Summary on Tau Leptonic Branching Ratios and Universality

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The large samples of τ decays available from CLEO and the four LEP experiment have resulted in new, precise measurements of the leptonic branching ratios of the τ. The experimental techniques to obtain these results are reviewed with special emphasis on the DELPHI measurement. World averages are found to be $B(\tau \rightarrow e\nu\bar{\nu}) = (17.81 \pm 0.06\%)$ and $B(\tau \rightarrow \mu\nu\bar{\nu}) = (17.36 \pm 0.06\%)$. These results are consistent with universality in the charged current couplings to a precision of about 0.25\%. The branching ratio measurements can also be used to constrain the "low energy parameter" $\eta$. It is shown that the sensitivity to $\eta$ depends on details of the momentum acceptance for muon identification in the different experiments. Assuming universality in the couplings, the estimate $\eta = 0.012 \pm 0.024$ is obtained.

1. INTRODUCTION

Univerality in the couplings of the three lepton families to the gauge bosons is a fundamental assumption of the standard model. As any observed deviation from universality would imply the presence of physics beyond the standard model, it is important to test this assumption as precisely as possible. Due to the accurate predictions of the standard model, the only new measurement which has not been discussed in a separate presentation at this conference. Brief mention will also be made of

$$\Gamma(\tau \rightarrow l\nu_\tau\bar{\nu}) = \frac{B_l}{\tau_\tau} = \frac{G^2_l m^5_\tau}{192\pi^2} f(x_l^2) r_{RC}, \quad (1)$$

measurements of the branching fractions $B_e = B(\tau \rightarrow e\nu\bar{\nu})$ and $B_\mu = B(\tau \rightarrow \mu\nu\bar{\nu})$ will, together with the $\tau$ and muon masses and lifetimes, give unambiguous tests of universality in the couplings of all three lepton families to the $W$-boson. In eq. 1, $\tau_\tau$ is the lifetime of the $\tau$-lepton, and the function $f(x_l)$, with $x_l = \frac{m_\tau}{m_\ell}$, is a phase space factor with $f(x_e) = 1$ and $f(x_\mu) = 0.9726$. The factor $r_{RC}$ accounts for radiative corrections, which, for practical purposes can be taken to be of equal magnitude for the rates $\Gamma_e$ and $\Gamma_\mu$.

In the standard model, the coupling $G_{l\tau}$ is given by

$$G^2_{l\tau} = \frac{g^2_l g^2_\tau}{32 m^4_W} \quad (2)$$

and equals the Fermi coupling constant if universality holds. As seen from equation 1, the ratio

$$\frac{B_\mu}{B_e} = \frac{g^2_\mu}{g^2_e} \cdot \frac{f(x_\mu^2)}{f(x_e^2)} \quad (3)$$

eliminates $g_\tau$, giving a direct comparison between $g_e$ and $g_\mu$.

A comparison of $g_\tau$ to the couplings to the two lighter leptons requires $\tau$ and muon masses and lifetimes. Using the analogue of equation 1 for muon decays, $g_\tau$ can be eliminated to give a test of $\tau-\mu$ universality:

$$B_\mu = \frac{g^2_\mu}{g^2_e} \left[ \frac{f(x_\mu^2)r^e_{RC}}{f(x_e^2)r^\mu_{RC}} \right] \frac{m^5_\tau}{m^5_\mu} \frac{1}{\tau_\tau} \cdot \tau_\tau \quad (4)$$

and finally $\tau-e$ universality is tested in confronting:

$$B_e = \frac{g^2_e}{g^2_\mu} \left[ \frac{f(x_e^2)r^\mu_{RC}}{f(x_\mu^2)r^e_{RC}} \right] \frac{m^5_\mu}{m^5_\tau} \frac{1}{\tau_\tau} \cdot \tau_\tau \quad (5)$$

with measurement. Here $x_\mu = \frac{m_\mu}{m_\tau}$. This talk reviews the newest measurements of $B_e$ and $B_\mu$ and uses the results to give estimates of the ratios between the couplings of the weak charged current to the different leptons. Special attention will be made to the new DELPHI result, the only new measurement which has not been discussed in a separate presentation at this conference. Brief mention will also be made of
other ways to test universality of the charged current, comparing the precision of these to the precision obtained with leptonic \( \tau \) decays. Finally a remark will be made concerning the sensitivity of the branching ratio measurements to the "low energy parameter", \( \eta \), and an estimate of this parameter will be made.

2. THE MEASUREMENTS

The world averages for \( B_c \) and \( B_\mu \) are dominated by the results from CLEO and the four LEP experiments. In the following attention will be given to the differences in the techniques used to extract the branching ratios. These differences are a consequence of the much smaller centre of mass energy at CESR compared to that at LEP. The current CLEO and ALEPH best estimates are published [2], [3], while L3, OPAL and DELPHI values are still preliminary. With the separate presentation of the new OPAL \( B_c \) measurement at this workshop [4], all measurements have been presented in this workshop series [5], except the new DELPHI results [6], and the OPAL \( B_\mu \) measurement which was available already last year [7]. Particular attention is thus given to the DELPHI measurements here (sect. 2.3).

2.1. The CLEO measurement

In \( e^+e^- \) collisions at the \( \Upsilon(4s) \) energy, it is difficult to distinguish a produced \( \tau^+\tau^- \) pair from a q\( \bar{q} \) pair unless requirements are made on the decay of at least one of the two \( \tau \)-leptons produced. The CLEO measurement [2], thus makes a selection of 1-prong 1-prong \( \tau^+\tau^- \) decays as a starting point for their analysis. The data are divided into classes \( ab \), where the first \( \tau \) has the decay mode \( \tau_1 \rightarrow a \), and the second \( \tau \) decays as \( \tau_2 \rightarrow b \). It is noted that the number of events, \( n \), in class \( ab \) is given by the product:

\[
B_a \times B_b = \frac{n \times (1-f)}{\varepsilon \times N_{\tau\tau} \times (2-\delta_{ab})}
\]

(6)

Here, \( f \) is the fractional background, \( \varepsilon \) is the efficiency of selection and \( \delta_{ab} \) is Kronecker delta. The impressive number of around three million \( \tau \) pairs produced, \( N_{\tau\tau} \), is determined from the theoretical cross section and the integrated luminosity. The analysis selects \( \tau \) decays with at most one \( \pi^0 \) present and uses the CLEO value for \( B(\tau^\pm \rightarrow \pi^\pm \pi^0) \) as input to perform a simultaneous determination of \( B_c \), \( B_\mu \) and \( B_k = B(\tau \rightarrow \pi(K)\nu) \), as well as the ratios \( B_\mu/B_c \) and \( B_k/B_c \). Due to the very large sample of \( \tau \)-leptons, the analysis obtains world record statistical precision on \( B_c \) and \( B_\mu \). The largest source of uncertainty to the measurements is in \( N_{\tau\tau} \), which has a relative precision of about 1.4%, contributing to the systematic uncertainty in the branching ratio estimates with about 0.7% (relative). This causes the overall uncertainty of the branching ratio measurements to be dominated by their systematic errors. Much of this is in common and cancels in the ratios.

2.2. Measurements at LEP

The high multiplicity of quark jets at LEP makes a generic \( \tau^+\tau^- \) preselection possible, and a simple multiplicity requirement (e.g. the number of charged tracks to be below 7) rejects most q\( \bar{q} \) events. The remaining sample of events is mainly composed of \( e^+e^- \), \( \mu^+\mu^- \) and \( \tau^+\tau^- \) pairs, and appropriately exploiting the presence of at least two unseen neutrinos in the two \( \tau \) decays permits the selection of \( \tau^+\tau^- \) pairs with high efficiency and purity, irrespective of the decay modes. Hence, the following expression can be used for computing the branching ratio:

\[
B(\tau \rightarrow l\nu\bar{\nu}) = \frac{N_l}{N_\tau} \frac{1-b_l}{1-b_\tau} \frac{\epsilon_\tau}{\epsilon_\ell},
\]

(7)

where \( N_\tau \) is the number of \( \tau \) decays preselected, \( N_l \) is the number of identified leptons out of this sample, \( \epsilon_\tau \), \( \epsilon_\ell \) are \( \tau \) and lepton selection efficiencies; and \( b_\tau \), \( b_l \) are background fractions in \( \tau \) and lepton samples. With a completely unbiased \( \tau^+\tau^- \) preselection, the value of \( \epsilon_\tau \) would be irrelevant, and the lepton identification efficiency, \( \epsilon_\ell^d \), as computed with respect to the selected sample of tau pairs could have replaced the total efficiency \( \epsilon_\ell \). However, the effect of a possible bias in the preselection procedure is not negligible and has to be evaluated. It is appropriate to factorize the identification efficiency into \( \epsilon_\ell = \epsilon_\ell^d \times \epsilon_\ell^i \), where \( \epsilon_\ell^i \) is the efficiency of the \( \tau^+\tau^- \) preselection procedure for the decay mode \( \tau \rightarrow l\nu\bar{\nu} \). Then the the effect of the preselection requirements on the bias factor, \( \beta_\ell = \epsilon_\ell^i/\epsilon_\ell^d \) should be evaluated as a
source of systematic uncertainty. The systematic uncertainties in the LEP measurement stem from the precision of cross checks between data and simulation, and are thus statistics driven. Apart from the common uncertainty in the background of the preselection sample, the systematic uncertainties in $B_e$ and $B_\mu$ are mainly uncorrelated. However, as the branching ratios are derived from a common sample of $\tau^+\tau^-$ pairs, there is statistical anticorrelation between the results obtained in a given experiment.

2.3. The DELPHI measurement

This is a brief account of the analysis described in [6]. Correct assignment of the charged and neutral particles to a specific $\tau$ is ensured by dividing the event into two hemispheres by a plane perpendicular to the event thrust axis. The preselection of $\tau^+\tau^-$ pairs is restricted to events where at least one of the two leading charged tracks in each hemisphere have a polar angle between 43 and 137 degrees. Some additional restrictions to fiducial volume are required depending on channel to be identified. To reject $e^+e^-\rightarrow qq$ events, the charged track multiplicity in the event is restricted to be between 2 and 6. Events from two photon interactions are rejected by asking a visible energy of at least 0.175 times the centre of mass energy. Furthermore it is required that the isolation angle between tracks in different hemispheres should be larger than 160 degrees. For events with two charged particles it is required that the missing transverse momentum in the event should be larger than 0.4 GeV/c, and that the acollinearity should be larger than 0.5 degrees.

After this, significant amounts of $e^+e^-$ and $\mu^+\mu^-$ pairs are still present in the sample. These are dealt with by exploiting the fact that much of the energy in the $\tau^+\tau^-$ events is not seen due to the neutrinos. The variables $p_{rad} = \sqrt{p_1^2 + p_2^2}$ and $E_{rad} = \sqrt{E_1^2 + E_2^2}$ are required to be smaller than the beam momentum and beam energy respectively. Fig. 1 shows the distributions in these variable. Backgrounds are measured from data by extrapolation from regions in the cut variables which are dominated by a particular background type.

Having adjusted the backgrounds, remaining discrepancies between data and simulation are assumed to affect the efficiency of the $\tau^+\tau^-$ preselection, and possibly affect the bias factor, $\beta_l$. The dependence of $\beta_l$ on a given selection variable is determined by varying a cut around its chosen value, and a systematic uncertainty is assigned based on the level of discrepancy observed comparing the number of events rejected in the data to the corresponding number from simulation.

The identification of muons is done by requiring hits in the muon chambers, or alternatively, requiring a significant energy deposition in the outermost layer of the hadron calorimeter. The efficiency of these requirements are measured with
respect to each other. Adjustments of the simulation efficiencies were found to be necessary. After these adjustments, good data-simulation agreement is found in the estimated efficiency of the combined requirement, as shown in fig. 2.3. The actual value for the efficiency obtained by this procedure is only valid for muons penetrating the whole detector, and dimuon events were used to verify the overall efficiency.

In order to reduce backgrounds from hadrons further, it was required that the average response per layer in the hadron calorimeter was compatible with a minimum ionizing particle. Further suppression of hadrons with the presence of a $\pi^0$ was ensured by limiting the total electromagnetic energy deposited in an 18 degree cone around the charged particle to 2 GeV. Selecting muons using tight requirements on the muon chamber response gave a very clean sample which was used for direct measurement of the efficiency of all background suppression requirements. The levels of the remaining backgrounds were verified by lifting one cut to get a sample enhanced with a particular background and comparing the effect of all other requirements on the data with the simulation result. Furthermore the muon momentum distribution was studied with and without a specific requirement to check that the data behaved as the simulation. The final muon momentum distribution obtained is shown in fig. 2.3.

In order to identify electrons, pull variables defined from the energy loss in the TPC ($\Pi_{E/dx}^{\pi}$, where the superscript refers to the particle hypothesis), and from the ratio between the electromagnetic energy and the particle momentum ($\Pi_{E/p}$) were defined. Again the redundancy of the two requirements could be used to check the efficiencies of these requirements, though only for momenta between 0.05 × $p_{\text{beam}}$ and 0.5 × $p_{\text{beam}}$. In this region, requiring either the energy loss should be incompatible with that expected for a pion, or a value of $\Pi_{E/p}$ compatible with that expected for an electron ensured a high, even and well controlled efficiency over this momentum range. For higher momenta, the efficiency requirement $\Pi_{E/p} > -1.5$ was checked with a sample of Bhabhas. Finally, all electron candidates should have
an energy loss compatible with that of an electron, by requiring $\Pi_e^e dE/dx$ larger than -2. Fig 2.3 shows the distributions of the relevant identification variables.

Much of the remaining background from hadrons was rejected by requiring that no energy should be deposited beyond the first layer of the hadron calorimeter, and tau decays with the presence of pions were rejected by vetoing candidates where the total electromagnetic energy deposition in a cone around the track exceeded 3 GeV. Here, neutral energy which could stem from bremsstrahlung of the electron, was excluded from the sum. The efficiency of all requirements except the $\Pi_e^e dE/dx$ requirement, could be well measured by choosing a clean sample of electrons, requiring $\Pi_e^e dE/dx > 0$. The efficiency of the $\Pi_e^e dE/dx > -2$ requirement was measured using bhabhas. Finally, to reject $e^+ e^- \rightarrow (e^+ e^-) e^+ e^-$ further, events where the momenta of both leading tracks were below $0.2 \times p_{beam}$, and compatible with electrons were discarded from the sample. As fig 5 shows, the resulting momentum distribution of the final sample of electron candidates is well described by simulation.

Table 2.3 summarizes the numbers entering the computation of the branching ratios for 93-95 data. For the $B_e$ measurement, the largest source of systematic errors is due to the uncertainty in the identification efficiency estimate, while the systematic error for the $B_\mu$ measurement is dominated by the uncertainty in the bias factor. The results from this analysis of 93-95 data are combined with the previously published DELPHI result using data from 1991 and 1992 to give the final DELPHI estimates of $B_e$ and $B_\mu$ (shown in table 2.3).

### 3. SUMMARY OF RESULTS AND UNIVERSALITY TESTS

TAU98 has three updated values for $B_e$ and $B_\mu$ compared to PDG98 [9] where nine measurements are used in each mode to compute averages. Hence the number of measurements per mode is still nine. Of these, CLEO and the four LEP experiments have similar precisions and combined they carry about 96.5 % of the weights for both

![Figure 4. Variables for electron identification.](image)

Large arrows show the cut values for identification. Regions to the left of the small arrows are used to check the background levels.

![Figure 5. a) Momentum distribution for identified electrons b) the ratio between data and expectation from simulation.](image)
Table 1
Number of candidates, efficiencies and backgrounds for the DELPHI 93-95 analysis and resulting $B_e$ and $B_\mu$ estimates.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\tau \rightarrow \mu \nu \bar{\nu}$</th>
<th>$\tau \rightarrow e \nu \bar{\nu}$</th>
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</thead>
<tbody>
<tr>
<td>$N_{\tau+\tau-}$</td>
<td>68655</td>
<td>68668</td>
</tr>
<tr>
<td>$\epsilon_\tau$ (%)</td>
<td>52.57 ± 0.04</td>
<td>50.87 ± 0.04</td>
</tr>
<tr>
<td>$b_\tau$</td>
<td>3.09 ± 0.11</td>
<td>3.05 ± 0.11</td>
</tr>
<tr>
<td>$N_l$</td>
<td>21040</td>
<td>18273</td>
</tr>
<tr>
<td>$\epsilon_l$ (%)</td>
<td>46.12 ± 0.11</td>
<td>36.79 ± 0.14</td>
</tr>
<tr>
<td>$b_l$</td>
<td>3.65 ± 0.16</td>
<td>5.23 ± 0.30</td>
</tr>
<tr>
<td>Branching ratio, 93-95 (%)</td>
<td>17.37 ± 0.11stat ± 0.07sys</td>
<td>17.98 ± 0.12stat ± 0.09sys</td>
</tr>
<tr>
<td>Branching ratio, 91-95 (%)</td>
<td>17.32 ± 0.10stat ± 0.07sys</td>
<td>17.92 ± 0.11stat ± 0.10sys</td>
</tr>
</tbody>
</table>

modes. Almost the full dataset of the LEP experiments is now analyzed, with the exception of ALEPH, where the analysis of only about half the LEP-I data is completed, and L3, where 1995 data are not included in their measurements. The measurements are summarized in figs. 6 and 7. The TAU98 world averages become:

$$B(\tau \rightarrow e\nu\bar{\nu}) = (17.81 \pm 0.06)\%$$

and

$$B(\tau \rightarrow \mu\nu\bar{\nu}) = (17.36 \pm 0.06)\%.$$ 

These branching ratios are thus now known to a relative precision of 0.33%, and are about 25% more precise than the PDG98 averages. The $\chi^2$ confidence for the combination of the $B_\mu$, is unnaturally high, about 99.7%. Disregarding psychological effects, an explanation for this can hardly be found, unless some measurements have assigned too large values for the systematic uncertainty. It may be noted that if one uses only the five most recent (and most precise) measurements, the $\chi^2$ confidence is relatively normal, at 91.3%. The level of agreement between the various $B_e$ measurements is perfectly normal.

To ensure the correct precision of the $e-\mu$ universality test as given by eq. 3, it is necessary to account for correlations in the measurements observed by the different experiments, in particular in the CLEO measurements. The evaluation below therefore averages the ratios $g_\mu/g_e$ when they are given by experiments. For the remaining measurements, averages of $B_e$ and $B_\mu$ are computed, and $g_\mu/g_e$ is deduced from eq. 3. The following

$$B(\tau \rightarrow e\nu\bar{\nu})$$

measurements

<table>
<thead>
<tr>
<th></th>
<th>$HRS$</th>
<th>$CLEO$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>17.0 ± 0.5 ± 0.6</td>
<td>19.1 ± 0.4 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>$ALEPH$ 89-90</td>
<td>18.09 ± 0.45 ± 0.45</td>
</tr>
<tr>
<td></td>
<td>$ARGUS$</td>
<td>17.5 ± 0.3 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>$ALEPH$ 91-93</td>
<td>17.79 ± 0.12 ± 0.06</td>
</tr>
<tr>
<td></td>
<td>$CLEO$</td>
<td>17.76 ± 0.06 ± 0.17</td>
</tr>
<tr>
<td></td>
<td>$DELPHI$ 91-95, prel.</td>
<td>17.92 ± 0.11 ± 0.10</td>
</tr>
<tr>
<td></td>
<td>$L3$ 91-94, prel.</td>
<td>17.67 ± 0.14 ± 0.13</td>
</tr>
<tr>
<td></td>
<td>$OPAL$ 91-95, prel.</td>
<td>17.81 ± 0.09 ± 0.06</td>
</tr>
<tr>
<td>$PDG98$</td>
<td>17.78 ± 0.08</td>
<td></td>
</tr>
<tr>
<td>$TAU98$</td>
<td>17.81 ± 0.06</td>
<td></td>
</tr>
</tbody>
</table>

$\chi^2$/d.o.f. $= 5.98$ (CL = 0.65)

$ACDLO \chi^2$/d.o.f. $= 1.174$ (CL = 0.85)

Figure 6. $B(\tau \rightarrow e\nu\bar{\nu})$ measurements for the world average.
Figure 7. $B(\tau \to \mu \nu \bar{\nu})$ measurements for the world average.

This precision approaches the one obtained from pion decays, where a comparison of the world average ratio: $R_{\text{exp}} = B(\pi \to e\nu(\gamma))/B(\pi \to \mu\nu(\gamma)) = (1.230 \pm 0.004) \times 10^{-4}$ [9] to its theoretical prediction assuming universality, $R_{\text{th}} = (1.2352 \pm 0.0005) \times 10^{-4}$ [10], leads to

$$\left(\frac{g_{\mu}}{g_{e}}\right)_{L} = 1.0021 \pm 0.0016$$

Here, the subscript signals that this tests the coupling to a longitudinal $W$, and is hence slightly different from the test from tau leptonic decays.

To test $\tau-e$ and $\tau-\mu$ universality, $\tau$ and muon masses and lifetimes are required. Using the theoretical prediction assuming universality, masses and lifetimes are required. Using the different from the test from tau leptonic decays.

Here, the subscript signals that this tests the coupling to a longitudinal $W$, and is hence slightly different from the test from tau leptonic decays.

To test $\tau-e$ and $\tau-\mu$ universality, $\tau$ and muon masses and lifetimes are required. Using the

TAU98 world average lifetime $\tau_{\tau} = 290.5 \pm 1.0$ fs [11], and PDG98 numbers for the other quantities, the values

$$\frac{g_{\tau}}{g_{\mu}} = 1.0002 \pm 0.0025$$

and

$$\frac{g_{\tau}}{g_{e}} = 1.0013 \pm 0.0025$$

are obtained. The uncertainties here are about equally shared between the uncertainty in the branching ratios and the $\tau$ lifetime.

Finally, if $e-\mu$ universality is assumed, the two branching ratios $B_{e}$ and $B_{\mu}$ can be combined to give:

$$B_{1} = (17.83 \pm 0.04\%)$$

where the phase space suppression of $B_{\mu}$ is corrected for. This gives the ratio:

$$\frac{g_{\tau}}{g_{e,\mu}} = 1.0007 \pm 0.0022$$

Another test of $\tau-\mu$ universality can be derived by comparing $B(\tau \to h\nu)$ to $B(h \to \mu\nu)$. Radiative corrections are now known to a precision of a few permille in these ratios, both for $h = \pi$ and $h = K$ [13], [14]. Uncertainties due to the hadron decay constant and CKM element cancel when forming the ratio of these two numbers. As it is difficult to make the distinction between pions and kaons, CLEO [2] recasts the ratio by forming a ratio of $B_{h} = B(\tau \to \pi\nu) + B(\tau \to K\nu)$ and the sum $H_{\pi} + H_{K}$. Here $H_{\pi}$ and $H_{K}$ are proportional to $B(\pi \to \mu\nu)$ and $B(K \to \mu\nu)$ respectively, given by

$$H_{h} = \frac{1 + \delta_{h}}{\tau_{h}m_{h}} \left(\frac{1 - m_{\mu}^{2}}{1 - m_{h}^{2}}\right)^{2} B(h \to \mu\nu)$$ (8)

with $h = \pi$ or $K$. The constants in front of the branching ratios are chosen such that the ratio

$$\frac{B_{\pi} + B_{K}}{H_{\pi} + H_{K}} = \frac{\tau_{\pi}m_{\pi}^{3}}{2m_{\mu}^{2}} \left(\frac{g_{\tau}}{g_{\mu}}\right)^{2}$$ (9)

is independent of the pion and kaon decay constants ($f_{\pi(K)}$) and CKM elements. With TAU98
average values for $B_h$ [12] and the $\tau$ lifetime [11] one obtains

$$\left( \frac{g_\tau}{g_\mu} \right)_L = 1.0037 \pm 0.0042$$

a precision which is approaching the precision obtained in leptonic $\tau$ decays. Again, the subscript signals that the spin structure of the coupling is different here compared to the tests based on purely leptonic decays.

Universality is also tested in a very direct way through the decay modes of real $W$ bosons [15] [16]. The sensitivity is presently about a factor of ten worse than in $\tau$ decays, but will greatly improve as data come in from LEPII and future hadron colliders.

4. THE $\eta$ PARAMETER

Departure from the standard model predictions for $B_e$ and $B_\mu$ does not necessarily imply non-universal couplings to the standard model $W^\pm$. The four fermion interaction can be written in general form in terms of the Michel parameters, and a non zero value of the $\eta$ parameter will affect the rates:

$$\Gamma_l = \frac{G_F^2 m_l^5}{192\pi^3} \left[ f(x_l^2) + 4x_l g(x_l^2) K \eta \right] r_{RC}.$$  \hspace{1cm} (10)

This expression is found in e.g. [17], but here a factor $K$ is included to account for experimental acceptance effects. $K = 1$ when such effects are neglected. Since the mass ratio $x_\mu$ is relatively large, $x_\mu = m_\mu/m_\tau \approx 1/17$, considerable sensitivity is obtained by forming the ratio:

$$B_\mu/B_e = 0.9726 + 0.217 K \eta.$$  \hspace{1cm} (11)

The parameter $\eta$ is the real part of the sum of interference terms between couplings of different Lorentz structure. The sum includes the product between the standard model coupling and the coupling of the $\tau$ to a charged scalar field. If an extra Higgs doublet is present, there is a relation between $\eta$ and the mass of the charged Higgs, given by [18],[19]:

$$\eta = -\frac{m_\tau m_\mu \tan^2 \beta}{2 m_H^2}$$  \hspace{1cm} (12)

where $\tan \beta$ is the ratio of the vacuum expectation values associated to the two Higgs doublets. Other contributions to $\eta$ have to be very small, as they would show up as the product of two non-standard couplings.

$\eta$ is often called the low energy parameter, as laboratory momentum spectra are mainly distorted at the low end for non-zero $\eta$. This is just where experiments have problems identifying muons, and the identification efficiency drops to zero at some momentum cutoff, $p_c$. Other experimental effects are probably not having a significant impact on the sensitivity factor $K$, and a crude estimate of the factor can be made based on the values of $p_c$ given in the different branching ratio analyses. If the standard model shape is

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Branching ratios plotted versus the $\tau$ lifetime. a) $B_e$, b) $B_\mu$, c) $B_l$, the combined leptonic branching ratio. The region between the dashed lines shows the expectation from equations 4 (fig. a) and c)) and 5 (fig. b), where the width of the band is given by the uncertainty of the $\tau$ mass.}
\end{figure}
used to correct for the number of events lost due to the cutoff, the sensitivity to $\eta$ is reduced by:

$$K = \frac{\int_{p}^{p_{max}} h_{\eta}(p) dp}{\int_{p}^{p_{max}} h_{s}(p) dp}$$

(13)

where $h_{s}(p)$ is the standard model normalized momentum distribution and $h_{\eta}(p)$ is the normalized distribution to multiply $\eta$.

For the branching ratio measurements, all four LEP experiments make a momentum cut in the range 2 to 2.5 GeV/c, typically at a momentum equal to 5 % of the beam momentum. In the CLEO analysis, all muon candidates are required to have a momentum of at least $0.28 \times \frac{p_{beam}}{}$, corresponding to about 1.5 GeV/c.

Generator level momentum distributions are shown in fig. 4 and based on these distributions the sensitivity factors are evaluated at: $K_{LEP} = 0.96$ and $K_{CLEO} = 0.72$ Including this, eq. 11 leads to the estimate:

$$\eta = 0.012 \pm 0.024$$

The assumption $K = 1$ would have given a 10 % smaller value for the uncertainty.

5. CONCLUSIONS

Including the measurements presented at this conference, the averages for the $\tau$ leptonic branching ratios are:

$$B(\tau \rightarrow e\nu\bar{\nu}) = (17.81 \pm 0.06)\%$$

$$B(\tau \rightarrow \mu\nu\bar{\nu}) = (17.36 \pm 0.06)\%$$

The present world averages tests $e\mu$ universality to a precision of 0.24 % at the level of the couplings. Furthermore, using lifetime and mass information, $e\tau$ and $\mu\tau$ universality tests have a precision of 0.25 %. The measurements also have implications for the Michel parameter $\eta$; although some care must be taken in evaluating the sensitivity, one can conclude that $\eta$ is compatible with zero to a precision of 2.4 %. No deviation from the standard model predictions is found.

The precision of these world averages should improve by about 10 % when all LEP data are fully analyzed. After this, little improvement

![Figure 9. Laboratory momentum distribution of muons from $\tau \rightarrow \mu\nu\bar{\nu}$ at LEP and CESR energies for the two hypotheses $\eta = 0$ and $\eta = 1$. Muons with momenta in the hatched regions are not identified by the experiments.](image)

is expected in the foreseeable future, as CLEO and the B factory measurements probably will be dominated by systematics at the present level. However, there should be room for considerable improvements in the precision of ratios between different branching fractions, for instance in $B_e/B_\mu$, with corresponding improvement in $g_e/g_\mu$.

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