Dbrane boundstate wavefunctions

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Abstract

A simple WKB approximation gives explicit information about D0brane boundstate wavefunctions, suggesting that at large $N$ each individual D0brane has a wavefunction $\exp(-c_{r^9/2}N^{-1/2})$. Thus the velocity dependent interaction energy $v^4r^{-7}$ leads to an effective confining potential that grows as $r^7$.

Dbrane boundstate wavefunctions play a prominent role in recent advances in non-perturbative string theory [1]. The form of these wavefunctions seems to be quite elusive, though the existence of the boundstates has been proved in some cases cases [2,3]. Polchinski has obtained results about the boundstate wavefunctions by other methods [4].

I show in this note that a standard physics calculation gives a surprisingly simple intuitive form for the wavefunction. The $N$ dependence of this wavefunction is automatically consistent with holography, as pointed out in section 7 of [1] by a scaling argument, but the explicit form of the wavefunction we will find below appears to be new. Important earlier work that is relevant background for the simple analysis presented here is that of Danielsson, Ferretti and Sundborg [5], and Kabat and Pouliot [6]. These references studied the problem by quite different methods. While there is some similarity to what we will find, the form of the wavefunction found here does not appear in these papers [5,6].

I will consider only D0branes in the following. $N$ static D0branes preserve half the supersymmetries of type IIA string theory so their energy is independent of their relative positions. Consider a configuration of $N$ D0branes at position 0 and 1 D0brane at position $r$ along the $x^1$ axis. We want to compute the matrix element

$$\langle N, x^1 = 0; 1, x^1 = r| \exp(-HT)|N + 1, x^1 = 0\rangle$$

where $H$ is the Hamiltonian of the system. We expect on elementary grounds that for large $T$ this matrix element should be dominated by the state(s) $|\psi_0\rangle$ in the Hilbert space with vanishing energy, giving us some insight into the overlap $\langle \psi_0|N + 1, x^1 = 0\rangle$ relative to the overlap $\langle \psi_0|N, x^1 = 0; 1, x^1 = r\rangle^*$.

On the other hand, we can compute eq. (1) by using Euclidean functional integrals, evaluated at saddle-point trajectories that take us from one point in the classical configuration space $C_1 \equiv (N, x^1 = 0; N_1, x^1 = r)$ to another point $C_2 \equiv (N + 1, x^1 = 0; N_1 - 1, x^1 = r)$. Both these configurations have the same classical energy, so this problem is similar to familiar ‘tunneling’ calculations. Of course, there is a crucial difference in that there is actually no ‘potential’ energy in the system. The only energy of interaction that appears in the system is when the D0branes have non-zero relative velocities, but since there must be some non-vanishing relative velocity for motion from $C_1$ to $C_2$, this interaction energy is non-vanishing and leads to a simple explicit result. As I shall show, the velocity-dependent interaction energy leads effectively to a wavefunction similar to that of a particle moving in a potential
Thus the potential is much flatter than that of a harmonic oscillator at small values of $r$ and much steeper at larger values of $r$.

The action of a test D0brane in the configuration $C_1$ takes the form

$$S \approx \int dt \left[ \frac{1}{2g} \dot{x}^2 + \frac{15N}{16} \frac{\dot{x}^4}{r^7} \right] \equiv \int dt \left[ \frac{1}{2g} \dot{x}^2 - V \right],$$

where $r$ is the distance of the test D0brane from the cluster of D0branes. This action is only valid when $r$ is large enough, and velocities small enough, but we are interested in the Euclidean action which has the same form with $15/16 \rightarrow -15/16$. The Euclidean motion then corresponds to a repulsion from the cluster of D0branes, so the velocity of the test D0brane vanishes as it approaches the cluster. It is a non-perturbative consequence of supersymmetry that the force must vanish in the limit of vanishing velocity, so we may be able to trust the action for the Euclidean motions of interest even though we cannot trust it for general Minkowski motions. This is nevertheless a point that needs to be carefully considered since the supersymmetry itself is not easily continued to Euclidean signature. The conserved quantity associated with Euclidean motions is $\epsilon_E = \frac{1}{2g} \dot{x}^2 + 3V_E$. The Euclidean action for a solution of the equations of motion with the desired boundary conditions is then

$$S_E = T\epsilon_E + 2 \int V_E dt.$$  

$\epsilon_E = 0$ for the motion with the smallest Euclidean action, so we see that

$$\dot{x}^2 = \frac{45N}{8} \frac{\dot{x}^4}{r^7}$$

which in particular shows that as the test D0brane approaches the cluster of zero-branes its velocity decreases as $r^{7/2}$. Finally

$$S_E = \int_0^r dx \frac{2\sqrt{2}}{9\sqrt{15gN}} x^{7/2}dx \propto r^{9/2}N^{-1/2}.$$  

The determinants for fluctuations about this solution should (mostly) cancel due to supersymmetry, so we are left with

$$\langle \psi_0 | N, x^1 = 0; 1, x^1 = r \rangle \propto \exp \left( -cr^{9/2}N^{-1/2} \right).$$

The coefficient $c$ in eq. (6) depends on higher order terms which could be included in eq. (2) but the important point is that for large $N$ the explicit wavefunction shows that the test D0brane is essentially confined to a flat box of size $N^{1/9}$. This power of $N$ is as expected from the discussion in section 7 of [1]. However, the power of $r$ in eq. (6) was not explicitly computed in [1]—it is amusing to find that a velocity dependent interaction energy that falls off as $v^4r^{-7}$ leads to a confining potential that grows as $r^7$. The picture is therefore in accord with a bag model of gravitons with D0branes as constituents, the parton intuition given in [1]. The elementary analysis presented here is somewhat different from the sophistication of other approaches [2,3], but it is hoped that some intuition into the structure of Dbrane bound states can be gained from extensions of this calculation.
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