Chiral kaon dynamics in heavy ion collisions

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Abstract

The influence of the chiral mean field on the collective motion of kaons in relativistic heavy ion reactions at SIS energies is investigated. We consider three types of collective motion, i.e. the transverse flow, the out-of-plane flow (squeeze-out) and the radial flow. The kaon dynamics is thereby described with a relativistic mean field as it originates from chiral lagrangians. For the $K$ mesons inside the nuclear medium we adopt a covariant quasi-particle picture including scalar and vector fields and compare this to a treatment with a static potential like force. The comparison to the available data ($K^+$) measured by FOPI and KaoS strongly favor the existence of an in-medium potential. However, using full covariant dynamics makes it more difficult to describe the data which might indicate that the mean field level is not sufficient for a reliable description of the kaon dynamics.

1 Introduction

In recent years strong efforts have been made towards a better understanding of the medium properties of kaons in dense hadronic matter. This feature is of particular relevance since the kaon mean field is related to chiral symmetry breaking [1]. The in-medium effects give rise to an attractive scalar potential inside the nuclear medium which is in first order, i.e. in mean field approximation, proportional to the kaon-nucleon Sigma term $\Sigma_{\text{KN}}$. A second part of the mean field originates from the interaction with vector mesons [1, 2, 3]. The vector potential is repulsive for kaons $K^+$ and, due to G-parity conservation, attractive for antikaons $K^-$. A strong attractive potential for antikaons may also favor $K^-$ condensation at high nuclear densities and thus modifies the properties of neutron stars [4].

One has extensively searched for signatures of these kaon-nucleus potentials in heavy ion reactions at intermediate energies [5, 6, 7]. In particular the collective motion of kaons in the dense hadronic environment is expected to be influenced by such medium effects [4, 8, 9, 10, 11, 12]. In this work we discuss the in-medium kaon dynamics for $K^+$ and $K^-$ with respect to the three prominent types of
collective motion in heavy ion reactions, namely the radial flow [12], the emission out of the reaction plane (squeeze-out) [13] and the in-plane flow (transverse flow) [8, 10, 14]. The interaction of the kaons with the dense hadronic medium is thereby described on the mean field level based on chiral models [1, 8]. Following Ref. [14] we discuss effects which originate from a fully covariant description of the kaonic mean field in a relativistic quasi-particle picture and compare this approach to the standard treatment with static potentials.

2 Covariant kaon dynamics

Due to its relativistic origin, the kaon mean field has a typical relativistic scalar–vector type structure. For the nucleons such a structure is well known from Quantum Hadron Dynamics [15]. This decomposition of the mean field is most naturally expressed by an absorption of the scalar and vector parts into effective masses and momenta, respectively, leading to a formalism of quasi-free particles inside the nuclear medium [15].

From the chiral Lagrangian [1] the field equations for the $K^\pm$-mesons are derived from the Euler-Lagrange equations [8]

$$\left[ \partial_\mu \partial^\mu \pm \frac{3i}{4f_\pi} j_\mu \partial^\mu + \left( m^2_K - \frac{\Sigma_{KN} f_\pi}{f_\pi^2} \rho_s \right) \right] \phi_{K^\pm}(x) = 0 \quad .$$

(1)

Here the mean field approximation has already been applied. In Eq. (1) $\rho_s$ is the baryon scalar density and $j_\mu$ the baryon four-vector current. Introducing the kaonic vector potential

$$V_\mu = \frac{3}{8f_\pi^2} j_\mu$$

(2)

Eq. (1) can be rewritten in the form [14]

$$\left[ (\partial_\mu + iV_\mu)^2 + m^2_{K^\pm} \right] \phi_{K^\pm}(x) = 0 \quad .$$

(3)

$$\left[ (\partial_\mu - iV_\mu)^2 + m^2_{K^\pm} \right] \phi_{K^-}(x) = 0 \quad .$$

(4)

Thus, the vector field is introduced by minimal coupling into the Klein-Gordon equation. The effective mass $m^*_K$ of the kaon is then given by [3, 14]

$$m^*_K = \sqrt{m^2_K - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_\mu V^\mu} \quad .$$

(5)

Due to the bosonic character, the coupling of the scalar field to the mass term is no longer linear as for the baryons but quadratic and contains an additional contribution originating from the vector field. The effective quasi-particle mass defined by Eq. (5) is a Lorentz scalar and is equal for $K^+$ and $K^–$. It should not be mixed up with the quantity, i.e. kaon energy at zero momentum $\omega(k = 0) = m^*_K \pm V_0$ for $K^\pm$, which is sometimes denoted as in-medium mass [2, 8, 16, 9] and which determines the shift of
Figure 1: In-medium kaon mass in nuclear matter for various mean fields derived from chiral lagrangians. The right panel shows the kaon energy at zero momentum which is often denoted as in-medium mass in the literature (the upper lines refer to $K^+$, the lower ones to $K^-$). The left panel shows the quasi-particle mass given by Eq. (5) which is equal for $K^+$ and $K^-$. 

The corresponding production thresholds. These two quantities, namely the energy at zero momentum and the in-medium quasiparticle mass $m_K^*$ are compared in Fig.1. For the modification of the kaon in-medium properties we parameterize results obtained from coupled channel calculation in chiral perturbation theory [16] (ChPT). We also compare them to two more simple mean field models of the type of Eq. (1) suggested by Li and Ko [8] (MF) as well as by Brown and Rho [2] (MF2).

As can be seen from Fig. 1 the quasi-particle mass $m_K^*$ is equal for $K^+$ and $K^-$ and generally reduced inside the nuclear medium. However, in the approach of Ref. [16] (Waas et al.) this reduction is much weaker than in the simple MF parameterization where the scalar field is in first order proportional to the scalar nucleon density.

Introducing an effective momentum $k_\mu^* = k_\mu \mp V_\mu$ for $K^+(K^-)$, the Klein-Gordon equation (3,4) reads in momentum space

\[ \left[k^{*2} - m_{K^*}^2\right] \phi_{K^\pm}(k) = 0 \]  

which is just the mass-shell constraint for the quasi-particles inside the nuclear medium. These quasiparticles can now be treated like free particles. In nuclear matter at rest the spatial components of the
vector potential vanish, i.e. \( V = 0 \), and Eqs. (3,4) reduce to the expression already given in Ref. [8].

The covariant equations of motion for the kaons are obtained in the classical (testparticle) limit from the relativistic transport equation for the kaons which can be derived from Eqs. (3,4). They are analogous to the corresponding relativistic equations for baryons and read

\[
\frac{dq^\mu}{d\tau} = \frac{k^\nu}{m_K^*}, \quad \frac{dk^\nu}{d\tau} = \frac{k^\mu}{m_K^*} F^{\mu\nu} + \partial^\mu m_K^* .
\] (7)

Here \( q^\mu = (t, q) \) are the coordinates in Minkowski space and \( F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \) is the field strength tensor for \( K^+ \). For \( K^- \) where the vector field changes sign the equation of motion are identical, however, \( F^{\mu\nu} \) has to be replaced by \(-F^{\mu\nu}\). The structure of Eqs. (7) may become more transparent considering only the spatial components

\[
\frac{dk^*}{dt} = -\frac{m_K^*}{E^*} \frac{\partial m_K^*}{\partial q} \pm \frac{k^*}{E^*} \times \left( \frac{\partial}{\partial q} \times V \right)
\] (8)

where the upper (lower) signs refer to \( K^+ \) (\( K^- \)). The term proportional to the spatial component of the vector potential gives rise to a momentum dependence which can be attributed to a Lorentz force, i.e. the last term in Eq. (8). Such a velocity dependent (\( v = k^*/E^* \)) Lorentz force is a genuine feature of relativistic dynamics as soon as a vector field is involved. If the equations of motion are, however, derived from a static potential

\[
U(k, \rho) = \omega(k, \rho) - \omega_0(k) = \sqrt{k^2 + m_K^2} - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_0^2 \pm V_0 - \sqrt{k^2 + m_K^2}
\] (9)

as, e.g. in Refs. [8, 4, 9], the Lorentz-force like contribution is missing. The same holds for non-relativistic approaches [10, 12] where the Lorentz force has also not yet been taken into account. Such non-covariant treatments are formulated in terms of canonical momenta \( k \) instead of kinetic momenta \( k^* \) and then the equations of motion (8) read [14, 17]

\[
\frac{dk}{dt} = -\frac{m_K^*}{E^*} \frac{\partial m_K^*}{\partial q} \pm \frac{v_i}{E^*} \frac{\partial V_i}{\partial q} = -\frac{\partial}{\partial q} U(k, \rho) \pm v_i \frac{\partial V_i}{\partial q}
\] (10)

with \( v = k^*/E^* \) the kaon velocity. Thus Eqs. (8) and (10) are more general than the treatment in the static potential \( U \), Eq.(9). Effects which arise from the full dynamics will be discussed in the following.

3 Collective flow of kaons

3.1 Radial flow

One way to obtain information on the collective motion is to investigate particle multiplicities as a function of the transverse mass \( m_t \) [12]. In the left panel of Fig. 2 the transverse mass spectrum of \( K^+ \) mesons emitted at midrapidity (-0.4 < y_{c.m.} < 0.4) is shown for a central (\( b=3 \) fm) Au+Au reaction at 1 A.GeV. In the presence of a kaon potential (MF2, taken from [2]) the kaon \( m_t \) spectrum
clearly exhibits a "shoulder-arm" shape which deviates from a pure thermal picture. On the other hand the calculation without any potential can be well described by a Boltzmann distribution, which is more or less a straight line if plotted logarithmically. Thus, the "shoulder-arm" structure is caused by the mean field rather than by a collective expansion of the kaon sources. The kaons experience an acceleration due to the repulsive potential as they propagate outwards from the participant region. We want to mention that these calculations have been performed with the QMD model where the kaon dynamics was treated non-covariantly, i.e. applying Eq. (10) and neglecting thereby the contribution of the spatial vector field components. However, including these Lorentz forces we find that the results for the $m_t$ spectrum are hardly affected since the kaons stem from the equilibrated fireball zone where the collective motion of the nucleons relative to the kaons is rather small. To extract the collective component from the kaon spectrum, we fit the QMD results including a common radial velocity to the
standard Boltzmann distribution

\[
\frac{d^3N}{d\phi dy dm_t} \sim e^{-\left(\frac{p^2}{T^2} - \frac{\alpha}{\gamma T}\right) \left(\frac{E^2}{p^2} + 1\right) + \frac{(\alpha T)^2 E^2}{p^2} \sqrt{\frac{(\gamma E - \alpha T)^2 - m^2}{p}}} \tag{11}
\]

where \(E = m_t \cosh y, p = \sqrt{p_t^2 + m_t^2 \sinh^2 y}, \alpha = \gamma \beta p / T, \gamma = (1 - \beta^2)^{-1/2}.\) The fit yields \(\beta = 0.11\) and \(T = 6.2\,\text{MeV}.\)

The attractive potential for \(K^-\) leads on the other hand to an enhancement of low energetic particles and an concave spectrum seen on the right panel of Fig.2. Turning the sign of the flow velocity \(\beta\) in Eq. (11) the calculation can also be described by an radial imploding source, i.e. by an virtual radial flow [12]. Thus the effect of the kaon potential on the collective motion can clearly be distinguished from the thermal contribution.

### 3.2 Squeeze-out

In this subsection we study the azimuthal asymmetry of \(K^+\) and \(K^-\) emission [13]. At SIS energies shadowing dominates the reaction of nucleons and pions and leads to an enhanced particle emission out of the reaction plane. The phenomenon is usually called ”squeeze-out”. For \(K^+\) mesons the shadowing effect can be expected to be small due to the moderate absorption cross section \(\sigma_{K^+N} \approx 10\,\text{mb}.\) The absorption cross for the \(K^-\) is much larger, \(\sigma_{K^-N} \approx 50\,\text{mb},\) and thus the \(K^-\) dynamics can be expected to be dominated by shadowing effects [13]. In Fig. 3 we investigated the squeeze-out for mid-rapidity \((-0.2 < (Y/Y_{\text{proj}})_{\text{cm}} < 0.2)\) \(K^+\) mesons in a semi-central (b=6 fm) Au+Au reaction at 1 A.GeV incident energy. In addition a transverse momentum cut of \(P_T > 0.2\,\text{GeV/c}\) has been applied in order to compare with the KaoS data [18]. The QMD calculations are performed for three different cases: (1) without any in-medium effects, (2) including the \(K^+\) potential \(U(\rho, k)\), Eq. (9), but neglecting the space-like components of the repulsive vector potential \(V\) and (3) with covariant in-medium dynamics, i.e. retaining also the space-like vector contribution in Eq. (10). The potential is again taken as in Ref. [2] (MF2). First of all it can be seen that an enhanced out-of-plane emission of \(K^+\) mesons is mainly a result of the kaon potential. Without any medium effects the \(K^+\) emission is nearly azimuthally isotropic. Very similar effects were also found be the Stony Brook group [7, 18]. However, the influence of the space-like components of the repulsive vector potential destroys the preferential emission of \(K^+\) mesons out of the reaction plane and thus also the agreement with the data. A similar effect will be also observed for the transverse flow of \(K^+\) [14] (see below).

Concerning \(K^-\) mesons the situation is just opposite to the case of \(K^+\). In the left panel of Fig. 3 the azimuthal distributions of \(K^-\)s emitted at midrapidity \((-0.2 < (Y/Y_{\text{proj}})_{\text{cm}} < 0.2)\) in a semi-peripheral (b=8 fm) Au+Au reaction at 1.8 A.GeV are shown using a \(P_T\) cut of \(P_T > 0.5\,\text{GeV}.\) Indeed, \(K^-\)s are strongly scattered or absorbed in the nuclear medium. If there is no in-medium potential acting on the \(K^-\)s, the emission at midrapidity exhibits an out-of-plane preference much
like pions [19]. Now the in-medium potential (the space-like components of the attractive vector part are again neglected) reduces dramatically the out-of-plane $K^-$ abundance ($\phi = 90^0$ and $270^0$), and leads thereby to a nearly isotropic emission. Hence, the medium effects act opposite on $K^+$ and $K^−$ – at least in the non-covariant description – and comparing the out-of-plane emission for both types of mesons should yield rather conclusive information form the experimental side.

### 3.3 Transverse flow

In Fig. 4 the transverse flow of $K^+$ mesons in Ni+Ni collisions at 1.93 A.GeV is compared to the FOPI data [5]. The results are obtained for impact parameters $b \leq 4$ fm and with a transverse momentum cut $P_T/m_K > 0.5$. Without medium effects a clear flow signal is observed which reflects the transverse flow of the primary sources of the $K^+$ production. The dominantly repulsive character of the in-medium potential (9) tends to push the kaons away from the spectator matter which leads to a zero flow around
Figure 4: Influence of the in-medium potential on the $K^+$ transverse flow. The left panel compares calculations for $K^+$ for a ($b$=4 fm) Ni+Ni reaction at 1.93 A.GeV to the FOPI data [5]. The calculations are performed including (solid) and without (dashed) the kaon potential and using full covariant kaon dynamics, i.e. the kaon potential with Lorentz forces (dotted). The left panel shows the flow for the same reaction at $b$=3 fm for two types of kaon mean fields, again with full covariant kaon dynamics (bottom) and using only the potential part (top).

midrapidity. The situation changes, however, dramatically when the full Lorentz structure of the mean field is taken into account according to Eqs. (8,10). The influence of the repulsive part of the potential, i.e. the time-like component, on the in-plane flow is almost completely counterbalanced by the velocity dependent part of the interaction. Hence, no net effect of the potential is any more visible. This feature is rather independent on the actual strength of the potential as can be seen from the right panel of Fig. 4 where the same calculation is performed for Ni+Ni at 1.93 A.GeV and an fixed impact parameter $b=3$ fm. Although the two types of potentials vary considerably in the strength of the kaonic vector field which gives rise to significantly different results in the case of the quasi-potential treatment (top), this effect is completely counterbalanced by the Lorentz-force contribution included in the lower figure. Although this Lorentz force vanishes in nuclear matter at rest, it is clear that this force generally contributes in heavy ion collisions. Kaons are produced in the early phase of the reaction where the relative velocity of projectile and target matter is large. Thus the kaons feel a
non-vanishing baryon current in the spectator region, in particular in non-central collisions.

The cancelation effects on the flow can be understood from Eq. (10). The vector field is generally proportional to the baryon current \( j_\mu = (\rho_B, u\rho_B) \) where \( u \) denotes the streaming velocity of the surrounding nucleons. Let us for the moment assume that \( u \) is locally constant, then the total contribution of the vector field in Eq. (10) can be written as
\[
\mp \frac{3}{8f} (1 - |v| |u| \cos \Theta) \frac{\partial \rho_B}{\partial q}.
\]
Now the angle \( \Theta \) between the kaon and the baryon streaming velocities determines the influence of the Lorentz force. Since in our case and also in the calculations of Refs. [4, 11] the \( K^+ \)s initially follow the primordial flow of the nucleons we have \( \cos \Theta \sim 1 \) which gives rise to the cancelation. However, the value of \( \Theta \) and also the magnitude of the kaon velocity are also related to the rescattering of the \( K^+ \) mesons with the nucleons. An enhanced rescattering as well as a different primordial \( K^+ \) flow might reduce the cancelation effects from the Lorentz force. Hence, the complete description of the in-plane \( K^+ \) flow is still an open question and further theoretical studies seem to be necessary [17].

4 Conclusions

We investigated the influence of chiral mean fields on the dynamics of \( K \) mesons in heavy ion reactions. To apply such mean fields in the framework of relativistic transport theory a covariant quasi-particle formulation is used. The vector part of the kaon self-energy which is of leading order in the chiral expansion and originates form the Weinberg-Tomozawa term is subsummizied into effective, kinetic four-momenta whereas the scalar part due to the next-to-leading order kaon-nucleon-sigma term enters into an effective mass. As a consequence of the full relativistic dynamics a Lorentz-force term appears which is missing in a description with a potential-like force only. In agreement with other works we find that the collective motion of kaons is strongly influenced by the mean field in dense matter. The comparison with data for the \( K^+ \) squeeze-out and the \( K^+ \) transverse flow strongly favors the existence of such in-medium potentials as predicted form the chiral models. However, applying the full relativistic dynamics the description of the data is more difficult since we observe strong cancelation effects connected to the Lorentz-force contribution. Thus we conclude that the present mean field description might be too simple and higher order terms from the chiral expansion should be taken into account as well. The radial motion of the kaons stemming from the fireball region is not affected by such questions, i.e. the Lorentz-forces, but gives also rise to a clear signal for the in-medium dynamics. Here deviations from the pure thermal spectrum are observed which can be attributed to an radial flow of \( K^+ \) and an ”virtual”, inverse radial flow of \( K^- \).
References


[18] P. Senger, this proceeding.
