D. Baleanu*

SYMMETRIES OF THE DUAL METRICS

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*Institute for Space Sciences, POB MG-36, Bucharest-Magurele, Romania
E-mail: baleanu@thsun1.jinr.ru; baleanu@roifa.ifa.ro
1 Introduction

The symmetries of space-times were systematically investigated in terms of the motion of pseudo-classical spinning point particles described by the supersymmetric extension of the usual relativistic point particle. Such a supersymmetric theory possesses a supercharge $Q$ generating the supersymmetry transformation between the particle's position $x^a$ and the particle's spin $\chi^a$, which must be introduced to forbid the negative norm states of spin due to indefinite Lorentz metric $\eta_{ab}$. It was a big success of Gibbons et al. [1] to have been able to show that the Killing-Yano tensor [2], which had long been known to relativistic as a rather mysterious structure, can be understood as an object generating "a non-generic" supersymmetry, i.e., a supersymmetry appearing only in specific spacetimes [1]. In a geometrical setting, symmetries are connected with isometries associated with Killing vectors, and more generally, Killing tensors on the configuration space of the system. An example is the motion of a point particle in a space with isometries [3], which is a physicist's way of studying the geodesic structure of a manifold. The non-generic symmetries were investigated in the case of Taub-NUT metric [4] and extended Taub-NUT metric [5]. The geometrical interpretation of Killing tensors was investigated in [6]. Recently Holten [7] have presented a theorem concerning the reciprocal relation between two local geometries described by metrics which are Killing tensors with respect to one another. If $K^\mu\nu$ are the contravariant components of a Killing tensor with respect to the inverse metric $g^{\mu\nu}$, then $g^{\mu\nu}$ must represent a Killing tensor with respect to the inverse metric defined by $K^\mu\nu$. The physical interpretation of the dual metrics was not clarified. In this paper we investigate the geometric duality which exist between $g_{\mu\nu}$ and $K_{\mu\nu}$. The main aim of this paper is to investigate generic and non-generic symmetries of the dual metrics $K_{\mu\nu}$ and to construct the dual spinning space. Spinning space was constructed without introduction of torsion. The introduction of torsion is not crucial here.

The plan of this paper is the following:

In Section 2 we analysed the generic and non-generic symmetries of the metric $K_{\mu\nu}$ which is dual to $g_{\mu\nu}$. In Section 3 the dual spinning space was constructed. In Section 4 we present our conclusions.

2 Symmetries and geometric duality

Let a space with metric $g_{\mu\nu}$ admits a Killing tensor field $K_{\mu\nu}$. A Killing tensor is a symmetric tensor which satisfies the following relation:

$$D_\lambda K_{\mu\nu} + D_\mu K_{\nu\lambda} + D_\nu K_{\lambda\mu} = 0$$  \hspace{1cm} (1)
where $D$ represent the covariant derivative. The equation of motion of a particle on a geodesic is derived from the action

$$ S = \int d\tau \left( \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu \right). $$

(2)

The Hamiltonian is constructed in the following form $H = \frac{1}{2} g_{\mu\nu} p^\mu p^\nu$ The Poisson brackets are

$$ \{x_\mu, p^\nu\} = \delta^\nu_\mu. $$

(3)

The equation of motion for a phase space function $F(x, p)$ can be computed from the Poisson brackets with the Hamiltonian

$$ \dot{F} = \{F, H\}. $$

(4)

From the covariant component $K_{\mu\nu}$ of the Killing tensor we can construct a constant of motion $K$,

$$ K = \frac{1}{2} K_{\mu\nu} p^\mu p^\nu. $$

(5)

We can easy verified that

$$ \{H, K\} = 0. $$

(6)

If we denote $D$ to be the covariant derivative in respect to $g_{\mu\nu}$ and $\hat{D}$ the covariant derivative with respect to $K_{\mu\nu}$ then:

$$ D_\lambda g_{\mu\nu} = 0 $$

$$ D_\lambda K_{\mu\nu} + D_\mu K_{\nu\lambda} + D_\nu K_{\lambda\mu} = 0 $$

(7)

$$ \hat{D}_\lambda K_{\mu\nu} = 0 $$

$$ \hat{D}_\lambda g_{\mu\nu} + \hat{D}_\mu g_{\nu\lambda} + \hat{D}_\nu g_{\lambda\mu} = 0. $$

(8)

On the other hand between the connections $\hat{\Gamma}_\nu^\mu$ and $\hat{\Gamma}_\nu^\mu$ is an interesting relation

$$ \hat{\Gamma}_\nu^\mu = \Gamma_\nu^\mu - K_\nu^\delta D_\delta K_\mu^\lambda. $$

(9)

As is well known for a given metric $g_{\mu\nu}$ the conformal transformation is defined as

$$ \hat{g}_{\mu\nu} = e^{2U} g_{\mu\nu}, \quad \hat{g}^{\mu\nu} = e^{-2U} g^{\mu\nu}, \quad U = U(x). $$

(10)

From (10) the relation between the corresponding connections is

$$ \hat{\Gamma}_\mu^\lambda = \Gamma_\mu^\lambda + 2\delta_\mu^\gamma U^\gamma - g_{\mu\nu} U^\nu \Gamma_\nu^\lambda $$

(11)

where $U^\nu = \frac{dU}{dx}$. 

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After calculations we conclude that the dual transformation (9) is not a conformal transformation.

It is easy to check that the metric \(K_{\mu\nu} = f_{\mu\lambda} f_{\nu}^{\lambda}\) has an inverse if the dimension of manifold are even.

The next step is to investigate the symmetries of the dual manifold \(K_{\mu\nu}^\prime\).

**Theorem 1**

The metric and the dual metric have the same generic symmetries if

\[
D_\delta K_{\mu\nu} \chi^\delta = 0. \tag{12}
\]

**Proof.**

Let \(\Gamma^\mu_{\nu\lambda}\) and \(\hat{\Gamma}^\mu_{\nu\lambda}\) be the connections corresponding to \(g_{\mu\nu}\) and respectively \(K_{\mu\nu}\). We have the following very important relation

\[
\hat{\Gamma}^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda} - K^{\mu\delta} D_\delta K_{\nu\lambda}. \tag{13}
\]

Taking into account (13) Killing’s vectors equations in the dual space are the following form

\[
D_\mu \hat{\chi}_\nu + D_\nu \hat{\chi}_\mu + 2 K^{\delta\sigma} D_\delta K_{\mu\nu} \hat{\chi}_\sigma = 0 \tag{14}
\]

where \(\hat{\chi}_\sigma\) are Killing vectors in the dual space and \(D\) represents the covariant derivative on manifold \(g_{\mu\nu}\). Let \(\chi_\sigma\) be the Killing vectors corresponding to \(g_{\mu\nu}\). When we suppose that \(\hat{\chi}_\sigma = \chi_\sigma\) from (12) and (14) we can obtain that

\[
D_\delta K_{\mu\nu} \hat{\chi}^\delta = D_\delta K_{\mu\nu} \chi^\delta = 0. \tag{15}
\]

If we assume that (12) is valid from (14) we can deduce that \(\chi_\sigma = \hat{\chi}_\sigma\). Q.E.D.

An interesting case arises when \(K_{\mu\nu}\) satisfies the following relation \(D_\lambda K_{\mu\nu} = \eta_\lambda K_{\mu\nu}\). Then we have the same generic symmetries for given manifold and for dual manifold if

\[
\eta_\delta \hat{\chi}^\delta = \eta_\delta \chi^\delta = 0. \tag{16}
\]

Another very interesting case arises when \(D_\lambda K_{\mu\nu} = 0\). Then we have the same generic symmetries for the manifold and its dual.

**Theorem 2**

The metric and the dual metric have the same non-generic symmetries if

\[
f_{\nu\delta} K^{\delta\sigma} D_\sigma K_{\mu\lambda} + 2 f_{\sigma\lambda} K^{\sigma\delta} D_\delta K_{\mu\nu} + f_{\mu\sigma} K^{\sigma\delta} D_\delta K_{\nu\lambda} = 0. \tag{17}
\]

**Proof.**
We suppose that metric $g_{\mu\nu}$ admits a Killing-Yano tensor $f_{\mu\nu}$. A Killing-Yano tensor is an antisymmetric tensor[7,8] which satisfies the following equations

$$D_\lambda f_{\mu\nu} + D_\mu f_{\nu\lambda} = 0.$$  \hspace{1cm} (18)

Because

$$K_{\mu\nu} = f_{\mu\beta} f^{\beta}_{\nu} = f_{\nu\alpha} f^{\alpha}_{\mu} = \delta^{\alpha}_{\mu} \delta_{\nu\alpha}$$

we have that

$$K_{\mu\nu} = f^{\alpha}_{\mu} f_{\nu\alpha} = \delta^{\alpha}_{\mu} \delta_{\nu\alpha}.$$  \hspace{1cm} (19)

If we take the dual in (20) we obtain that

$$g_{\mu\nu} = \epsilon^{\alpha}_{\mu} \epsilon_{\nu\alpha} = \delta^{\alpha}_{\mu} \delta_{\nu\alpha}.$$  \hspace{1cm} (21)

Taking into account that

$$f_{\mu\nu} = f_{\mu\sigma} \epsilon^{\sigma}_{\nu}$$

from (20), (21) and (22) we deduce

$$f_{\mu\nu} = - \delta^{\nu}_{\mu}.$$  \hspace{1cm} (23)

From (18) and (23) the metric and its dual must have the same Killing-Yano tensors.

On the other hand from (13) Killing-Yano equations in the dual space have the form

$$\hat{D}_\nu f_{\mu\lambda} + \hat{D}_\mu f_{\nu\lambda} = D_\mu f_{\nu\lambda} + D_\nu f_{\mu\lambda} + f_{\nu\delta} K^{\delta\gamma} D_\gamma K_{\mu\lambda} + 2 f_{\nu\gamma} K^{\gamma\delta} D_\delta K_{\mu\lambda} = 0$$

where $D$ represents the covariant derivative corresponding to $g_{\mu\nu}$. If we suppose that (17) is satisfied then $\hat{f}_{\mu\nu} = - f_{\mu\nu}$ is a Killing-Yano tensor. If we suppose that $\hat{f}_{\mu\nu} = - f_{\mu\nu}$ is a Killing-Yano tensor then from (21) we deduce (17).

When this Theorem is satisfies the metric and its dual have the same non-generic symmetries. QED

A solution of Eq.(31) is $f_{\mu\nu} = b_{\mu\nu}$ where $b_{\mu\nu}$ is a constant antisymmetric tensor.

If the manifold admits a Killing tensor $K_{\mu\nu}$ satisfying $D_\lambda K_{\mu\nu} = 0$ then dual manifold has the same symmetries like initial manifold.

An interesting example is Kerr-Newmann metric [1].

The Kerr-Newmann geometry describes a charged spinning black hole; in a standard choice of coordinates the metric is given by the following line element:

$$ds^2 = -\frac{\Delta}{\rho^2} \left[ dt - a \sin^2(\theta) d\varphi \right]^2 + \frac{\sin(\theta)^2}{\rho^2} \left[ (r^2 + a^2) d\varphi - adt \right]^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2.$$  \hspace{1cm} (25)
Here
\[ \Delta = r^2 + a^2 - 2Mr + Q^2, \quad \rho^2 = r^2 + a^2 \cos^2 \theta \]  
(26)

with $Q$ the background electric charge, and $J = Ma$ the total angular momentum. The expression for $ds^2$ only describes the fields outside the horizon, which is located at
\[ r = M + (M^2 - Q^2 - a^2)^{1/2}. \]  
(27)

The Killing Yano tensor for the Kerr-Newman is defined by [1]
\[ \frac{1}{2} f_{\mu\nu} dx^\mu \wedge dx^\nu = a \cos \theta \, dr \wedge (dt - a \sin^2 \theta \, d\phi) + r \sin \theta \, d\theta \wedge [-a \, dt + (r^2 + a^2) \, d\phi]. \]  
(28)

The Kerr-Newman metric admits a second-rank Killing tensor field. It can be described in this coordinate system by the quadratic form

\[
dK^2 = K_{\mu\nu} dx^\mu dx^\nu = \frac{a^2 \cos^2 \theta \Delta}{\rho^2} [dt - a \sin^2 \theta \, d\varphi]^2 +
\frac{r^2 \sin^2 \theta}{\rho^2} [(r^2 + a^2) d\varphi - a \, dt]^2 - \frac{\rho^2}{\Delta a^2 \cos^2 \theta} \, dr^2 + \frac{\rho^2}{r^2} \, d\theta^2.
\]  
(29)

Its contravariant components define the inverse metric $\tilde{g}^{\mu\nu}$ of the dual geometry. From [7] we know that the dual line element is

\[
d\tilde{s}^2 = \frac{\Delta}{\rho^2 a^2 \cos^2 \theta} [dt - a \sin^2 \theta \, d\varphi]^2 + \frac{\sin^2 \theta}{\rho^2 r^2} [(r^2 + a^2) d\varphi - a \, dt]^2 - \frac{\rho^2}{\Delta a^2 \cos^2 \theta} \, dr^2 + \frac{\rho^2}{r^2} \, d\theta^2
\]  
(30)

providing an explicit expression for the dual metric.

Kerr-Newman metric admits two Killing vectors $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial \varphi}$. Because $K_{\mu\nu}$ is dual to $g_{\mu\nu}$ and $K^{\mu\nu}$ are dual to $g^{\mu\nu}$ is interesting to investigate the generic and non-generic symmetries in this two cases. Using Theorem 1 the corresponding Killing vectors for dual metrics (29) and (30) are found to be the same as for Kerr Newman metric(25).

We use now Theorem 2 for investigation non-generic symmetric of dual metrics (29) and (30). The equations (24) have no solutions for $f_{\mu\nu}$ given by (28). This means that the metric $K_{\mu\nu}$ do not admits Killing Yano tensors.

The Killing Yano equations corresponding to dual metric from (30) are [2]
\[
D_{\mu} f_{\nu\lambda} + D_{\nu} f_{\mu\lambda} + f_{\rho\delta} D_{\delta} K_{\mu\lambda} + 2 f_{\beta\gamma} D_{\gamma} K_{\nu\mu} + f_{\rho\delta} D_{\delta} K_{\nu\lambda} = 0.
\]  
(31)
Here $D_\mu$ represents the covariant derivative corresponding to $K_{\mu\nu}$. After very tedious calculations we found that the equations (31) have no solutions.

In conclusion the dual metrics (29) and (30) have no non-generic symmetries.

Now we want to investigate the connection between Riemann curvature tensor, Ricci, Ricci scalar of manifold $g_{\mu\nu}$ and the corresponding expressions for the dual metrics $K_{\mu\nu}$. We know that

$$R^\beta_{\nu\rho\sigma} = \Gamma^\beta_{\nu\sigma,\rho} - \Gamma^\beta_{\nu\rho,\sigma} + \Gamma^\alpha_{\nu\sigma} \Gamma^\beta_{\alpha\rho} - \Gamma^\alpha_{\nu\rho} \Gamma^\beta_{\alpha\sigma}. \quad (32)$$

then the corresponding dual Riemann curvature tensor $\hat{R}^\beta_{\nu\rho\sigma}$ has the following expression

$$\hat{R}^\beta_{\nu\rho\sigma} = R^\beta_{\nu\rho\sigma} + R'^\beta_{\nu\rho\sigma} \quad (33)$$

where $R'^\beta_{\nu\rho\sigma}$ is given by

$$R'^\beta_{\nu\rho\sigma} = -(K^{\beta\delta} D_\delta K_{\nu\sigma})_{\nu} + (K^{\beta\gamma} D_\gamma K_{\nu\rho})_{\sigma} - \Gamma^\alpha_{\nu\sigma} K^{\beta\delta} D_\delta K_{\alpha\rho} - \Gamma^\alpha_{\nu\rho} K^{\beta\gamma} D_\gamma K_{\alpha\sigma} + K^{\alpha\delta} D_\delta K_{\nu\rho} K^{\beta\delta} D_\delta K_{\alpha\sigma}. \quad (34)$$

The explicit expression for the Ricci tensor is

$$R_{\mu\nu} = \Gamma^\alpha_{\mu\alpha,\nu} - \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\nu} \Gamma^\beta_{\alpha\beta} + \Gamma^\alpha_{\mu\beta} \Gamma^\beta_{\nu\alpha}. \quad (35)$$

The dual Ricci tensor can be written as

$$\hat{R}_{\mu\nu} = R_{\mu\nu} + R'_{\mu\nu} \quad (36)$$

where

$$R'_{\mu\nu} = -(K^{\alpha\gamma} D_\gamma K_{\mu\nu})_{\nu} + (K^{\alpha\gamma} D_\gamma K_{\mu\nu})_{\alpha} - \Gamma^\alpha_{\mu\nu} K^{\beta\delta} D_\delta K_{\alpha\beta} - \Gamma^\alpha_{\mu\beta} K^{\beta\gamma} D_\gamma K_{\nu\alpha} + K^{\alpha\delta} D_\delta K_{\mu\nu} K^{\beta\delta} D_\delta K_{\alpha\beta}. \quad (37)$$

If the metric $g_{\mu\nu}$ satisfies Einstein's equations in vacuum

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \quad (38)$$

then the dual metric $K_{\mu\nu}$ satisfies the following equation

$$(\hat{R}_{\lambda\sigma} - R'_{\lambda\sigma})(\delta_{\lambda\mu}\delta_{\sigma\nu} - \frac{1}{2} g_{\mu\nu} g^{\lambda\sigma}) = 0 \quad (39)$$

where $\hat{R}_{\mu\nu}$ and $R'_{\mu\nu}$ are given by (36) and (37).
3 Dual spinning space

The purpose of this section is to construct the dual spinning space. The non-
generic symmetries on the manifolds are related to the existence of Killing-
Yano tensors [3]. Let a theory described by the vielbein \( e^\mu_a \) and having a Killing-
Yano tensor \( f^\mu_{\nu a} = f^\mu_{\nu a} e_{\nu a} \). This theory has a supersymmetry described by the
supercharge \( Q = e^\mu_a P^a \psi^a \), where the momentum \( P^\mu \) has the form \( P^\mu = \dot{x}^\nu g_{\nu\mu} \).
The noncovariant Poisson bracket read

\[
Q_f = f^\mu_a P^a \psi^a + \frac{i}{6} c_{abc} \psi^a \psi^b \psi^c \tag{40}
\]

where

\[
c_{abc} = e_a^\mu e_b^\nu e_c^\lambda D_\lambda f_{\mu\nu} . \tag{41}
\]

By construction \( Q \) and \( Q_f \) satisfies

\[
\{ Q, Q_f \} = 0 \tag{42}
\]

where noncovariant Poisson brackets are

\[
\{ F, G \}_I = \frac{\partial F}{\partial x^\mu} \frac{\partial G}{\partial P_\mu} - \frac{\partial F}{\partial P_\mu} \frac{\partial G}{\partial x^\mu} + i(-1)^{\mu\nu} \frac{\partial F}{\partial \psi^a} \frac{\partial G}{\partial \psi_a} + \frac{1}{\psi^a} \frac{\partial F}{\partial \psi_a} . \tag{43}
\]

The dual theory is described by a vielbein \( e^\mu_a \) and the Killing-Yano tensor
\( \hat{f}_{\mu\nu} = -f_{\mu\nu} \). The momentum \( \hat{P}^\mu \) is given by

\[
\hat{P}^\mu = \dot{x}^\nu K_{\mu\nu} . \tag{44}
\]

The noncovariant Poison bracket are in this case

\[
\{ F, G \}_II = \frac{\partial f}{\partial x^\mu} \frac{\partial G}{\partial \hat{P}_\mu} - \frac{\partial F}{\partial \hat{P}_\mu} \frac{\partial G}{\partial x^\mu} + i(-1)^{\mu\nu} \frac{\partial F}{\partial \hat{\psi}^a} \frac{\partial G}{\partial \hat{\psi}_a} . \tag{45}
\]

The supercharges has the form

\[
\hat{Q} = \psi^a e^\mu_a \hat{P}^\mu \tag{46}
\]

the new supercharge has the form

\[
\hat{Q}_f = \psi^a \hat{f}_\mu^a \hat{P}_\mu + \frac{i}{6} \psi^a \psi^b \psi^c \hat{c}_{abc} \tag{47}
\]

with

\[
\{ \hat{Q}, \hat{Q}_f \}_II = 0 . \tag{48}
\]

Because we have \( \hat{f}_{\mu\nu} = -f_{\mu\nu} \) the supercharges \( \hat{Q} \) and \( \hat{Q}_f \) have the forms

\[
\hat{Q} = (f^{-1})^a \dot{x}^\nu \dot{\psi}^a \tag{49}
\]
and
\[
\dot{Q}_i = (f^{-1})^b_i c^a_b (f^{-1})^a_c \dot{x}^c \psi_a + \frac{i}{3} f^a_i f^b_i f^c_i H_{\mu \nu \lambda} \psi^a \psi^b \psi^c.
\] (50)

Here
\[
H_{\mu \nu \lambda} = \frac{1}{3} \left( \frac{\partial f_{\mu \nu}}{\partial x^\lambda} + \frac{\partial f_{\nu \lambda}}{\partial x^\mu} + \frac{\partial f_{\lambda \mu}}{\partial x^\nu} \right).
\] (51)

The dual spinning space was constructed naturally without introduction of torsion. In [7] the torsion was considered but the consistency condition (21) were not taking into account.

4 Conclusions

Recently Holten [7] has presented a theorem concerning the reciprocal relation between two local geometries described by metrics which are Killing tensors with respect to one another. Unfortunately Holten found the geometric duality between \(g^{\mu \nu}\) and \(K_{\mu \nu}\).

In this paper we have investigate the geometric duality between \(g_{\mu \nu}\) and a Killing tensor \(K_{\mu \nu}\). We found an interesting relation between connections corresponding to \(g_{\mu \nu}\) and \(K_{\mu \nu}\). Relation (11) is important from geometrical point of view because it can be generalized when \(K_{\mu \nu}\) is not a Killing tensor. Symmetries of the dual manifolds \(K_{\mu \nu}\) were investigated. The manifold \(g_{\mu \nu}\) and its dual metric \(K_{\mu \nu}\) have the same Killing-Yano tensors \(f_{\mu \nu}\) if and only if (17) is zero.

The dual manifold \(K_{\mu \nu}\) has the same Killing vectors as \(g_{\mu \nu}\) in a particular case when \(D_\lambda K_{\mu \nu} = 0\) or \(D_\lambda K_{\mu \nu} = \eta_\lambda K_{\mu \nu}\).

The dual spinning space was constructed without introduction of torsion. Dual Kerr-Newmann metric has the same Killing vectors as Kerr-Newmann metric but has no Killing-Yano tensors. In this case we can not construct a dual spinning space. This result is different from those obtained in [7].

A special case arises when we investigate dual Einstein's metrics. If \(g_{\mu \nu}\) belongs to type D and N in the Petrov's classification, then it has Killing-Yano tensors. A special attention will be devoted to Taub-NUT because it has one Killing-Yano tensor and three complex structures [9].

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References


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Балеану Д.
Симметрии двуальных метрик

Исследуются геометрическая двуальность между метрикой $g_{\mu\nu}$ и тензором Киллинга $K_{\mu\nu}$. Найдены условия, при которых симметрии метрики $g_{\mu\nu}$ и ее двуальной $K_{\mu\nu}$ — одинаковы. В случае, когда метрика $g_{\mu\nu}$ оказывается эйнштейновской, найдены соответствующие уравнения для $K_{\mu\nu}$.

Двуальное спинорное пространство построено без введения кручения. Общие результаты применены в случае метрики Керра—Ньюмана.

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Baleanu D.
Symmetries of the Dual Metrics

We study the geometric duality between the metric $g_{\mu\nu}$ and a Killing tensor $K_{\mu\nu}$. We found the conditions when the symmetries of the metric $g_{\mu\nu}$ and the dual metric $K_{\mu\nu}$ are the same. Dual spinning space was constructed without introduction of torsion. The general results are applied to the case of Kerr–Newmann metric.

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