Hybrid inflation with running inflaton mass

Laura Covi

Department of Physics,
Lancaster University,
Lancaster LA1 4YB. U. K.

E-mail: l.covi@lancaster.ac.uk

Abstract

We realize and study a model of hybrid inflation in the context of softly broken
supersymmetry. The inflaton is taken to be a flat direction in the superfield space
and, due to unsuppressed couplings, its soft supersymmetry breaking mass runs
with scale. Both gauge and Yukawa couplings are taken into account and different
inflationary scenarios are investigated depending on the relative strengths of the
couplings and the mass spectrum.

1 Introduction

One of the well–known problems of implementing the usual slow–roll inflation picture1
in the framework of supersymmetric theories is the fact that the second of the flatness
conditions,

\[ \epsilon \equiv \frac{1}{2} \left( \frac{V''}{V} \right)^2 \ll 1 \]  \hspace{1cm} (1)

\[ |\eta| \equiv \left| \frac{V''}{V} \right| \ll 1, \]  \hspace{1cm} (2)

where \( V \) is the inflaton potential and we have set \( M_p \equiv (8\pi G)^{-1/2} \equiv 1 \), is usually violated
by supergravity corrections [2]. All the scalar particles’ squared masses (and therefore
also the inflaton’s) receive a contribution at least of order \( V \) that spoils completely the
flatness of the potential.

Few proposals exists in literature able to solve this problem without fine tuning. We
will in the following consider an inflationary model of the type proposed by Stewart [3, 4],

\footnote{For a general discussion and references on inflation see [1].}
where quantum corrections to the inflaton mass flatten a region of the potential. The original model [3, 4], studied also in [5], considered the case of a charged inflaton field, where the dominant corrections were due to the gauge coupling and neglected the role of the Yukawa coupling. Since the model is of the hybrid type, and the end of inflation is determined by the Yukawa couplings, such a procedure has to be justified. We will see in an explicit case that this approximation is indeed acceptable in the case of small Yukawa coupling, but that in general such couplings can play a fundamental role in the flattening so that also the case of a singlet inflaton is possible.

In the next section we will briefly review Stewart model with a charged inflaton and introduce the formalism for taking into account one loop quantum corrections; in section 3 we will describe an explicit model based on $SU(N)$ non-abelian gauge group. In section 4 we will write the one-loop Renormalization Group equations for the parameters involved. In section 5 we will consider the case of very small Yukawa coupling, while in section 6 and 7 the case of a Yukawa coupling respectively of the order and much greater than the gauge coupling. In the limit of negligible gauge coupling, we will recover also the case where no gauge symmetry is present. Finally we will discuss the results and the naturalness of the model and comment on the experimental signatures. We will not address in detail this last issue, but refer to [5, 6] for a model independent analysis.

\section{The running-mass model}

In the model proposed by Stewart [4] (see also the review [1]), slow-roll inflation occurs, with the following Renormalization Group improved potential for the canonically normalized inflaton field $\phi$:

\begin{equation}
V = V_0 + \frac{1}{2} m_\phi^2(\phi) \phi^2 + \frac{1}{2} m_\psi^2(\phi) \psi^2 + \frac{1}{4} \lambda(\phi) \phi^2 \psi^2 \cdots .
\end{equation}

The constant term $V_0$ is supposed to dominate at all relevant field values. Non–renormalizable terms, represented by the dots, give the potential a minimum at large $\phi$, but they are supposed to be negligible during inflation. The last two terms also vanish during inflation, since $\psi = 0$, but are responsible for the exit from the inflationary period. In fact at some critical value $\phi_c$, the $\psi$ field effective mass $m_\psi^2(\phi_c) + 1/2 \lambda(\phi_c) \phi_c^2$ becomes negative and the fields roll towards the true vacuum with vanishing cosmological constant, characterized by

\begin{align}
\langle \phi \rangle &= 0 \quad (4) \\
\langle \psi \rangle &= V_0^{1/2}/|m_\psi| . \quad (5)
\end{align}

The inflaton mass-squared and all the other parameters depend on the renormalization scale $Q$, and following [3, 4] we have taken

\begin{equation}
Q = \phi ,
\end{equation}

2
where now $\phi$ denotes the classical v.e.v of the inflaton field during inflation. Such choice for the renormalization scale minimizes the one loop correction to the potential: in fact the main one loop contribution comes from the fields that acquire a mass proportional to $\phi$ and therefore goes like $\ln(\phi/Q)$ for $\phi$ larger than any other scale. If this is not the case some other $Q$ will be appropriate and the simplification given by eq.(6) is no more viable. We will assume that the inflaton v.e.v. is the dominant scale up to the end of inflation.

At the Planck scale, $m_\phi^2(M_{Pl})$ is supposed to be negative, with the generic magnitude

$$|m_0^2| = |m_\phi^2(M_{Pl})| \sim V_0 \quad (7)$$

coming from supergravity corrections [2, 5].

If there were no running, this would give $|\eta| \sim 1$, preventing slow–roll inflation. But at field values below the Planck scale, the RGE’s drive $m_\phi^2(\phi)$ to small values, corresponding to $|\eta(\phi)| \ll 1$, and slow–roll inflation can take place there\(^2\). We have in fact

$$\epsilon = \phi \left[ m_\phi^2 + \frac{1}{2} \frac{d m_\phi^2}{d \ln \phi} \right] \quad (8)$$

$$\eta = m_\phi^2 + \frac{3}{2} \frac{d m_\phi^2}{d \ln \phi} + \frac{1}{2} \frac{d^2 m_\phi^2}{(d \ln \phi)^2} \quad (9)$$

Since in this model the $\eta$ parameter changes considerably as $\phi$ decreases, slow-roll inflation is assumed to continue until some epoch $\phi_{\text{end}}$, when $\eta(\phi)$ becomes of order 1. Then $\phi$ start a brief phase of fast–roll up to the critical value $\phi_c$, where the mass term of the other field $\psi$ becomes negative and inflation finally ends some number $N_{\text{fast}}$ of $e$-folds after the end of slow-roll inflation, when the fields settle down in the true vacuum.

### 3 The model

Let us consider the case of the superpotential

$$W = \lambda S \text{Tr} (\phi_1 \phi_2) \quad (10)$$

where $S$ is a singlet chiral superfield, while $\phi_i$ are chiral superfield in the adjoint representation of the gauge group $SU(N)$. We can exclude other terms in the superpotential invoking some kind of discreet R-symmetry forbidding terms like $\text{Tr} (\phi_i^2)$ or higher powers of $S$. One such example could be $W \to e^{i\alpha} W$, $S \to e^{i\alpha} S$, $\phi_{1/2} \to e^{\pm i\alpha/4} \phi_{1/2}$.

In this case we can easily compute the scalar potential given by (10) in the limit of unbroken supersymmetry and, writing the adjoint fields in the fundamental basis\(^3\)

\(^2\)We are not addressing the problem of initial conditions ad assume that the inflaton tunnels in the small $\eta$ region from other values, for example the minimum given by the non-renormalizable terms at large field values.

\(^3\)We define the fundamental representation of $SU(N)$ $t_a$ such that $\text{Tr} (t_at_b) = \frac{1}{2} \delta_{ab}$ and $[t_a, t_b] = f_{abc} T_c$, while for the adjoint representation, e.g. $T_{ij}^a = f_{aij}$, we have $\text{Tr}(T_a T_b) = N \delta_{ab}$. 

3
\( \phi_i = \phi_i^a t_a \), it is given by:
\[
V = \frac{\lambda_2^2}{4} |\phi_1^a| |\phi_2^a|^2 + \frac{\lambda_4^2}{4} |S|^2 (|\phi_1^a|^2 + |\phi_2^a|^2) + \frac{|D_a|^2}{2}
\]
(11)

where \( S, \phi_i \) indicate now the scalar components of the chiral multiplets, summation over \( a \) is implicit and
\[
D_a = i \frac{g}{2} f_{abc} (\phi^{b*}_1 \phi^c_1 + \phi^{b*}_2 \phi^c_2)
\]
(12)

with \( g \) denoting the \( SU(N) \) gauge coupling.

We see clearly that a flat direction exists for
\[
S = 0
\]
(13)
\[
\phi_1^a \phi_2^a = 0
\]
(14)
\[
f_{abc} \phi_1^{b*} \phi_1^c = 0.
\]
(15)

This is not the most general case and other flat directions are present, parametrized by gauge invariant polynomials [9].

For our purposes we will anyway consider the case when the inflaton is one of the components of the charged fields, i.e. we will take \( \phi = Re \phi_1^a \) for example and all the other fields driven to zero. Given some dominant non-zero component \( \phi_1^a \) the potential is such that \( S \) and \( \phi_2^a \) are driven to zero by the effective mass term \( \lambda_2^2 |\phi_1^a|^2 \). The gauge group \( SU(N) \) is broken by the inflaton v.e.v. and some combinations of the other charged fields components, depending on the rank of the residual gauge group, are eaten up by the gauge bosons through the Higgs mechanism. For what concerns the other fields, either they are driven to zero by the D term or they do not interact with the inflaton\(^4\).

Then for the field \( \phi \) the potential reduces to zero and the only contribution is from the soft susy breaking terms [7]
\[
V_{ssb} = V_0 + m_S^2 |S|^2 + m_1^2 |\phi_1^a|^2 + m_2^2 |\phi_2^a|^2 + \left( \frac{Y}{2} \lambda S \phi_1^a \phi_2^a + h.c. \right)
\]
(16)

where \( m_S, m_1/2 \) are respectively the singlet and the charged fields susy breaking masses and \( Y \) denotes the supersymmetry breaking trilinear term. From supergravity, we expect all the susy breaking scalar masses to be of order of \( V_0 \) and the trilinear parameter to be of order \( V_0^{1/2} \). \( V_0 \) is a cosmological constant that is generated by some other sector of the theory and is cancelled in the true vacuum by the v.e.v. of some field in our sector, playing the role of the \( \psi \). Notice that if we consider the tree level potential, such

\(^4\)In the case of \( SU(2) \) we will have a \( U(1) \) residual symmetry and two of the gauge bosons would acquire mass together with two physical scalars and the corresponding fermions giving 2 massive vector multiplets. The remaining scalar \( Im \phi_1^a \) will also obtain a large mass from the D-term. For larger groups, it depends on the pattern of the breaking, e.g. \( SU(5) \) could be broken to the SM group (with 12 massive gauge bosons) or to \( SU(4) \times U(1) \) (with 8 massive gauge bosons) depending on which direction the inflaton field points.
field will have to develop a v.e.v. of the order of the Planck scale, since $V_0/|m_\psi^2| \simeq M_P^2$ reintroducing the Planck mass.

We see that along our flat direction, the potential reduces exactly to the form of eq. (3). For writing the RGE improved potential, we will need to consider the one loop renormalization group equations for our parameters.

### 4 Renormalization Group Equations

Following [8], we write down the equations for our particle content. The gauge field strength $\alpha = g^2/(4\pi)$ and the gaugino mass satisfy

\[
\frac{d\alpha}{dt} = \frac{\beta}{2\pi} \alpha^2 \tag{17}
\]

\[
\frac{d\tilde{m}}{dt} = \frac{\beta}{2\pi} \tilde{m} \tag{18}
\]

where $t = \ln(Q)$ is the renormalization scale and $\beta = -N$ in our case of $SU(N)$ with two matter superfields in the adjoint representation ($\beta = -3N + n_{adj}$).

This two equations are independent from the others and their solution is

\[
\alpha(t) = \frac{\alpha_0}{1 - \frac{\beta}{2\pi} \alpha_0 t} = \frac{\alpha_0}{1 + \tilde{\alpha}_0 t} \tag{19}
\]

\[
\tilde{m}(t) = \frac{\tilde{m}_0}{\alpha_0} \alpha(t) \tag{20}
\]

where $\tilde{\alpha}_0 = N\alpha_0/(2\pi)$.

For the Yukawa coupling, which we can always take real absorbing its phase in the definition of the singlet field $S$, we have instead

\[
\frac{d\lambda}{dt} = -N\frac{\alpha}{\pi} \lambda + \frac{\lambda}{16\pi^2} (N^2 + 1)|\lambda|^2 \tag{21}
\]

while the soft susy breaking masses follow the equations

\[
\frac{dm_i^2}{dt} = \frac{N^2 - 1}{16\pi^2} |\lambda|^2 \left[2m_i^2 + 2(m_i^2 + m_S^2) + |Y|^2 \right] \tag{22}
\]

\[
\frac{dm_i^2}{dt} = \frac{|\lambda|^2}{16\pi^2} \left[2m_i^2 + 2(m_i^2 + m_j^2) + |Y|^2 \right] - \frac{2N\alpha}{\pi} \tilde{m}_j^2 \tag{23}
\]

where $i \neq j$, $m_S, m_i$ are respectively the masses of $S, \phi_i$ and $Y$ is the trilinear susy breaking term.

We can simplify this system of first order equations by considering instead than the charged particle masses, other variables, i.e.

\[
m_{1-2}^2 = m_1^2 - m_2^2 \tag{24}
\]

\[
m_{1-S}^2 = m_1^2 - \frac{1}{N^2 - 1} m_S^2 \tag{25}
\]
we can then recast the system into the form:

\[
\frac{dm_{1-2}^2}{dt} = 0 \tag{26}
\]

\[
\frac{dm_{1-S}^2}{dt} = - \frac{2N\alpha}{\pi} \tilde{m}^2 \tag{27}
\]

\[
\frac{dm_S^2}{dt} = \frac{N^2 + 1}{8\pi^2} |\lambda|^2 m_S^2 + \frac{N^2 - 1}{8\pi^2} |\lambda|^2 \left[ 2m_{1-S}^2 - m_{1-2}^2 + \frac{|Y|^2}{2}\right]. \tag{28}
\]

The trilinear term will have instead the equation

\[
\frac{dY}{dt} = \frac{1}{32\pi^2} (N^2 + 1) |\lambda|^2 + \frac{2}{\pi} N\alpha \tilde{m}. \tag{29}
\]

These are a system of coupled differential equations. We will in the next sections consider approximate solutions in three different cases and obtain the running inflaton mass.

5 Small Yukawa coupling: \( \lambda^2 \ll \alpha \)

This case is the simpler and has been already considered in a model independent form in [5]. In this approximation we can neglect the \( \lambda^2 \) terms compared with the \( \alpha \) ones and inflaton mass running does not depend on the Yukawa coupling. We have then

\[
m_i^2(t) = m_i^2,0 - A_0 \left[ 1 - \frac{1}{(1 + \tilde{\alpha}_0 t)^2}\right] \tag{30}
\]

where \( A_0 = 2\tilde{m}_0^2 \). Notice that eq.(30) gives in general a solution of eq.(27), in the case of non negligible Yukawa.

For this expression for the running mass, the potential has a form studied in [5]: the inflaton mass becomes positive as \( \phi \) decreases and near the region where it vanishes the flatness condition \( 9 \) is satisfied. We will not consider here the inflaton dynamics in this potential, but analyze instead the condition for the end of inflation, namely the fact that the critical value does depend on the Yukawa.

Assuming universality in our sector, i.e. that all the scalar masses \( m_S^2, m_i^2 \) are equal at the Planck scale, we see that naturally the role of the \( \psi \) field is played by the singlet field \( S \), whose mass will stay practically constant and negative.

Two issues are to be considered here; first the fact that in the limit \( \lambda \to 0 \) the critical value becomes very large and practically no inflation takes place along our flat direction. We have that at tree level, from eq. (11),

\[
\phi_c^2 = \frac{-4m_{S,0}^2}{\lambda_0^2} \simeq \frac{4V_0}{\lambda_0^2}. \tag{31}
\]
and therefore a lower bound on the Yukawa couplings appears if we ask to have sufficient inflation.

Following the discussion in [5] we will consider that slow roll inflation ends before the critical value is reached and consider the bound on the Yukawa coupling coming from such condition, neglecting for the moment the running of both $\lambda, m_2^2$ (we will check later the consistency of our assumption). The end of slow roll is given by $\eta(\phi_{\text{end}}) = 1$, i.e. using (30) and (9),

$$\phi_{\text{end}}^2 \simeq \exp \left[ \frac{-2}{\tilde{\alpha}_0}\left(1 - \frac{1}{\sqrt{1 + \frac{V_0 + |m_{1,0}^2|}{A_0}}}\right)\right],$$

in Planck units; we have then the bound

$$\lambda_0^2 \geq 4V_0 \exp \left[ \frac{2}{\tilde{\alpha}_0}\left(1 - \frac{1}{\sqrt{1 + \frac{V_0 + |m_{1,0}^2|}{A_0}}}\right)\right].$$

(32)

(33)

As we can see this bound is very sensitive to the value of the gauge coupling and also $V_0$. For value of the parameters in the acceptable range given by [5], we have for example, for $|m_{1,0}^2| = V_0, A_0 = 2V_0, V_0 = 10^{-32}$,

$$8 \times 10^{-7} \leq \lambda_0^2 \ll \alpha_0 = 0.01\frac{2\pi}{N},$$

(34)

so that in this case an acceptable range of Yukawa couplings exists where our approximation is valid and slow roll ends before the critical value. We see anyway that the gauge coupling has to be not too small, otherwise the bound (33) is not consistent with our initial assumption $\lambda_0^2 \ll \alpha_0$. We can find a lower bound on $\alpha_0$ for any choice of the other parameters from the inequality:

$$\frac{1}{2}\alpha_0 \ln \left(\frac{\pi\alpha_0}{2N\sqrt{V_0}}\right) \geq 1 - \frac{1}{\sqrt{1 + \frac{V_0 + |m_{1,0}^2|}{A_0}}}.$$

(35)

The second issue is to check how our initial assumption is modified by the running of the relevant quantities, which even if small is there. To estimate it let us consider the approximate equation

$$\frac{d\lambda}{dt} = -N\frac{\alpha}{\pi}\lambda;$$

(36)

the solution to such equation is

$$\lambda = \lambda_0\frac{\alpha^2}{\alpha_0^2}.$$

(37)

We see therefore that

$$\frac{\lambda}{\alpha} = \frac{\lambda_0}{\alpha_0}\frac{\alpha}{\alpha_0}.$$

(38)
so that our initial assumption $\lambda_0^2 \ll \alpha_0$ remains valid as long as we do not enter the region $\alpha \gg \alpha_0$, which anyway will coincide to the non perturbative region for reasonable values of $\alpha_0$.

Finally let us consider how the definition of the critical value is modified by the running; we have already obtained the running of the Yukawa coupling in eq. (37). We can see clearly from eq. (28) that the corrections to $m_S^2$ due to the running are of order $\lambda_0^2$ or higher. To lowest order in $\lambda_0^2$ we have therefore

$$\phi_c^2 = -\frac{4m_{S,0}^2}{\lambda_0^2(t_c)} \simeq \frac{4V_0}{\lambda_0^2}(1 + \tilde{\alpha}_0 t_c)^4;$$

we see that the correction to the tree level relation is of order $\alpha_0$ and therefore small in the perturbative regime.

6 Fixed point region: $\lambda^2 \geq \alpha$

Let us consider now the case when $\lambda^2 \simeq \alpha$ and we can assume that the Yukawa coupling is quickly driven towards the fixed point solution

$$\lambda^2 = \frac{12\pi N}{N^2 + 1} \alpha.$$  

In this case the equation for the singlet mass and the trilinear term become

$$\frac{dm_S^2}{dt} = \frac{3N}{2\pi} \alpha m_S^2 + \frac{3N(N^2 - 1)}{2\pi(N^2 + 1)} \alpha \left[ 2m_{1-s}^2 - m_{1-2}^2 + \frac{1}{2} |Y|^2 \right]$$

$$\frac{dY}{dt} = \frac{3N}{8\pi} \alpha Y + \frac{2N}{\pi} \tilde{\alpha} \tilde{m};$$

they are easily solved, and we have for a real $Y_0$:

$$Y(t) = (Y_0 + \frac{16}{7} \tilde{m}_0)(1 + \tilde{\alpha}_0 t)^{3/4} - \frac{16}{7} \tilde{m}_0 \frac{1}{1 + \tilde{\alpha}_0 t}$$

$$m_S^2(t) = \left[ m_{S,0}^2 + (N^2 - 1)(B_0 + C_0 + D_0 + E_0) \right] (1 + \tilde{\alpha}_0 t)^3 - (N^2 - 1)B_0 (1 + \tilde{\alpha}_0 t)^{3/2}$$

$$-(N^2 - 1)C_0 - \frac{D_0(N^2 - 1)}{(1 + \tilde{\alpha}_0 t)^{1/4}} - \frac{E_0(N^2 - 1)}{(1 + \tilde{\alpha}_0 t)^2}$$

where

$$B_0 = \frac{1}{N^2 + 1} \left[ \frac{16}{7} \tilde{m}_0 + Y_0 \right]^2$$

$$C_0 = \frac{1}{N^2 + 1} \left[ m_{1,0}^2 + m_{2,0}^2 - \frac{2}{N^2 - 1} m_{S,0}^2 - 4 \tilde{m}_0^2 \right]$$

$$D_0 = -\frac{192}{91(N^2 + 1)} \tilde{m}_0 \left[ \frac{16}{7} \tilde{m}_0 + Y_0 \right]$$

$$E_0 = \frac{972}{245(N^2 + 1)} \tilde{m}_0^2$$

8
Figure 1: This figure shows the dependence of the masses and lambda coupling on the scale $t = \ln(\phi)$ for universal masses $-m_0^2$ at the Planck scale for $SU(2)$. We have taken in both cases $\tilde{\alpha}_0 = 0.01$, $\tilde{m}_0 = -Y_0 = m_0$ and rescaled the masses with respect to the initial absolute value. The lines labelled $A$ refer to the small Yukawa coupling case with $\lambda_0 = 10^{-5}$, while those labelled $B$ to the fixed point case $\lambda_0 = 0.388$. The full lines are the mass of the charged field $m_1^2(t)/m_0^2$, the dashed lines the singlet mass $m_S^2(t)/m_0^2$ and the dotted lines the Yukawa coupling $\lambda$.

are constant given in terms of the initial values.

From this solutions it is easy to obtain the inflaton mass in our case

$$m_1^2(t) = m_{1-s}(t) + \frac{m_S^2(t)}{N^2-1}$$

$$= m_{1,0} - \frac{m_{S,0}}{N^2-1} - A_0 - C_0 + \frac{A_0 - E_0}{(1 + \tilde{\alpha}_0 t)^2} - \frac{D_0}{(1 + \tilde{\alpha}_0 t)^{1/4}} - B_0 (1 + \tilde{\alpha}_0 t)^{3/2} + \left( \frac{m_{S,0}}{N^2-1} + B_0 + C_0 + D_0 + E_0 \right) (1 + \tilde{\alpha}_0 t)^3.$$  \hspace{1cm} (50)

We show the behaviour of the inflaton and singlet mass on Figure 1 for the case of $SU(2)$ and universal masses at Planck scale. Both the curves for the case of negligible Yukawa, eq. (30), and fixed point solution, eq. (50), are shown for comparison. Notice that the running is slightly weaker in the second case due to the presence of the Yukawa and trilinear terms, but that the inflaton mass tends to positive values also in this case. Moreover the singlet mass $m_S^2$, after becoming positive in a small region, returns negative, so that an hybrid end of inflation in this direction is always possible. The critical value in this case will be defined implicitly by

$$\phi_c^2 = \frac{-4m_S^2(t_c)}{\lambda(t_c)}.$$  \hspace{1cm} (51)
If we relax universality of the susy breaking masses at the Planck scale, taking \( m_{2,0}^2 \ll m_{1,0}^2 \) and \( m_{S,0}^2 \) sufficiently positive, the role of the \( \psi \) field can be played instead by the other charged field \( \phi_2^a \) and the true vacuum would correspond to a broken gauge group.

7 Large Yukawa coupling: \( \lambda^2 \gg \alpha \)

In this case we can neglect the terms proportional to \( \alpha \) with respect to those proportional to \( \lambda^2 \) and the equations become similar to those for uncharged fields. We can therefore consider at the same time the model where \( \phi_i \) are just two singlet fields substituting in the following \( N^2 \to 2 \). This substitution amounts to consider only one degree of freedom instead of the \( N^2 - 1 \) of a field in the adjoint representation of \( SU(N) \).

The Yukawa coupling follows the equation:

\[
\lambda^2 = \frac{\lambda_0^2}{1 - \frac{N^2+1}{2\pi^2} \lambda_0^2 t}; \tag{52}
\]

while the trilinear term is given by

\[
Y^2 = \frac{Y_0^2}{\sqrt{1 - \frac{N^2+1}{2\pi^2} \lambda_0^2 t}}. \tag{53}
\]

All the scalar masses have a behaviour similar to that of \( m_S^2 \):

\[
m_S^2(t) = \frac{N^2 - 1}{N^2 + 1} \left[ (m_{S,0}^2 + m_{1,0}^2 + m_{2,0}^2 + Y_0^2) \frac{1}{1 - \frac{N^2+1}{2\pi^2} \lambda_0^2 t} - \frac{Y_0^2}{\sqrt{1 - \frac{N^2+1}{2\pi^2} \lambda_0^2 t}} \right] - \frac{1}{N^2 - 1} \left( m_{1,0}^2 - m_{2,0}^2 + \frac{2}{N^2 - 1} m_{S,0}^2 \right) \tag{54}
\]

\[
m_i^2(t) = m_i^2 + \frac{1}{N^2 - 1} (m_S^2(t) - m_{S,0}^2). \tag{55}
\]

Note that in case of universality and \( SU(N) \), only the singlet mass changes sign due to the running; the other charged particle masses run slower since they interact with less particles. So a natural scenario is that where the role of the inflaton is played by the field \( S \), while some other of the fields, whose mass stays negative, is \( \psi \). Figure 2 displays the behaviour of the masses in case of a common initial value.

In the case of three singlet fields, all masses run similarly and never change sign, therefore the only way of having an hybrid inflationary scenario, without invoking other fields and Yukawa couplings, would be to relax universality. One possibility for inflation in this case, is to have negative initial masses and let one of them become positive like for the \( SU(N) \) case. Another option is that initial different positive masses are driven negative, or very small for what regards the inflaton, by the Yukawa coupling like in the case of the radiative EW breaking in the MSSM. In such a picture not only would the
quantum corrections be responsible for the flattening of the potential, but also for the triggering of the hybrid–type end of inflation.

The last two possibilities are shown in figure 3 for specific choices of the parameters.

8 Observational constraints

The spectral index behaviour in the running mass models is more easily understood using the linear approximation [6]. Cosmological scales must leave the horizon when the inflaton v.e.v. lies in the flat region of the potential, i.e. in the neighborhood of the point where $\epsilon$ vanish, $\epsilon(t_\star) = 0$ and $\ln(\phi_\star) = t_\star$. As described in [6], all the observational constraints on the spectral index and COBE normalization can be cast in a simple form considering the linear approximation for the running inflaton mass around $\phi_\star$ and defining the three parameters:

$$c = -\frac{dm_\star^2}{dt}|_{t=t_\star} = 2m_\star^2(t_\star)$$  \hspace{1cm} (56)$$

$$\tau = -|c| \ln(\phi_\star)$$  \hspace{1cm} (57)$$

$$\sigma = \lim_{\phi \to \phi_\star} c e^{N(\phi)} \ln(\phi_\star/\phi),$$  \hspace{1cm} (58)$$
Figure 3: The running of the different masses in case of non-universality of the initial mass scale for $\lambda_0 = 1, Y_0 = -m_0$ in the three singlet scenario. Case A: the initial masses are positive such that $m^2_{i,0} = m^2_{S,0}/2$; the dashed line line gives the behaviour of the $S$ mass, while the dotted line of the other two masses. Case B: the initial masses are negative and $m^2_{S,0} = m^2_{i,0}/2$; the full line is $m^2_S(t)$ while the dashed line is $m^2_i(t)$.

where $N(\phi)$ is the number of e-folds from $\phi$ to $\phi_{end}$ in the slow roll approximation:

$$N(\phi) = \int_{\phi_{end}}^{\phi} \frac{V}{V'} d\phi.$$  (59)

Computing this quantities in the specific cases would make possible to give the behaviour of the spectral index scale dependence and the bound on $V_0$ from COBE normalization and restrict the initial parameter space. We will not do this here for our particular model, but refer to [6] for a model independent analysis on the allowed parameter space for $c, \tau$ and $\sigma$. Let us note that in the language of that reference, the case of a negative inflaton mass at high scales corresponds to models (i) and (ii), while a positive inflaton mass at $M_P$, as mentioned in the last section, leads to models (iii) and (iv).

A clear experimental signature of this kind of running mass models is anyway the scale dependence of the spectral index $n$,

$$\frac{n - 1}{2} = \sigma e^{-cN} - c$$  (60)

that will be within the reach of the future Planck satellite measurements [10] in a large part of the parameter space.

## 9 Conclusions

We have described a particular model of hybrid inflation with a running mass and found that an interesting regime for inflation is present for any hierarchy of the initial couplings.
In most cases the potential is flattened by the gauge interaction and a charged inflaton is needed, but for large Yukawa coupling also this kind of corrections can be sufficient to give a small $\eta$ and a singlet inflaton can be considered.

Further investigations will be required for delimiting precisely the region in parameter space compatible with the present observational constraints and determine the naturalness of this scenarios. One point we would like to make is that a sizable region has been found for the case of a negligible Yukawa coupling [5]. The introduction of a Yukawa coupling near the fixed point solution, as discussed in section 6, should enlarge such region since $\lambda$ slows the running of the inflaton mass and therefore weakens the scale dependence of the spectral index. For what concerns the large Yukawa and the singlet inflaton cases, a more detailed analysis is needed, in particular in order to see how large $\lambda$ has to be to flatten sufficiently the potential. Notice that contrary to what happens in the usual inflationary picture, in our case the inflaton has to have sizeable couplings to other fields in order for the one loop correction to flatten the potential.

Another issue we would like to mention here is the identification of the sector the inflaton and the other fields belong to; since no matter fields in the adjoint representation appear in the MSSM, we are obliged to extend its particle content.

One possibility could be to identify our gauge group with colour $SU(3)$ and assume that the colour symmetry is broken during inflation to be restored at the end. Since at that point our adjoint fields would acquire a mass of order $M_P$ from the $S$ v.e.v., such scenario would not modify our low energy phenomenology. In that case all the particle of the MSSM would have to be taken into account and the RGE equations would be different from those studied here. In particular the colour gauge group would be non–asymptotically free due to the quarks contribution to $\beta$. A successful inflationary scenario can be constructed anyway in the non–asymptotically free case [6], but the picture would be certainly more complicated than described here due to the many fields involved and less attractive from the theoretical point of view.

Otherwise, we could think of accommodate our fields into some Gran Unified Theory, like $SU(5)$, or in some hidden sector. In the first case, the “right” symmetry breaking has to be considered in order to end in the MSSM minimum through a charged $\psi$ field and care has to be taken to insure that all the GUT particles masses are at the right scale. It would be probably difficult to realize such a picture without some sort of tuning.

In the second case instead, the model is certainly less constraint. Reheating or pre-heating could in this case be difficult to realize if the only interaction between the hidden and visible sector is gravitational. In this case anyway, the role of the $\psi$ field could be played by $S$ and reheating the universe to our visible sector could proceed through interactions of this singlet field with SM fields. If such singlet could be identified with a bulk modulus that could be achieved and a v.e.v. of the order of the Planck mass would be natural.

All this possibilities will be the subject of further studies.

Let us conclude here recalling that in any case the experimental signature of this kind of models, i.e. the scale dependence of the spectral index, is usually non negligible and
within the reach of the next satellite experiments.

Acknowledgements

The author would like to thank D. Lyth for many discussions on this subject, encouragement and comments on the manuscript and L. Roszkowski for discussions and encouragement; she is also grateful to M. Bastero-Gil, E. Copeland, A. C. Davis, S. Davis, X. Meng, G. Ross and S. Sarkar for discussions during the XIXth UK Institute for Theoretical High Energy Physicists in Oxford. The author is supported by PPARC grant GR/L40649.

References