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soumis à Physics Letters

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GANIL P 98 29

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The role of isospin as a degree of freedom and as an observable in heavy ion reaction experiments is analyzed in the theoretical framework of an isospin dependent Lattice Gas Model. We show that the critical temperature is only slightly dependent on the isospin asymmetry and finite size effects go in the direction of increasing the τ critical exponent respect to the asymptotic limit of the three dimensional Ising universality class. An interesting signature of the liquid-gas phase transition and a possible observable to localize the multifragmenting system in the $T - \rho$ plane is given by the isotopic composition of fragments. We show that in the coexistence region the vapor fraction is more asymmetric than the liquid fraction and that this signal should not be washed out by secondary decays.

It is generally believed that heavy ion collisions may provide an experimental tool to explore the properties of nuclear matter under different thermodynamical conditions. Then an important experimental issue is to extract informations on the nuclear equation of state (EOS) from reaction data. In particular, one of the most important challenges of heavy ion physics at intermediate bombarding energies is the identification and characterization of the expected nuclear liquid-gas phase transition.

Numerous studies of the transition have been performed using only one type of particles. This is the case of mean field studies of spin and isospin saturated systems as well as percolation and lattice gas simulations using a unique type of site occupancy [1-3]. However, since nucleons are characterized by several quantum numbers, a more realistic approach should be to consider phase transitions in systems with several conserved charges. Concerning the isospin degree of freedom this might be an important issue since most of the multifragmenting systems produced by heavy ion collisions are far from being isospin symmetric. From a theoretical point of view it is well known that a system with several conserved charges may present different phase transition phenomenologies. Mean field calculations for nuclear systems with isospin and baryon number degrees of freedom [5,6] show that indeed the nature of the liquid-gas phase transition can be modified: the transition can be continuous, phase separation can be driven by isovector as well as isoscalar instabilities, the value of the critical temperature is dependent of the considered asymmetry. However as far as phase transitions are concerned the mean field approximation is known to be a rather poor approach and one may worry about the validity of these results. Moreover mean field approaches do not give access to the actual partitions while fragment observables are an interesting tool to study phase transitions.

Because of the universality of second order phase transition the general features of the observables and in particular the value of the critical exponents do not depend on the details of the interaction but only on general prop-

erties such as the dimensionality of the space and of the order parameter. This is why generic informations on the transition can be obtained within schematic models as the Lattice Gas Model [1-4]. Within this framework we shall show that the critical parameters of the phase transition depend only very slightly on the asymmetry and that this dependence is probably non detectable for experimentally available N/Z ratios. On the other hand the phase coexistence constraints in a binary fluid imply very specific features for the fragment isotopic distribution; the isospin degree of freedom can be therefore viewed as an extra observable that, together with size distributions, can help to assess nuclear thermodynamics by localizing the system in the temperature - density plane and providing other signals of the occurrence of the transition.

The Lattice Gas Model of Lee and Yang [7], where the grandcanonical partition function of a gas with one type of atoms is mapped into the canonical ensemble of an Ising model for spin 1/2 particles in a magnetic field (dipolar interaction), has successfully described the liquid-gas phase transition for atomic systems. This same model has already been applied to nuclear physics for isospin symmetric systems in the grandcanonical ensemble [1,3], with an approximate sampling [8] of the canonical ensemble [2,4], and for isospin asymmetric infinite nuclear matter in the mean field approximation [6]. The isospin dependent Lattice Gas Model is isomorphous to a spin 1 Ising model with dipolar and quadrupolar interactions [9]. The n sites of the lattice are characterized by an occupation number τ which is defined as $\tau = 0$ for a vacancy, and $\tau = 1(-1)$ for a proton(neutron). The magnetization $M = \sum_{i=1}^n \tau_i/n$ and quadrupolar moment $Q = \sum_{i=1}^n (\tau_i)^2/n$ of the system have here the physical interpretation of isospin asymmetry y and density ρ via

$$\frac{\rho}{\rho_0} = \frac{A_0}{n} = Q; \quad y = \frac{N_0 - Z_0}{A_0} = \frac{M}{Q} \quad (1)$$

where N_0, Z_0, A_0 are the neutron, proton and mass number of the system and ρ_0 is normal nuclear matter dens-

ity. Particles occupying nearest neighboring sites interact with an energy $\epsilon_{\tau_i, \tau_j}$ according to the hamiltonian

$$H = \sum_{i=1}^n \frac{p_i^2}{2m} (\tau_i)^2 + \sum_{i \neq j} \epsilon_{\tau_i, \tau_j} \tau_i \tau_j$$

$$= \sum_{i=1}^n \frac{p_i^2}{2m} Q_i - \sum_{i \neq j} \frac{\epsilon_d + \epsilon_s}{2} Q_i Q_j - \sum_{i \neq j} \frac{\epsilon_d - \epsilon_s}{2} \tau_i \tau_j \quad (2)$$

For our applications we have fixed the coupling constants $\epsilon_{\tau_i, \tau_j}$ to nuclear phenomenology. The choice $\epsilon_{1, -1} \equiv \epsilon_s = 5.5$ MeV guarantees the correct saturation properties of nuclear matter, while $\epsilon_{1, 1} = \epsilon_{-1, -1} \equiv \epsilon_d = 0$. MeV is fixed such as to reproduce the symmetry coefficient of the liquid drop model parameterization in the mean field approximation.

In the calculations shown below the numerical realization of the model is a three-dimensional cubic lattice with periodic boundary conditions characterized by a linear size $L = 8$, a density ρ , a concentration y and a temperature T . Statistical averages are taken over 10000 events obtained with a Metropolis sampling of the lattice in the canonical ensemble. The accuracy of the energy sampling is estimated to be less than 0.1 MeV [10].

The definition of two possible order parameters by eq.(1) implies the potential existence of tricritical points (i.e. phase transition critical points of third order) as in the case of the normal to superfluid transition of $^3\text{He}-^4\text{He}$ binary mixtures [11]. However this is not the case in nuclear physics since the interaction ϵ_s between protons and neutrons is more attractive than the interaction ϵ_d between protons and protons or neutrons and neutrons, i.e. no phase separation between protons and neutrons is possible. In this sense at most a second order phase transition is expected. The density difference between the liquid and the vapor phase $\rho_L - \rho_G$ is the only relevant order parameter, and we expect the system to belong to the same universality class than in the homogeneous case and therefore observe the same critical exponents for any value of y . Finite size effects can however act differently in the two models; as an example it has recently been shown [12] that the two fluid model has smaller residual corrections to scaling on most observables respect to the standard Ising model.

The critical exponents as well as the critical temperature can be extracted from the cluster size distribution at a given density by means of finite size scaling [13]. The definition of clusters in the lattice gas combines a site and bond percolation algorithm [2,3]: two particles are bonded if they a) occupy neighboring sites and b) the kinetic energy of their relative motion $\vec{p}_r^2/2\mu$ does not exceed the binding energy $\epsilon_{\tau_i, \tau_j}$. This definition of clusters (equivalent to the Coniglio-Klein algorithm [14] if the kinetic distribution is Maxwellian [3]) guarantees that for an infinite system the thermodynamical critical point exhibits critical fragment size distributions.

The critical exponents (e.g. τ, σ) and temperature T_c are extracted from the simulated events by using the fact that at any density ρ [10,15] the cluster size distribution scales as

$$\frac{dN}{dA}(A, T) = A^{-\tau} f(A^\sigma (T - T_c)) \quad (3)$$

where f is a scaling function.

FIG. 1. Critical parameters as a function of the isospin asymmetry of the system for a cubic lattice with $A_0 = 256$ particles at $\rho = \rho_0/2$. Open symbols: isospin independent interaction.

Finite size scaling as expressed by equation (3) can be inferred from renormalization arguments only for the infinite system and in the close proximity of the critical point : in this case the critical exponents are given by the Ising universality class ($\tau = 2.2088, \sigma = 0.6397$ [12]). In finite systems in principle this equation should be modified. However, introducing a finite size correction to the critical parameters, eq.(3) is expected to be still valid for a limited range of fragment sizes. It was shown [10,15] that for a system of typical nuclear sizes eq.(3) describes rather well the intermediate mass fragment (IMF) production. Moreover finite size scaling was observed [10,15] over a wide range of densities $0.01 \leq \rho/\rho_0 \leq 0.99$ so that a complete line of critical temperatures $T_c(\rho)$ for IMF production is expected. In the present paper we shall restrict the considered densities to $\rho = \rho_c = \rho_0/2$ in order to concentrate on the isospin degree of freedom.

The extraction of critical parameters from the size distribution proceeds as follows. The production yield of

each species of size A as a function of temperature is seen to pass through a maximum $dN_{max}(A)$ at a temperature $T_{max}(A)$ which corresponds to a maximum of the scaling function f . The double logarithmic slope of dN_{max} as a function of A is then identified as the τ exponent; the critical temperature is obtained as the temperature at which the power law fit to the size distribution dN/dA gives the same exponent τ ; another power law fit of the variable $T_{max}(A) - T_c$ allows to get the exponent σ . Finally the quality of the fit is verified by checking that the scaled cluster size distribution $A^\tau dN/dA$ gives a unique function for all sizes $4 \leq A \leq 30$ and temperatures $2\text{ MeV} \leq T \leq 20\text{ MeV}$ when plotted as a function of $(T - T_c)A^\sigma$. For details about the numerical resolution and the intrinsic precision of the method see refs. [10].

The results of this analysis for the critical exponents and temperature are shown in figure 1 at different values of isospin asymmetry. For comparison, the values obtained in a system of the same size with isospin independent interactions $\epsilon_s = \epsilon_d = 5.5\text{ MeV}$ are also given. The same results within error bars are obtained if the parameters are extracted using instead of the total mass of the fragment the total charge or neutron number.

As expected from universality the critical exponents do not depend on the isospin asymmetry. The observed deviation of τ respect to the isospin independent case is due to the different influence of finite size effects. This can be easily understood since in an isospin dependent case the fragments lie close to the bottom of the stability valley $N \approx Z \approx A/2$ while in the isospin independent case no restrictions are expected on the partitioning of A nucleons into Z protons and N neutrons. For $A/A_0 \ll 1$ this reduction of the partitions number is an increasing function of A which means that from a combinatorial point of view it is harder to make a big cluster in the isospin dependent system even if the system is isospin symmetric. The difference vanishes as $A \rightarrow \infty$ but for the finite system IMF size distributions are naturally steeper, *i.e.* τ is larger than the expected value of the Ising universality class. This result is qualitatively in agreement with the findings of ref. [4].

The critical temperature shows an approximate quadratic dependence on asymmetry (lower part of figure 1) similar to the mean field calculations of refs. [5,6] $T_c(y) - T_c(0) \propto y^\xi$ with $\xi = 2$. It should be noticed that since y does not correspond to an order parameter in the considered case the coefficient ξ should not be considered as a critical exponent. Considering the limited variation of y in experiments, even for very exotic nuclei and extremely precise measurements the asymmetry dependence of the critical temperature would be barely visible. On the other hand, the results shown in figure 1 imply that fragmentation data obtained from systems with different asymmetries can be analysed together for a better determination of critical parameters, isospin observables being used to pin down the phase transition

and measure thermodynamical quantities.

FIG. 2. Ratio between neutron N and proton Z number of produced clusters with $N > 40$ as a function of temperature for a fragmenting system with $N_0 = 160$ and $Z_0 = 96$ at $\rho = \rho_0/2$. Upper (lower) part: Coniglio-Klein (Ising) clusters. Left part: interaction energy $\epsilon_s = \epsilon_d = -5.5\text{ MeV}$. Right part: interaction energy $\epsilon_s = -5.5, \epsilon_d = 0\text{ MeV}$.

Indeed isospin can be not only viewed as a degree of freedom but also as an observable: we can hope that next generation detectors for multifragmentation, measuring masses as well as charges of the reaction products, will supply us with richer two dimensional informations reducing the ambiguities of the existing signals for the phase transition. In this context one intriguing possibility is the expectation that for an $y \neq 0$ system inside the coexistence region the vapor phase should be more asymmetric than the liquid phase [5].

The isotopic composition of heavy fragments (*i.e.* of the liquid fraction) is displayed in the upper part of figure 2 as a function of temperature for a system with $y = 0.25$. The horizontal lines indicate the N/Z of the fragmenting source and the line of maximum binding of fragments in the isospin dependent case ($N/Z = 1$). If the interaction energy is assumed independent of isospin (left panel), at all temperatures the distribution is centered on the N/Z of the source. This effect is purely combinatorial and the broadening of the distribution approaching the critical temperature reflects the maximum of fluctuations at the critical point. If symmetry energy is taken into account (right panel), the distribution is centered on a value the closer to stability the higher the temperature.

Remark that the Coniglio-Klein algorithm for cluster definition guarantees that the excitation energy per nucleon is lower than the threshold of particle emission [3,4], i.e. the fragments produced are relatively cold and secondary evaporation is (in average) taken into account. One can then wonder if the observed proton richness (relative to the isotopic composition of the source) of the fragments does not trivially reflect secondary decay. If we construct clusters simply by connecting neighbour occupied sites (Ising clusters), then the average isotopic distribution is given by the lower part of figure 2, which is centered on the combinatorial expectation except at very low temperature. Therefore, even if the cluster proton richness in the model is a genuine sign of phase coexistence one can argue that any model including secondary decay should exhibit the same feature and it may be difficult to disentangle the two effects.

A clearer signal of phase coexistence with two conserved charges can be obtained from the analysis of the vapor isotopic composition. Figure 3 shows the ratio between the production yields of different light isobars as a function of temperature with and without isospin dependent interactions. Due to the lack of quantum nuclear structure in the model one does not expect that the emission probability of a cluster of mass say $A = 3$ in the model really corresponds to the measurable production rate of tritons or helium 3. However since we have chosen isobars with very similar binding energies the production probability ratio should be meaningful. One can see that only at supercritical temperatures the combinatorial value $R = N/Z = 1.67$ is approached, and the trend is opposite respect to the expected effect of secondary decay. As a matter of fact the same trend is visible for Ising clusters (dashed-dotted line) showing that the neutron richness of the vapor fraction is already present in the statistical sampling of configuration space.

In this paper we have shown that the isospin degree of freedom can provide novel observables to assess thermodynamical behaviours in heavy ion collisions.

As a matter of fact most existing exclusive data on multifragmentation [16–18] show a very satisfactory agreement with the thermodynamics of phase coexistence as far as the charge distributions of intermediate mass fragments are concerned, but deviations are systematically seen in the dynamical variables. These deviations can be ascribed to preequilibrium contaminations and/or the superposition to equilibrium of a collective motion, but they can also be interpreted as the signature of a completely non equilibrium process for fragment production [19]. This ambiguity of interpretation is amplified by the fact that two independent non measurable state variables (T, ρ) have to be extracted from the data. It is therefore extremely important to have new independent static observables in order to better constraint statistical models and convincingly establish if statistical equilibrium is effectively reached in the experiments.

FIG. 3. Relative production yield of light isobars as a function of temperature for a fragmenting system with $N_0 = 160$ and $Z_0 = 96$ at $\rho = \rho_0/2$. Full (dashed-dotted) lines: interaction energy $\epsilon_s = -5.5$, $\epsilon_d = 0$ MeV; Coniglio-Klein (Ising) clusters. Dashed lines: interaction energy $\epsilon_s = \epsilon_d = -5.5$ MeV; Coniglio-Klein clusters.

From mean field theory it is known that if a binary mixture is undergoing a liquid-gas phase transition the vapor fraction is more asymmetric than the liquid fraction; our exact lattice calculations on finite systems show that an important neutron excess in the light fragment isotopic distributions could be a signature of the coexistence region which is not washed out by secondary decay. Even if qualitatively this signal has already been experimentally observed [20], for a realistic application to fragmentation data many important physical ingredients have to be added to the model. Surface effects have to be studied by replacing periodic boundary conditions with a confining Lagrange potential, the persistence of signals must be verified respect to the Coulomb interaction, and the possible influence on the results of the chosen thermodynamical ensemble (microcanonical versus canonical) has to be checked [21].

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