx l \quad (k, l = \text{integer}), \quad (1)$$

where ν_x and ν_y are the horizontal and vertical betatron tunes respectively, and ν_s is the synchrotron tune. A significant feature of the proposed schemes is their simplicity; in other words, we need nothing new to develop. For instance, in the dispersive coupling scheme, all one have to do is simply the installation of an ordinary accelerating cavity at a position where there is finite dispersion. Clearly, it would be possible to test the validity of the basic idea in the TSR and/or ASTRID ring only by making minor modifications to the lattice parameters. As a matter of fact, the effectiveness of our resonant coupling method has been partially confirmed in a very recent experiment [8].

In the following sections, we evaluate the effectiveness of the resonant coupling schemes described in Refs. [6] and [7]. To give specific, well trustable results, we perform numerical simulations exactly considering the lattice parameters of several existing cooler rings. After giving a brief review of the coupling schemes, we present a possible design of a coupling RF cavity and, then, show single-particle simulation results obtained with the tracking code “SAD (Strategic Accelerator Design)” [9]. The molecular dynamics (MD) approach is further employed to incorporate the effect of Coulomb interactions between stored ions. Finally, we attempt to answer the questions such as “What is the lowest transverse temperature achievable with the coupling method?”, “Why is the imbalance of final beam temperatures between the longitudinal and transverse directions so large in TSR and ASTRID?”, “How much cooling rate is necessary in an ideal situation to reach the possible lowest temperature?”, “Is laser cooling of fast stored beam really only longitudinally powerful?”, etc.

II. THE 3D LASER COOLING SCHEMES

A. Dispersive coupling in an RF cavity

Consider an ordinary storage ring consisting of dipole magnets, quadrupole magnets, and a regular RF cavity. The single-particle Hamiltonian governing this dynamical system can be written, under the smooth approximation, as [7]

$$H_1 = \frac{1}{2} (p_x^2 + \nu_x^2 x^2) + \frac{1}{2} (p_y^2 + \nu_y^2 y^2) - \frac{\eta W^2}{2} + \Gamma_q y \left(x - \frac{D_x}{R} W \right) \delta_p(\theta - \theta_q) - \frac{h_b q V_b}{p_0 \beta_0 c} \cos \left[\psi - \frac{1}{R} \left(D_x p_x - \frac{dD_x}{d\theta} x \right) \right] \times \delta_p(\theta - \theta_b), \quad (2)$$

where the canonical variables $(x, p_x, y, p_y, \psi, W)$ have been scaled to be dimensionless [10], $\delta_p(z)$ is the periodic delta function with the periodicity of 2π , and the independent variable θ has been introduced as $\theta = s/R$ with R and s being, respectively, the average radius of the ring and path length along the design orbit. Other parameters appearing in Eq. (2) are:

- η = phase slip factor,
- p_0 = design kinetic momentum,
- q = charge state of stored ions,
- h_b = harmonic number of the RF field,
- V_b = RF voltage of the cavity,
- β_0 = Lorentz factor,
- D_x = horizontal dispersion function,
- Γ_q = skew coupling factor,
- c = speed of light,

θ_b = longitudinal coordinate at the RF cavity, and

θ_q = longitudinal coordinate at the skew quadrupole.

The skew quadrupole field, corresponding to the fourth term in the right hand side of Eq. (2), is needed to combine the vertical motion to the horizontal motion. When the axial length of the skew magnet is L_q , the coupling factor Γ_q can be defined by

$$\frac{\Gamma_q}{R} = \frac{L_q}{B\rho} \left(\frac{\partial B}{\partial y} \right), \quad (3)$$

where B is the magnetic field, and $B\rho$ is the magnetic rigidity. Instead of a skew quadrupole magnet, a solenoid can, of course, also be used for horizontal-vertical coupling [6].

Specifically, the RF potential represented by the last term in Eq. (2) yields a synchrotron coupling, provided that either the function D_x or θ -derivative of D_x is non-zero at the RF cavity location. Although the RF coupling involves not only linear but also nonlinear components, the linear contribution is particularly important. Through this coupling potential, the longitudinal dissipative force due to laser photons can be effectively extended to the transverse degrees of freedom under the condition in Eq. (1). Needless to say, the mixing of the three harmonic motions, i.e. synchrotron motion and betatron motions, is always present unless we turn off the coupling fields. However, when the operating point of a storage ring is far away from the coupling resonance, no dramatic cooling effect will take place in the transverse directions.

B. The coupling-cavity scheme

There is an alternate way to artificially generate a strong synchrotron coupling. It is mathematically evident, from the discussion above, that the key is the second order potential proportional to the product $x\psi$ (or $y\psi$). Such a term can be created by means of a special *coupling cavity* excited in TM₂₁₀ mode [6]. Assuming a simple rectangular pill-box cavity of the width of $2a$ and the height of $2b$, the vector potential for this mode is given by

$$\mathbf{A}_c = \left(0, 0, \frac{V_c}{\omega_c} \sin \left(\frac{\pi x}{a} \right) \cos \left(\frac{\pi y}{2b} \right) \sin(\omega_c t + \phi_c) \right), \quad (4)$$

where V_c and ω_c denote, respectively, the voltage and angular frequency of the coupling RF field, and ϕ_c is the initial phase. Equation (4) indicates that, in the paraxial approximation, the longitudinal electric field of the cavity certainly contains a linear dependence on the horizontal coordinate x and, thus, can be a source of a linear synchrotron coupling. Neglecting the dispersive effects for the sake of simplicity, the approximate Hamiltonian for the coupling-cavity scheme takes the form [11]

$$H_2 = \frac{1}{2} (p_x^2 + \nu_x^2 x^2) + \frac{1}{2} (p_y^2 + \nu_y^2 y^2) - \frac{\eta W^2}{2} - \frac{h_b q V_b}{p_0 \beta_0 c} \cos \psi \delta_p(\theta - \theta_b) + \Gamma_q x y \delta_p(\theta - \theta_q) - \frac{2\pi h_b \Gamma_c}{h_c} x \sin\left(\frac{h_c}{h_b} \psi\right) \delta_p(\theta - \theta_c), \quad (5)$$

where h_c is the harmonic number of the coupling cavity sitting at $\theta = \theta_c$, and Γ_c is the coupling strength fully dependent on the cavity design. Obviously, the lowest order term of the coupling-cavity potential is proportional to $x\psi$ as requested.

The coupling-cavity scheme has two unique features. Firstly, it is straightforward to develop a direct coupling between longitudinal and vertical direction by means of a *vertical* coupling cavity, i.e. the coupling cavity rotated by 90 degrees around the beam line. Secondly, the transverse cooling rates can be even more enhanced than those of the RF dispersive coupling method discussed in the last subsection. The main reason is that, in the coupling-cavity scheme, the RF dispersive coupling is also there since we always have to turn on a regular cavity as well in order for the resonance condition to be satisfied [12]. Consequently, we can anticipate the couplings originating from two separate sources unless momentum dispersion vanishes at the position of the regular cavity.

C. The width of the coupling resonance

We now briefly investigate the sensitivity of the transverse cooling process to the resonance condition. Considering only the longitudinal-horizontal dispersive coupling for brevity, the cooling rate per revolution can be derived as

$$\Lambda_x = \frac{(1-g)\Lambda}{2} \quad (\text{for the horizontal motion}), \quad (6a)$$

$$\Lambda_s = \frac{(1+g)\Lambda}{2} \quad (\text{for the longitudinal motion}), \quad (6b)$$

where we have assumed smooth longitudinal cooling force which has the friction coefficient Λ ,

$$g = \frac{|\cos(2\pi\nu_x) - \cos(2\pi\nu_s)|}{\sqrt{[\cos(2\pi\nu_x) - \cos(2\pi\nu_s)]^2 + \xi^2 \sin(2\pi\nu_x) \sin(2\pi\nu_s)}} \quad (7)$$

and the coupling constant ξ is defined by

$$\xi^2 = \frac{2\pi h_b q V_b}{p_0 \beta_0 c} \left(\frac{D_b}{R} \right)^2 \quad (8)$$

with D_b being the magnitude of dispersion in an RF bunching cavity. In this analysis, the higher-order terms with respect to ξ^2 and Λ have been dropped. Note that the sum of both rates in Eqs. (6) is equal to Λ because of the conservation of energy.

Equations (6) together with Eq. (7) point out that the horizontal cooling rate grows as the value of ν_s approaches the non-integer part of ν_x . The two cooling rates are equalized when $\cos(2\pi\nu_x) = \cos(2\pi\nu_s)$. The full width half maximum of this resonance peak with respect to ν_s can be obtained as

$$\delta\nu_s \approx \frac{\xi}{\sqrt{3\pi}}. \quad (9)$$

III. A POSSIBLE DESIGN OF THE COUPLING CAVITY

Before proceeding to numerical simulations, we present a possible design of the coupling cavity in this section. The first task is to determine the operating RF frequency. In order to reduce complicated, unnecessary nonlinear effects during the cooling process, the bucket length of the coupling-cavity field may better be shorter than or equal to that of the bunching RF field; namely, $h_c \leq h_b$. Taking practical convenience into consideration, we simply assume $h_c = h_b$ below [13].

The harmonic numbers used for bunched-beam laser cooling in TSR and ASTRID are not so large; typically, less than 30. In the application of the resonant coupling schemes, however, h_b need to be much greater to ensure a rather high synchrotron tune with a very low RF voltage. To be specific, let us here say that V_b of the order of 100 volts or less is desirable, recalling the past cooling experiments in ASTRID [14]. On the other hand, ν_s has to be as large as possible, such that one can have enough clearance from transverse integer stopbands under the condition in Eq. (1). To meet these two contradictory requirements, a beam of lower kinetic energy is preferred. While the minimum distance from the stopbands necessary for beam stability depends on the machine condition, we consider ν_s up to 0.1 to be on a safe side in this paper.

For the pill-box geometry taken into account in Eq. (4), the RF frequency $f_c (= \omega_c/2\pi)$ satisfies

$$\frac{1}{a^2} + \frac{1}{(2b)^2} = \left(\frac{2f_c}{c} \right)^2. \quad (10)$$

We readily recognize, from this equation, that the cavity dimension becomes huge unless a sufficiently high frequency is chosen. In fact, even for a frequency in the range of several hundred MHz, the cavity size is still of the order of 1 m. Assuming $^{24}\text{Mg}^+$ ions stored in the ASTRID ring at the total energy of 100 keV, the harmonic number will then be beyond a few thousand corresponding to the RF bucket length of only less than around 1 cm. Therefore, from a practical point of view, we better devise a special lumped circuit instead of a simple rectangular resonator, setting the design frequency much lower.

FIG. 1. Conceptual layout of a coupling cavity. RF power is supplied directly to the two electrodes by an external circuit. To generate a dipole-like electric field near the beam line, RF voltages of opposite signs are applied to the electrodes.

Designing a proper lumped circuit is actually not too difficult. Noting that the axial electric field of a coupling cavity is proportional to $x \cos(\omega_c t + \phi_c)$ near the beam line, it is expected that the cavity might have such a geometry as to produce a dipole-like field. We thus consider a simple structure as depicted in Fig. 1, which is basically a rectangular box with two electrodes standing across the beam line from each other. The longitudinal dimension of the cavity is roughly equal to $2\beta_0 c/f_c$. To excite the coupling field, the electrode poles are connected to an external circuit feeding RF power at the design frequency. The frequency is simply determined by the external electronics devices.

To show an explicit design of a coupling cavity, we again consider the ASTRID ring storing a 100 keV $^{24}\text{Mg}^+$ beam. As an example, we choose here $h_c = h_b = 260$, which is ten times larger than that of the usual ASTRID operation. The corresponding RF frequency is 5.78 MHz. At this harmonic number, the RF voltage required for $\nu_s = 0.1$ is only about 37 volts. In order to precisely analyze the electromagnetic field distribution of the coupling cavity, it is indeed necessary to employ a 3D Maxwell equation solver. However, since the cavity dimension is much smaller than the RF wave length, we can approximately treat the case as a static problem. We are thus allowed to use, e.g., the POISSON code [15] for a quick evaluation of the coupling field. Figure 2 displays a possible coupling-cavity design, where we have assumed that the structure is spatially homogeneous in the vertical direction. In deciding the geometrical parameters, we particularly demanded the following points: (a). the cavity size is modest, (b). the coupling coefficient Γ_c is as large

FIG. 2. A possible coupling-cavity design. One quarter of the cavity has been drawn. The solid curves inside the cavity represent the equi-potential lines obtained from the POISSON code, and the small arrows show the directions of the electric field.

as possible, and (c). the field disturbance by the end plates and the beam holes is negligible.

The use of the field data in Fig. 2 enables us to estimate how much coupling can be generated at a specific amount of supplied power. To evaluate the net effect of the cavity field on particle orbits, we introduce the x -dependent functions

$$\bar{V}_x = - \int_{-L_c/2}^{L_c/2} \frac{\partial V}{\partial x} \cos\left(h_c \frac{s}{R}\right) ds, \quad (11a)$$

$$\bar{V}_s = - \int_{-L_c/2}^{L_c/2} \frac{\partial V}{\partial s} \sin\left(h_c \frac{s}{R}\right) ds, \quad (11b)$$

where V is the scalar potential calculated from POISSON, L_c denotes the axial length of the cavity, and the origin of the longitudinal coordinate is located at the center of the cavity. These integrated-field functions for the geometry in Fig. 2 are plotted in Fig. 3 where the electrode voltage has been assumed to be ± 1 volt. Apparently, the longitudinal field component of the present coupling cavity geometry is linearly dependent on the horizontal coordinate x in the vicinity of the beam line as consistent with Eq. (4). Expanding Eqs. (11) into power series as

$$\bar{V}_j = V_c \left[\zeta_{j0} + \zeta_{j1} \left(\frac{x}{d}\right) + \zeta_{j2} \left(\frac{x}{d}\right)^2 + \zeta_{j3} \left(\frac{x}{d}\right)^3 + \dots \right] \quad (j = x \text{ or } s), \quad (12)$$

where d is the scaling constant chosen here to be 1 cm, the expansion coefficients are found as in Table I demonstrating that ζ_{s1} is most dominant. (The constant offset ζ_{x0} has only little effect on the beam dynamics.) In the present design, the distance between the electrodes has been chosen rather large to eliminate possible aperture

FIG. 3. The effective fields of the coupling cavity defined by Eqs. (11). The inter-electrode voltage has been set at 2 volts.

TABLE I. The expansion coefficients in Eq. (12) where the cavity geometry exhibited in Fig. 2 has been assumed.

	$V_x(j=x)$	$V_s(j=s)$
ζ_{j0}	-6.7606×10^{-2}	5.7765×10^{-4}
ζ_{j1}	6.7108×10^{-4}	2.8741×10^{-2}
ζ_{j2}	-6.4157×10^{-3}	-4.1517×10^{-3}
ζ_{j3}	-4.2928×10^{-6}	2.8549×10^{-3}

limitation. If the inter-electrode distance can be shortened, it is, of course, possible to increase the value of ζ_{s1} much more.

IV. SAD TRACKING RESULTS

In general, ion beams circulating in storage rings are quite hot and thin in all three degrees of freedom, which implies that intra-particle collisions through the Coulomb fields are initially negligible. For the study of this high-temperature regime, the MD approach is not very useful as it often needs a large amount of computing time. Instead, the tracking code “SAD” [9] is more suitable for systematically exploring the behavior of usual hot beams. In order to incorporate the effect of coupling cavities into SAD, we here employ the effective fields in Eq. (12) under the thin lens approximation. Furthermore, for the laser-cooling process, we have added another subroutine where a longitudinal dissipative force is applied to simulation particles in such a way as schematically expressed in Fig. 4. For the width of the laser cooling band Δw , we input some specific value generally much smaller than the RF bucket height. (Δw is associated with the limiting temperature reachable in the simulation.) The cooling band sits near the top of the bucket at the beginning and

FIG. 4. Longitudinal laser cooling model employed in the SAD code. $\delta p/p$ is the momentum deviation from the design value.

TABLE II. The main ASTRID parameters in the normal operating mode

Quantity	Value
Ring Circumference $2\pi R$	40 m
Betatron tunes (ν_x, ν_y)	(2.29, 2.73)
Dipole bending radius	6.366 m
Lattice superperiodicity N_l	4
Transition energy γ_T	4.58
RF harmonic number	26

is then scanned down to the $\delta p/p = 0$ axis at an adequate speed. During this scanning process, ions which happen to be inside the band when traversing the cooling section lose energy by the amount of Δw . In this way, the beam is eventually compressed into a tiny region around the origin unless the scanning speed of the cooling band is too fast compared to the synchrotron oscillation cycle.

Let us first study the ordinary operating mode of ASTRID where $\nu_x = 2.29$ and $\nu_y = 2.73$ (See Table II). The stored particles are 100 keV $^{24}\text{Mg}^+$ ions which have been actually used in recent cooling experiments in ASTRID. The harmonic number of the accelerating cavity is 26. The RF voltage is linearly increased from 0 to 10 volts within the first 10^3 turns in order to adiabatically capture the initial continuous beam into the RF buckets. After this beam capturing procedure, the laser cooling band is scanned in the subsequent 10^5 turns. We then obtain Fig. 5(a) in which no transverse cooling is visible. Now, we strengthen the quadrupole fields to let $\nu_x = 3.11$ and $\nu_y = 3.09$. A pure skew quadrupole magnet with $\Gamma_q = 0.48$ is also added to couple the horizontal and vertical motions. Moreover, we adjust the synchrotron tune to 0.1, increasing the harmonic number to 260. The SAD result based on this resonant situation is given in Fig. 5(b). The effectiveness of the coupling scheme is evident.

To figure out the dependence of cooling property on the resonance condition, we perform a series of simulations varying the synchrotron tune over a wide range.

FIG. 5. SAD simulation results. The time evolutions of root-mean-squared (rms) beam emittances are plotted. The case (a) corresponds to the ordinary operating mode of ASTRID while the lattice parameters have been modified in the case (b) such that the resonance conditions in Eq. (1) are roughly satisfied. Notice that the cooling process can be shortened in the case (b) because of the high synchrotron tune.

The lattice condition is taken exactly the same as that of Fig. 5(b). As indicated by the solid curves in Fig. 6, final transverse emittances rapidly grow in the region $\nu_s \leq 0.05$ due to the reduction of cooling efficiency. The approximate cooling rates evaluated from SAD simulations are also displayed in Fig. 7 where the great enhancement of transverse dissipation around the coupling resonance has been verified. The resonance width estimated by Eq. (9) is about 0.033, which agrees fairly well with this tracking result.

The coupling potential never disappears as long as an RF cavity is sitting at a dispersive position. In fact, the transverse cooling rates are quite low but still finite even if the working point is well distanced from the resonance. Therefore, we more or less expect the effect of synchrotron coupling in bunched-beam operation.

Fortunately, one can further enhance the transverse cooling efficiency in the off-resonant region by switching on a coupling cavity. The dotted curves in Fig. 6 correspond to the cases where the coupling cavity has been excited with different RF powers. We clearly see the remarkable improvement of final emittances. Concerning the electrode voltage, V_c of less than a few hundred volts seems to suffice for achieving the transverse temperatures

FIG. 6. The ratio of final to initial rms emittances vs. operating synchrotron tune. The solid curves represent the case where only the dispersive RF coupling is present. We have also tried three different cases of the coupling-cavity scheme (the dotted curves); i.e., $V_c = 348$ volts (open circles), $V_c = 696$ volts (triangles), $V_c = 1044$ volts (squares).

FIG. 7. Approximate cooling rates evaluated from SAD simulations where no coupling cavity has been activated. The simple linear friction model has been used. The horizontal and vertical cooling rates are identical.

comparable to the longitudinal final value. Note here that the value of V_c does not represent the actual potential felt by stored ions. For instance, when $V_c = 500$ volts, the maximum linear coupling potential at the horizontal position $x = 1$ cm is only 14.37 volts according to Table I. As the transverse dimension of a cooled beam is gener-

ally even much smaller than 1 cm, the coupling-cavity parameters considered in Fig. 6 should be practicable.

V. MOLECULAR DYNAMICS RESULTS

We are now ready to examine the effect of space charges on the cooling properties, employing the MD code “SOLID” [16]. In this code, we numerically iterate the equations of motion, incorporating the characteristics of actual storage rings like bending and straight sections, and AG focusing, as we did in the SAD simulations. Ewald-type [17] summation is performed in the azimuthal direction to evaluate the long-ranged Coulomb forces among particles and their image charges modeled in periodic “supercells” for computing efficiency. In the following MD simulations, we always set the initial beam temperatures to be a few hundred Kelvin in all three directions since this range of temperature can be accomplished, e.g., with the electron cooling technique. Furthermore, for the sake of reducing the computing time, a simplified cooling model is adopted; namely, we use the linear transformation

$$\left(\frac{\delta p}{p}\right)_{\text{out}} - \left(\frac{\delta p}{p}\right)_{\text{in}} = \lambda_s \left(\frac{\delta p}{p}\right)_{\text{in}}, \quad (13)$$

where $(\delta p/p)_{\text{in}}$ and $(\delta p/p)_{\text{out}}$ are the momentum deviation before and after the cooling section, and λ_s is a certain constant associated with the cooling strength. While the model is rather crude, no fundamental difference from SAD results has so far been identified in the high temperature regime.

The first example is the resonant ASTRID lattice studied in Section IV. Figure 8 illustrates the ν_s -dependence of the final beam temperature where the total number of stored $^{24}\text{Mg}^+$ ions is 2.6×10^4 . It has been demonstrated that the present coupling method works excellently, achieving the transverse temperatures of the order of 1K. Of particular interest is that the possible lowest temperature is almost independent of the synchrotron tune in this ultra-cold range where Coulomb coupling must be quite effective. At high temperature, however, the transverse cooling process is extremely slow unless the synchrotron tune is near the resonant value. As a matter of fact, to save the computing time when $\nu_s < 0.05$, we have been forced to employ the RF voltage of the optimum resonant value first until the beam gets sufficiently cold. (The voltage is adjusted to a proper value after the transverse temperature has reached around a few tens of Kelvin.)

As is well-known, the lattice of a storage ring suitable for beam crystallization must fulfill the so-called *maintenance condition* given by [18]

$$\max(\nu_x, \nu_y) < \frac{N_l}{2\sqrt{2}}, \quad (14)$$

FIG. 8. Equilibrium beam temperature vs. operating synchrotron tune, obtained from SOLID. The same resonant ASTRID lattice as employed in Fig. 6 has been considered; namely, $\nu_x = 3.11$, $\nu_y = 3.09$, $\Gamma_q = 0.48$, and $h_b = 260$. The coupling cavity has been switched off. The cooling strength parameter is $\lambda_s = 0.9$. The minimum transverse temperature reached with the coupling method is certainly much lower than that experimentally achieved in ASTRID. Notice also that there still exists significant temperature imbalance between the longitudinal and transverse degrees of freedom.

where N_l is the lattice superperiodicity [19]. Since neither ASTRID nor TSR meet this requirement, we here take, among a wide range of choice, the lattice parameters of TARN II [20] to investigate the role of the maintenance condition. The TARN II ring, whose circumference is 77.7 m, basically has a sixfold symmetry, and each lattice period contains four bending magnets within which the orbit curvature is 4.01 m. For vertical-horizontal coupling, we put a skew quadrupole magnet in every period, keeping the high superperiodicity. The coupling constant defined by Eq. (3) is 0.0124. Other main parameters used in the present TARN II simulations are summarized in Table III. In order for the betatron tunes to satisfy Eq. (14) and the resonance condition simultaneously, we change the gradients of the focusing quadrupole fields such that $(\nu_x, \nu_y) = (2.096, 2.104)$. Then, according to Eq. (1), the synchrotron coupling could be

TABLE III. The main TARN II parameters

Quantity	Value
Ring Circumference $2\pi R$	77.7 m
Betatron tunes (ν_x, ν_y)	(2.096, 2.104)
Dipole bending radius	4.01 m
Skew quadrupole strength	0.001 m^{-1}
Lattice superperiodicity N_l	6
Transition energy γ_T	2.258
Ion species	$^{24}\text{Mg}^+$
Kinetic energy	1 MeV
RF harmonic number ($h_b = h_c$)	1000

FIG. 9. Equilibrium beam temperature vs. operating synchrotron tune, obtained from SOLID. The TARN II parameters listed in Table III have been employed. The upper and lower figures show the equilibrium temperatures achieved by the dispersive coupling scheme and by the coupling-cavity scheme respectively. Here, λ_s has been set at 0.9, and the total stored ions are 10^5 .

most strengthened when $\nu_s \approx 0.1$.

The beam temperatures achieved in TARN II by means of the coupling method are plotted in Fig. 9. We see that the transverse temperatures have now become more than two orders of magnitude lower than those in the ASTRID case. Moreover, the strange temperature anisotropy observed in ASTRID has disappeared. The beam temperatures reached in these numerical experiments already go beyond the border where the phase transition of fast stored beam might begin to occur. Figure 10 displays the real-space profile of a $^{24}\text{Mg}^+$ beam which has arrived at an equilibrium state in TARN II. As anticipated, a 3D ordered structure has been formed.

Figure 9 also indicates that, similar to the ASTRID case in Fig. 8, a higher transverse cooling rate does not always yield a lower beam temperature. The equilibrium temperature tends to be even lowered at a smaller synchrotron tune, regardless of the fact that the transverse cooling efficiency certainly becomes much worse in the off-resonant region. Furthermore, it turns out that the coupling cavity does not improve the final temperature although it significantly shortens the cooling time. These results strongly suggest that transverse cooling rate itself may not be of essential importance for cold beams, at least, from the viewpoint of reachable temperature.

FIG. 10. Real-space profiles of a 1 MeV $^{24}\text{Mg}^+$ beam laser-cooled with the dispersive-coupling scheme in the TARN II ring.

In order to confirm this result, we explored cooling efficiency as functions of cooling power. We obtained the λ_s -dependence of minimum beam temperature as illustrated in Fig. 11. There is a sudden jump around $\lambda_s \approx 0.3$ [21]. Once λ_s exceeds the threshold value, the equilibrium temperature stays at an almost constant level. This observation supports the above expectation that the lowest temperature achievable in a particular direction might have no concern with the cooling rate of that direction as long as the beam is cold. The matter is

FIG. 11. Dependence of equilibrium beam temperature on longitudinal cooling power. The synchrotron tune has been fixed at 0.05, and the total number of stored ions is 10^5 . In the upper picture, only the dispersive coupling in the regular RF cavity has been considered while, in the lower picture, the coupling cavity has been excited simultaneously.

how much *total* cooling power one can apply to a stored beam. Finally, for reference, we show, in Fig. 12, equilibrium temperatures obtained with different number of stored ions. No noticeable dependence of temperature on the total number of ions has so far been found.

VI. DISCUSSIONS

In spite of the past experimental observations in TSR and ASTRID, the MD results given in the last section naturally lead us to the conclusion that longitudinal laser cooling of fast stored beams could be equally efficient in all three degrees of freedom [22]. If so, what has caused such a large imbalance between longitudinal and transverse temperatures in these storage rings? The answer is just behind Eq. (14). As self-consistently proven in Ref. [23], the mean property of a cold circulating beam can be approximately described by the same envelope equations as familiar in the standard theory for linear transport systems. It is straightforward to show that the envelope motion can be unstable due to the Mathieu resonance [23]. Equation (14) is actually nothing but the condition for extremely space-charge-dominated beams to avoid half-integer Mathieu stopbands [24]. Recalling that the maintenance condition has been broken in

FIG. 12. Equilibrium beam temperature vs. total number of stored ions at $\lambda_s = 0.1$ (upper) and 0.9 (lower). The TARN II parameters given in Table III have been assumed again.

both TSR and ASTRID, high-density ion beams laser-cooled in these machines are always inside a Mathieu stopband and, therefore, simply unstable transversely. Consequently, the transverse equilibrium temperatures can never be reduced to the TARN II level [25,26]. During the present MD study, we have also recognized that, unlike the TARN II case, the time-evolution of beam temperature in ASTRID is often quite spiky since captured ions tend to escape from the RF buckets. The chief origin of this spiky behavior may be attributed to the Mathieu instability rather than to binary Coulomb collisions. Hence, the temperature anisotropy experimentally detected in TSR and ASTRID should not be interpreted as an evidence that transverse motion of fast stored beams can only weakly be cooled by a longitudinal laser light.

In principle, a storage-ring system for low-energy ions is roughly equivalent to an RFQ ion-trap system, except for the existence of momentum dispersion induced by bending magnets. The contribution from dipole fields is actually almost separable in the stationary state of space-charge-dominated beams [23]. Although beam confinement field is time-independent in a storage ring, stored ions just feel an RF force as they are circulating around the system at great speed. Accordingly, what can happen in an RF trap could also happen in a storage ring. Following this understanding, it becomes even more apparent that the longitudinal cooling force could sympathetically produce strong transverse friction in cold beams. In fact,

we only need a single laser to make a crystal in an RFQ ion trap. It is, furthermore, suggestive that the stability of crystals in RF trap systems has been studied with a Mathieu-type equation [27].

The dispersive effect peculiar to storage-ring systems, however, imposes a special demand upon the nature of cooling force for stabilizing crystals. Since the revolution frequencies of all ions forming a crystalline beam are identical in order to maintain its ground-state structure, longitudinal laser must provide such a cooling force as to give a greater average velocity to a radially outer particle. This is referred to as *tapered cooling* [18,28]. If a too powerful *un-tapered* laser was applied, multi-dimensional crystalline structures would be easily destroyed since it simply tries to equalize the longitudinal velocities of all particles.

We now try to give a possible explanation as to why there is a jump of equilibrium temperature in Fig. 11. The heating-rate diagram shown in Fig. 13 is helpful for this purpose. Interestingly, the basic feature of the diagram is insensitive to the lattice design, stored ion species, beam energy, etc., according to the past MD studies [29]. For an ideal storage ring free from field imperfections, the peak heating rate $\tau_{\max}$ due to random Coulomb collisions has been found to be typically of the order of 0.1. To attain a crystalline beam, we must overcome this peak. When the maximum heating rate exceeds the cooling efficiency available, we will inevitably hit the right side of the heating-rate mountain in Fig. 13 and can never go beyond the peak to the other side. The achievable beam temperature is then higher than T_{peak} . On the other hand, we can reach a crystal level of temperature whenever the cooling rate per turn is greater than $\tau_{\max}$. This is the reason why the sudden change of equilibrium temperature occurs at a specific value of λ_s . As for the minimum cooling power required for beam crystallization, we need to check out the product of beam temperature and the heating rate of that temperature. This product usually becomes maximum at around $T \approx T_{\text{peak}}$. The energy removed by a single traversal of cooling section should thus be greater than, at least, $\tau_{\max} T_{\text{peak}}$ [K]. Since T_{peak} is 1 ~ 10 [K] and $\tau_{\max} \approx 0.1$ in most cases, the cooling power of 0.1 ~ 1 [K/turn] might be necessary for our final goal.

Finally, we again emphasize the practical importance of the resonant coupling method explored in this paper. Since the method does not rely on the mutual particle interactions, one can readily bring any high-temperature beam to an ultra-cold state within a very short cooling time, independent of the ion density. As for the sympathetic cooling, initial stored beam must be already dense enough to develop strong Coulomb coupling. Otherwise, the cooling time may become unreasonably long or it may be simply impossible to cool betatron motions. Therefore, we will need the artificial coupling in practice to instantly reduce the transverse temperature of initial hot beams. As demonstrated in the present MD simulations, it will be possible to realize a Coulomb ordering by means

FIG. 13. Schematic drawing of a typical heating-rate curve.

of the resonant coupling method if the storage-ring lattice is properly designed [30].

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- [30] Since 1D string structure has no transverse extent, it is ideally free from the Mathieu instability. It may thus be possible to create string crystals even in ASTRID and TSR. However, considering that sympathetic cooling is extremely slow or simply negligible for such a low density beam, the resonant coupling method must be applied.