Comments on “Differential cross section for Aharonov-Bohm effect with nonstandard boundary conditions”

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Abstract

We show that the violation of rotational symmetry for differential cross section for Aharonov-Bohm effect with nonstandard boundary conditions has been known for some time. Moreover, the results were applied to discuss the Hall effect and persistent currents of fermions in a plane pierced by a flux tube.

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In their recent paper [1], Štovíček and Váňa discuss the differential cross section \(d\sigma/d\theta\) for the Aharonov-Bohm effect with nonstandard boundary conditions. The latter mean in general that a wave function does not vanish at the position of a singular flux tube. Their main result is that, compared with the standard case, two new features occur: a violation of rotational symmetry and a more significant backward scattering. They discuss the violation of rotational symmetry in the rotationally invariant case (the case \(w' = 0\) in [1]) and for a new class of self-adjoint extensions which do not commute with the angular momentum (see also [2]). In the latter case, the scattering cross section depends on the incident direction \(\theta_0\) and the outgoing direction \(\theta\) separately and is not invariant under \(\theta_0, \theta \rightarrow \theta_0 + \gamma, \theta + \gamma\). In the rotationally invariant case, the scattering cross section depends on the scattering angle \(\varphi = \theta - \theta_0\), nevertheless, \(d\sigma(d\varphi)/d\theta \neq d\sigma(-d\varphi)/d\theta\) in general.

We should like to point out that the discussion of the scattering in the presence of an Aharonov-Bohm potential with the nonstandard boundary conditions has a much longer history (see, for example, [3]) than references in [1] tend to indicate. Also the violation of rotational symmetry for the differential cross section in the rotationally invariant case has been pointed out some time ago [4]. In Ref. [4], various physical quantities were calculated for a two-parameter family of self-adjoint extensions which respect the rotational invariance (i.e., no coupling between different angular momentum channels), including the local density of states, persistent currents for both spinless and spin one-half fermions in a plane pierced by a flux tube (see also [5]), the second virial coefficient of interacting anyons, etc., and the number of bound states for a flux tube of nonzero radius. Moreover, in contrast to the Štovíček and Váňa paper [1], the results were applied to discuss the Hall effect in the presence of magnetic vortices [4].

Let \(e, m, E\) be as in [1] the charge, the mass, and the energy of the scattered particle and let us concentrate on the rotationally invariant case. Let \(\alpha\) be the total flux through the flux tube in units of the flux quantum \(\Phi_0 = hc/|e|\). Let us write \(\alpha = n + \eta\), where \(n\) is an integer and \(\eta\) is the nonintegral part of \(\alpha\), \(0 \leq \eta < 1\). Under certain conditions [4], one ends up with a bound state in either one or in both channels \(l = -n\) and \(-n - 1\) [4]. If \(E_l\) is the corresponding bound state energy, the conventional phase shift \(\delta^0_l\) receives an additional contribution \(\triangle_l(E)\) and the resulting phase shift is

\[
\delta_l(E) = \delta^0_l + \triangle_l(E) = \frac{1}{2} \pi (|l| - |l + \alpha|) + \triangle_l(E),
\]

where \(\delta^0_l\) corresponds to the conventional Aharonov-Bohm scenario and

\[
\triangle_l = \arctan \left( \frac{\sin(|l + \alpha|\pi)}{\cos(|l + \alpha|\pi) - A_l^{-1}} \right).
\]

Here \(A_{-n} = (E/|E_{-n}|)^\eta\) and \(A_{-n-1} = (E/|E_{-n-1}|)^{1-\eta}\) are energy dependent, and \(A_l \rightarrow 0\) as \(\eta \rightarrow 0\).
In the limit $|E_l| \to 0$, $\delta_0^l \to -\delta_0^l$ (the phase-shift flip), while the conventional Aharonov-Bohm scenario corresponds to the limit $|E_l| \to \infty$ which yields $\triangle l = 0$. Surprisingly enough, the presence of a bound state manifests itself by a resonance at some positive energy, which depends on $\eta$ and $|E_l|$ [4]. For example, for $0 < \eta < 1/2$ the resonance appears at the $l = -n$ channel at $E_{res} = |E_{-n}|/[\cos(\eta \pi)]^{1/n} > 0$.

The differential cross section is then [4]

$$\left( \frac{d\sigma}{d\varphi} \right) (k, \varphi) = \left( \frac{d\sigma^0}{d\varphi} \right) (k, \varphi) + \frac{8\pi}{k} \sum_{l=-n-1}^{-n} \sin^2 \triangle l + \frac{4 \sin(\pi \alpha)}{k \sin(\varphi/2)} \times$$

$$\left[ \sin \triangle_{-n} \cos (\triangle_{-n} - \pi \alpha + \varphi/2) + \sin \triangle_{-n-1} \cos (\triangle_{-n-1} + \pi \alpha - \varphi/2) \right], \quad (3)$$

where $k = (2mE/h^2)^{1/2}$. The differential cross section remains periodic with respect to the substitution $\alpha \to \alpha \pm 1$, and, as can be easily verified, it becomes asymmetric with regard to $\varphi \to -\varphi$ (what is equivalent, with regard to $\alpha \to -\alpha$) as long as $\triangle_{-n} \neq -\triangle_{-n-1}$ (modulo $\pi$). However, as has been discussed in [4], this is generic. The breaking of the rotational symmetry is then simply a consequence of the fact that the nonstandard boundary conditions can only be imposed in an asymmetric way, namely for $\alpha \geq 0$ only in the channels $l = -n$ and $-n - 1$ with $l \leq 0$.

The asymmetric differential scattering cross section can give rise to the Hall effect. Let $n_v$ be the density of vortices and $n_e$ the density of electrons. The Hall resistivity calculated in the dilute vortex limit (by neglecting multiple-scattering contributions) is [4]

$$\rho_{xy} = \frac{4n_v}{n_e} \frac{\hbar^2}{e^2} \sin(\pi \alpha) \sin \triangle_{-n} \cos (\triangle_{-n} - \pi \alpha) + \sin \triangle_{-n-1} \cos (\triangle_{-n-1} + \pi \alpha). \quad (4)$$

Note that, with appropriate modifications, results in this paper apply also to scattering of sound waves in a vortex field [6], to the case of neutral particles with an anomalous magnetic moment in an electric field, and to exotic cases of scattering in the presence of a cosmic string and a gravitational vortex (see [4] for references).

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References


