Selection Effects, Biases, and Constraints in the Calán/Tololo Supernova Survey

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ABSTRACT

We use Monte Carlo simulations of the Calán/Tololo photographic supernova survey to show that a simple model of the survey’s selection effects accounts for the observed distributions of recession velocity, apparent magnitude, angular offset, and projected radial distance between the supernova and the host galaxy nucleus for this sample of Type Ia supernovae (SNe Ia). The model includes biases due to the flux-limited nature of the survey, the different light curve morphologies displayed by different SNe Ia, and the difficulty of finding events projected near the central regions of the host galaxies. From these simulations we estimate the bias in the zero-point and slope of the absolute magnitude-decline rate relation used in SNe Ia distance measurements. For an assumed intrinsic scatter of 0.15 mag about this relation, these selection effects decrease the zero-point by 0.04 mag. The slope of the relation is not significantly biased. We conclude that despite selection effects in the survey, the shape and zero-point of the relation determined from the Calán/Tololo sample are quite reliable. We estimate the degree of incompleteness of the survey as a function of decline rate and estimate a corrected luminosity function for SNe Ia in which the frequency of SNe appears to increase with decline rate (the fainter SNe are more common). Finally, we compute the integrated detection efficiency of the survey in order to infer the rate of SNe Ia from the 31 events found. For a value of \( H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}^{-1} \) we obtain a SN Ia rate of \( 0.21^{+0.30}_{-0.13} \) SNe. This is in good agreement with the value \( 0.16\pm0.05 \) SNe recently determined by Capellaro et al. 1997.

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1. Introduction

The Calán/Tololo (CT, hereafter) Survey was a classical photographic search for supernovae initiated in June 1990 with the principal goal of examining the Hubble diagram for Type Ia supernovae (SNe Ia) out to redshifts of $\sim0.1$ (Hamuy et al. 1993, hereafter referred to as Paper I). Upon its completion in November 1993, the survey had discovered 49 SNe, 31 of which were spectroscopically classified as SNe Ia (Hamuy et al. 1996c, hereafter referred to as Paper VII). Because the survey was carried out in a systematic and uniform way and complemented with followup photometric and spectroscopic observations of unprecedented quality, it provides a unique opportunity to study the properties of the Type Ia family.

In any survey, however, selection effects present a major problem in drawing firm conclusions from the observed data. Because the CT survey was flux-limited, we expect that Malmquist bias will have favored the discovery of the intrinsically-brightest objects of the supernova luminosity distribution and suppressed the detection of the intrinsically-faintest events over most of the volume surveyed.

Another bias is expected which arises from the different rates of evolution exhibited by SNe Ia. It is by now well established that the peak magnitudes of SNe Ia are correlated with the duration of the peak in their lightcurves (Phillips 1993, Hamuy et al. 1996a, hereafter referred to as Paper V). Because the survey was limited to relatively infrequent observations when compared with the length of time the SNe are at their brightest, a selection effect similar to Malmquist bias will occur in which the bright supernovæ are more likely to be detected not only because they are brighter but also because they last longer.

A possible consequence of these biases is a shift of the zero-point of the Hubble diagram, and hence the Hubble constant, to larger values. Because SNe Ia are seen to exhibit a large scatter ($\sim2$ mag in B) in their maximum-light luminosities (Phillips 1993, Paper V), this shift could be substantial. Using the peak luminosity-decline rate relation, initially found by Phillips 1993 from nearby SNe Ia, the observed peak magnitudes can be “corrected” to produce a Hubble diagram with significantly lower scatter $\sim0.10-0.15$ mag (Hamuy et al. 1996b, hereafter referred to as Paper VI; see also Riess et al. 1996 for a different approach to applying the same correction). Although we can expect the bias in the zero-point of this corrected Hubble diagram to be significantly reduced, concern has been expressed regarding the actual shape of the peak luminosity-decline rate relation (hereafter referred to as $M/\Delta m_{15}$ relation) which determines the amount of correction to be applied to the observed magnitudes. In fact, the relation determined from the CT sample appears to be shallower than that implied by the nearby SNe (Paper V), a possible consequence of selection effects in the photographic survey.
Both the zero-point and slope of the $M/\Delta m_{15}$ relation are relevant to the determination of the Hubble constant from intermediate-distance SN samples and of cosmological parameters (such as $\Omega_M$ and $\Omega_\Lambda$) from high-redshift SNe Ia (Perlmutter et al. 1995, Schmidt et al. 1998). The very existence of such a correlation, and its detailed form, is an important clue to understanding the nature of the explosions themselves.

The main goal of this work is to derive a simple model for the selection effects of the CT survey and for the parent population of SNe Ia that suitably reproduces the observational properties of the $\sim30$ discovered SNe Ia. This model may then be used to estimate the biases in the observational parameters derived from the CT sample and to infer intrinsic properties of the Ia family like the $M/\Delta m_{15}$ relation and the luminosity function.

After summarizing the observational data (Section 2), in Section 3 we use simulations to test the selection effects proposed and derive parameters which best fit a model for the parent population based on the observed $M/\Delta m_{15}$ relation and a flat luminosity function. After exploring its parameter space in some detail, in Section 4 we use this model to estimate the biases in the observed $M/\Delta m_{15}$, luminosity function, and galaxy type-redshift relation. With corrected versions of these functions, in Section 5 we iterate our simulations in order to test our revised assumptions about the intrinsic properties of SNe Ia. Armed with this improved model, in Section 6 we compute the detection efficiency of the survey and estimate the actual rate of SNe Ia.

2. Observations

The details of the CT Supernova Survey were extensively described in Paper I. In this section we summarize only those features of the survey necessary to modeling selection effects. The search phase of the survey was begun in June 1990 using the Cerro Tololo Interamerican Observatory (CTIO) Curtis Schmidt Camera which has a 19.05 cm x 19.05 cm field and a scale of 96.6 arcsec mm$^{-1}$, providing a useful sky coverage of 26.13 square degrees when used with photographic plates. The search was performed with hypersensitized (unfiltered) Iia-O plates which yielded a limiting magnitude for isolated sources of $m_{pg} \sim 19$ with an exposure of 20 min on moonless nights. The plate scanning was performed by experienced assistants at the Department of Astronomy of the University of Chile. A detailed comparison of each plate with a first-epoch exposure was carried out visually with a Zeiss-Jena blink comparator.

The survey began with observations of 60 fields located at high galactic latitudes covering from 0 to 24 h in right ascension. Given the large area covered by the photographic
plates, the resulting sample of galaxies surveyed in the program was not seriously biased to any particular Hubble type. Table 1 of Paper I lists the equatorial coordinates of the selected fields. During 1990 and 1991 each field was observed at \( \sim 30 \) day intervals. As of March 1992 the frequency of observations was increased to twice per month in order to discover SNe at earlier stages in their evolution. This required the elimination of 15 fields from the initial list; the remaining 45 fields were observed until the end of the search phase on November 1993. A total of 1,019 plates were obtained in the course of the program. Figure 1 shows the complete time sampling of the 60 fields.

The survey discovered a total of 49 SNe, of which 31 were spectroscopically confirmed as SNe Ia\(^1\). Table 1 gives a complete listing of the observed properties of these SNe Ia, of which useful follow-up CCD photometry was obtained for 26 events. These constitute our sample of “best-observed” SNe. 60% of these occurred in spiral galaxies.

Some of the characteristics of the CT sample can be appreciated from the histograms shown in Figure 2. The recession velocity distribution shows a systematic increase in the number of supernovae detected in redshift bins with velocities from 0 to \( \sim 15,000 \) km sec\(^{-1}\). This is expected from a spatially uniform distribution of SNe as the volume of each velocity (distance) bin \((4\pi r^2 dr)\) increases quadratically with distance. At larger distances the SNe become significantly harder to discover because they appear fainter to the observer, the objects remain above the detection threshold for a shorter time, and a larger fraction of the SNe appear projected against the over-exposed central parts of the host galaxies (the “Shaw” effect described below). The cutoff at \( \sim 30,000 \) km sec\(^{-1}\) is the signature of a flux-limited survey; no supernovae are bright enough to be detected at greater distances with the survey’s nominal limiting magnitude. Given the linear relation of the Hubble law between apparent magnitude and the logarithm of the redshift, the peak apparent magnitude distribution is similar to the velocity histogram, reaching a maximum at B\( \sim 17.5\).

The absolute magnitude distribution displays a sharp peak around \( M_{MAX}^B = -19.0 + 5\log(H_0/65) \), with a dramatic decline toward less luminous events. It is hard to ascertain at this stage whether the cutoff is real or due to statistical fluctuations arising from the small sample size. Clearly, the overall shape of this distribution does not correspond to the actual luminosity function of SNe Ia as intrinsically fainter events are less likely to be discovered by the search. The decline rate distribution displays a similar behavior to that of absolute magnitude, in accord with the linear relation between peak

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\(^1\)Note that in previous papers of this series an additional SN Ia (1992au) was included in the list of discoveries. This will not be considered in the statistical analysis of this paper since this object was found as part of a QSO search on an objective prism IIIaJ plate.
luminosity and decline rate displayed by the CT SNe (Paper V). A number of differences are evident, however. The decline rate distribution is somewhat flatter, with a shallow maximum around $\Delta m_{15}(B)=1.2$ with no obvious cutoff toward fast-declining (faint) SNe.

The angular separation histogram has a sharp maximum at $\sim 7$ arcsec and trails off toward larger offsets. The remarkable lack of objects discovered at projected angular separations less than 4 arcsec is a clear indication that the search fails to discover SNe at or near the center of the surveyed galaxies – it is difficult to imagine a physical effect which would lead to the absence of supernovae where the stellar density is greatest. This effect was noted originally by Shaw 1979, who interpreted it as an observational bias inherent to all photographic surveys; the central parts of the parent galaxies are over-exposed, making it impossible to detect the additional brightening from a supernova. The projected radial distance distribution is similar to the angular separation histogram. The distribution has a peak at $\sim 6$ kpc followed by a tail suggestive of the distribution of light from an exponential disk.

The ideal tool for studying the spatial distribution of the discovered events is the $V/V_{MAX}$ test. Unfortunately, this requires an estimate of the limiting magnitude at which supernovae could be discovered for each of the discovery plates. While it is in principle possible to measure the limiting magnitude for isolated sources for each observation, the threshold for SN detection is much harder to estimate from photographic plates since the SN are generally superimposed on the bright background of the host galaxies and a plethora of effects such as confusion and image saturation make the SN detection threshold a function of position and host galaxy morphology as well as the overall plate limit. We have therefore been unable to calculate $V/V_{MAX}$ values for the CT SNe.

3. Simulating the Survey

Our goal is to propose a simple model for the selection effects of the CT photographic survey which employs the fewest assumptions and parameters yet accounts as closely as possible for the observed distributions in Figure 2. To test the model, we have performed Monte Carlo simulations of a parent population of SNe with the following characteristics.

3.1. Sampling from the parent population

We assume that the SNe occur in galaxies. Although the galaxy distribution is clumpy, the CT survey covers a sufficient volume of space that “clumpiness” from large
scale structure should not have a significant effect. Each photographic plate covers \( \sim 230 \) (\( H_0/65 \))\(^{-1} \) Mpc in depth and \( \sim 20 \) (\( H_0/65 \))\(^{-1} \) Mpc on the plane of the sky (for an average redshift of 15,000 km sec\(^{-1} \)). This is several times larger than the galaxy clustering length of 8.3 (\( H_0/65 \))\(^{-1} \) Mpc obtained from analysis of the angular two-point correlation function (Peebles 1993). We therefore assume for the purpose of this investigation that the parent population of SNe is uniformly distributed in space, and to each generated SN we randomly assign a distance within a spherical volume. The radius of this sphere is chosen to be just larger than the maximum distance to which the most luminous event can be observed (given the limiting magnitude of the simulation). We parametrize the time of the SN explosion by the time of maximum light which we randomly sample within a period of 1,460 days, the 4 years during which observations were obtained. Because it is potentially possible to discover supernovae at times much later than maximum light, this time window begins on JD 2447866.5 which corresponds to 200 days before the first observation epoch of the CT survey. Because the risetime of the B lightcurve is \( \ll 18 \) days, the window ends 19 days after the last observing run.

To each generated SN we assign an absolute magnitude from the \( M/\Delta m_{15} \) relation found in Paper V, namely,

\[
M_{MAX}^B - 5\log(H_0/65) = a_0 + b_0[\Delta m_{15}(B) - 1.1],
\]

where \( a_0=-19.258 \) and \( b_0=0.784 \). Note that both of these coefficients are entirely determined from the observed sample of SNe Ia and do not depend on the choice of the Hubble constant (\( H_0 \)). The latter can be obtained from \( a_0, b_0, \) and the absolute magnitude of a SN Ia with known decline rate. If \( M_{1,1}^B \) denotes the absolute magnitude of a “standard” SN Ia, one corrected to a decline rate of \( \Delta m_{15}(B) = 1.1 \), equation (1) becomes,

\[
M_{1,1}^B - 5\log(H_0/65) = a_0,
\]

from which the value of \( H_0 \) can be obtained directly. In fact, this method was employed in Paper VI using the absolute magnitude of nearby SNe Ia recently determined by Sandage and collaborators (Sandage et al. 1996, and references therein) to infer a value of \( H_0(B) \sim 64 \). For the sake of generality, we have chosen in this paper to leave the Hubble constant as a free parameter in the simulations, parameterizing it with \( M_{1,1}^B \) according to equation (2).

\[^2\]Note that in Paper V (and the other papers of the series) the labels of the figures have the wrong sign for the logarithm. This is a purely cosmetic error which affects neither the content nor the conclusions of the papers.
There are several potential problems in adopting the observed \( M/\Delta m_{15} \) relation. First, this equation was obtained from a sub-sample of CT SNe with decline rates ranging between \( 0.87 \leq \Delta m_{15}(B) \leq 1.7 \). At decline rates larger than 1.7, the CT sample contains only one object, SN 1992K, with \( \Delta m_{15}(B)=1.93 \), which appears to be 2\( \sigma \) fainter than the value given by equation (1). Second, it may well be that the actual shape of this function departs from the assumed linear form, even on the range \( 0.87 \leq \Delta m_{15}(B) \leq 1.7 \). A recent re-examination of the \( M/\Delta m_{15} \) relation after applying corrections for host galaxy extinction suggests that the data support fitting this relation by a quadratic function (Phillips et al. 1998). In any case, as the underlying cause of the relation is largely unexplained, there is no physical reason to expect any particular functional form. A linear fit is an adequate approximation for the majority of objects observed and a convenient choice for the purpose of illustrating selection effects. In section 4.1 we will consider simulations with a quadratic function. The third possible problem with equation (1) is that the coefficients \( a_0=-19.258 \) and \( b_0=0.784 \) could be substantially biased by the selection effects of the search. The assumption of a \( M/\Delta m_{15} \) relation of the form of equation (1) for the parent population will be reviewed in detail in Section 4.1, after evaluating possible observational biases in the search.

The process of assigning an absolute magnitude to each generated SN starts by generating a uniform random value for the decline rate in the interval \( \Delta m_{15}(B)=0.87-1.93 \) which spans the entire range of decline rates of the best-observed SNe Ia. By construction, the decline rates of the CT SNe were also limited to this interval from comparison of the light curves of these objects to that of a family of six template lightcurves where SNe 1992bc (with \( \Delta m_{15}(B)=0.87 \)) and 1991bg (\( \Delta m_{15}(B)=1.93 \)) are at the extremes of the distribution (Hamuy et al. 1996d, hereafter Paper VIII). The assumption of a flat decline rate distribution in the simulation (which, since the relation between absolute magnitude and decline rate is assumed to be linear, corresponds to a flat luminosity function for SNe Ia) is just a hypothesis at this stage, one which is further examined in Section 4.2. Given the decline rate, we use equation (1) to calculate the SN absolute magnitude. To this we add a Gaussian scatter with \( \sigma_M=0.15 \) mag (consistent with the observational scatter of the corrected Hubble diagram). The distance and absolute magnitude are then used to calculate the peak apparent magnitude. The recession velocity is determined from Hubble’s law, the chosen value of the Hubble constant (computed from the absolute magnitude \( M_B^{1\sigma} \)), and an additional Gaussian scatter of \( \sigma_V=600 \) km sec\(^{-1} \) to allow for the peculiar velocity of the host galaxy.
3.2. Adding the selection effects

We define a successful SN discovery as an event which satisfies two criteria. The first is that at the time of observation the event has an apparent magnitude brighter than the limiting magnitude of the photographic plate. The second is that it has a minimum projected separation from the nucleus of the parent galaxy.

Because a SN is a transient event we not only need to know its peak apparent magnitude but also its brightness at the specific time of the observation. This requires us to model the luminosity evolution of events which have been found to exhibit a wide range of light curve morphologies. In Paper VIII we presented a family of six template light curves for SNe Ia for days (relative to B maximum) -5 through +80 and spanning the entire range of observed decline rates of the best-observed SNe Ia ($\Delta m_{15}(B)=0.87\text{--}1.93$). In this paper we adopt an expanded version of these templates which covers a wider range in age (from day -14 to +600). The new values, given in Table 2, were obtained from photometry of these same six SNe. In those cases where the photometric data were insufficient to span the desired age range, it was necessary to extrapolate the new values. The extrapolations to late epochs are fairly certain since SNe display a linear decline in magnitude after day $\sim 30$; as shown in Section 5 no supernovae were “discovered” in the simulations after day 255. Before maximum light, on the other hand, the templates are much steeper and the extrapolation suffers an additional uncertainty.

Armed with this family of six extended B templates, for each simulated SN we perform an interpolation in decline rate in order to produce a synthetic B light curve. Then we randomly select one of the CT field numbers in order to choose a time sampling (this does not constrain the SN candidate by its location in the sky). For the first observation epoch of the selected field (which is taken from the observing records of the survey) we calculate the apparent magnitude of the generated SN. Since the age of the SN at the specific observation epoch is known (from the randomly generated epoch of maximum and the time of observation), we interpolate in time within the synthetic light curve in order to calculate the magnitude difference of the SN between the observation epoch and peak. Because we know the peak apparent magnitude, we may finally calculate the present-epoch apparent magnitude and ask whether the SN is brighter than the limiting magnitude. If the SN is detected on the first plate, we add this object to the list of positive detections and move on to the next SN candidate. If the SN is not detected in the first epoch we continue with the subsequent epochs of observation for the selected field until the SN is either detected or the survey of that field is terminated.

Although we know the nominal limiting magnitude of the survey ($m_{pg} \sim 19$), the individual values for the complete set of plates are not available nor do we know the
limiting magnitude for SN discovery. Thus, in the simulations the limiting magnitude is a free parameter ($B_{lim}$) with a characteristic fluctuation of $\sigma_{B_{lim}}=0.3$ mag. This fluctuation allows for variations of the detection threshold between plates due to seeing, clouds, the phase of the Moon, plate quality, the difficulty of finding SNe projected on galaxies with different surface brightnesses, etc.

As we do not have light curve information before day -14, we assume that SNe younger than this age are not detected, and to increase the efficiency of the simulation we assume that SNe older than 600 days are too faint ever to be seen. These assumptions have very little effect on our results since only a very small fraction of the generated SNe are located close enough to the observer to be brighter than the detection threshold for ages outside these limits.

As seen in the angular separation histogram of Figure 2, the survey is clearly biased against the detection of SNe near the center of the host galaxy. This occurs because such SNe have low contrast with respect to the bright background (frequently overexposed) on which they lie, increasing the difficulty of discovery in a visual scan of the plate. For our simple model we assume that the SNe occur in idealized host galaxies consisting of smooth exponential disks characterized by a luminosity scale length (a free parameter in the simulations which we denote $r_0=1/\alpha$), with the radial distribution of SNe following that of the host galaxy light. The radial distribution is assumed to be the same for all SN Ia lightcurve morphologies. We assign random orientations to the disk galaxies with respect to the observer in order to calculate the projected angular offset. To account for the observed bias, a successful detection requires not only an apparent magnitude brighter than the limiting magnitude but also a minimum projected separation between the SN and the parent galaxy center of 4 arcsec (in agreement with the observed limit from Figure 2).

We have made the computationally expedient choice to represent all host galaxies as exponential disks. Although a significant number of events (40% of the total) occurred in ellipticals (see Table 1), this choice affects only the estimate of the SN rate (section 6). Given the other, larger uncertainties involved in this calculation such a simplification has only a marginal effect. When appropriate, in the following sections we will include in our discussion radial distributions following a de Vaucouleurs $r^{1/4}$ law characterized by an effective radius $r_{eff}$. As will be seen, the effect of changing the form of the radial distribution we adopt is small.

Of course, the real detection probability depends on many other factors such as the proximity to spiral arms or HII regions (confusion), or the presence of possibly strong and patchy obscuration by dust. We do not consider any such further effects as they are difficult to model without introducing numerous ill-defined free parameters. We shall find below
that our simple model, without such embellishments, is quite sufficient to reproduce the observations.

3.3. Exploring the parameter space

The results of the simulations depend upon a number of parameters which are listed in Table 3. While the determination of all of these may, in principle, be biased by the manner in which the survey was conducted, most either do not affect the results to any significant extent or are fixed to reasonable accuracy by observation. $a_0$, $b_0$, and $\sigma_M$ have been determined from the survey in Paper V. The choice of peculiar velocity dispersion ($\sigma_V$) is well determined by observations (Marzke et al. 1995) and proves to be a small fraction of the recession velocity for most of the discovered SNe. The value of the scatter in the limiting magnitude ($\sigma_{B_{lim}}$) is just a guess at this stage. The value of the angular separation cutoff seems well determined by the histogram in Figure 2. We will adopt these values (denoted “fixed” in Table 3) for present purposes and their sensitivity to selection effects will be explored in Section 4 ($a_0$, $b_0$, and $\sigma_M$) and Section 5 ($\sigma_{B_{lim}}$).

To begin our exploration of this space of parameters, we choose to vary the three most important as determined by numerical experiment: $M^{B}_{1.1}$, $B_{lim}$, and $\alpha$. We divide the range of plausible values for these parameters, $-19.758 \leq M^{B}_{1.1} \leq -17.958$, $17.8 \leq B_{lim} \leq 18.8$, and $0.05 \leq \alpha \leq 0.50$ kpc$^{-1}$, into ten intervals each and perform simulations for each of the 1000 combinations, stopping the simulation after 9,000 “supernovae” are detected.

We measure the relative merit of a given simulation by comparing the generated histograms in recession velocity and angular separation to those of the survey given in Figure 2. The probability that a given observed histogram is drawn from the same distribution as the synthetic data is calculated from a Kolmogorov-Smirnov (KS) statistic, and the overall success is measured by the joint probability that both distributions match. A first check on the model is that a satisfactory solution exists for some combination of these parameters.

Figure 3 shows the resulting contour levels of the KS probability for the angular separation distribution (thin lines) in the $B_{lim}$-$\alpha$ plane, for four different values of $M^{B}_{1.1}$. It is remarkable that these surfaces are well organized and exhibit simply-connected maxima within the adopted parameter range, and further remarkable that the maxima for the different probabilities overlap.

For a fixed value of $M^{B}_{1.1}$, the KS probabilities for the angular separations (thin lines) are most strongly affected by $\alpha$ which measures the scale length of the exponential disks.
The probabilities prove quite insensitive to the limiting magnitude of the simulation. It is interesting to note that the value of $\alpha$ that yields the highest KS probability for the angular separation distribution increases for fainter SN absolute magnitudes (i.e. the scale length decreases for fainter absolute magnitudes). This comes about because fainter absolute magnitudes imply that successful detections will occur in galaxies located at smaller distances. This is shown in the left panels of Figure 4 for two widely different values of $M_{l,1}^B$. Smaller values of $M_{l,1}^B$ make the characteristic angular size of the SN host galaxies increase, broadening the distribution of projected angular separations as can be seen in Figure 4 (right panels). The only way to compensate for this broadening and simultaneously match the observed distribution of angular separations (Figure 2) is to decrease the scale length of the disks (by increasing $\alpha$). What we learn from this test is that different combinations of $\alpha$ and $M_{l,1}^B$ values can give equally good KS probabilities for the angular separation; it is not possible to break this degeneracy with this test alone.

Returning to Figure 3, the thick lines show the contour levels of the KS probability for the recession velocity distributions. For a fixed value of $M_{l,1}^B$, these probabilities are quite sensitive to $B_{lim}$. This is to be expected since the upper cutoff of the recession velocity distribution of SNe is determined by the limiting magnitude for SN detection. These probabilities depend somewhat on $\alpha$ but very little on $M_{l,1}^B$. For a given value of $M_{l,1}^B$ it is possible to identify in this diagram the values of the $\alpha$ and $B_{lim}$ parameters which simultaneously yield the highest KS probabilities for the angular separation and recession velocity distributions. These “best-values” are indicated with the $\oplus$ symbol in Figure 3. For $B_{lim}$, we find a “best-value” of 18.3 for all values of $M_{l,1}^B$. For $\alpha$ we find a “best-value” between 0.15 and 0.3 kpc$^{-1}$, depending on $M_{l,1}^B$.

In order to examine the possibility of a bias in the determination of $\alpha$ from the inclusion of SNe in elliptical galaxies, we have explored the same parameter space with elliptical hosts removed from the CT sample (13 events). Although elliptical hosts dominate the events at large radial separations, the distributions of angular separation for ellipticals and spirals are very similar (see Figure 2). We thus do not expect the value of $\alpha$ we obtain to be sensitive to the inclusion of SNe with elliptical hosts and, indeed, the simulations confirm this. Another interesting test is to determine the effective radius for elliptical hosts employing a SN radial distribution following an $r^{1/4}$ law. An examination of the parameter space determined by the distribution of angular separations for the 9 CT events in early type galaxies yields a “best-value” for $r_{eff}$ between 3 and 7 kpc, depending on $M_{l,1}^B$ ($r_{eff}$ decreases for fainter absolute magnitudes). For $B_{lim}$ we find again a “best-value” of 18.3.

The “best-value” of 18.3 for $B_{lim}$ is reasonable since the nominal limiting magnitude of the plates for isolated sources is $\sim 19$ and the limiting magnitude for SN discovery must
be brighter than this given the additional difficulties in finding SNe described above. If we
pick an absolute magnitude consistent with that of Paper VI ($M_{B_{1.1}}=-19.258$; $H_0=65$),
the “best value” for $\alpha$ is 0.15 kpc$^{-1}$, which corresponds to a scale length of $r_0=6.7$ kpc for the
disks of the host galaxies. This value lies within the range (0.6-8.0 kpc) of observed scale
lengths for spiral galaxies, properly scaled to $H_0=65$ (van der Kruit 1988). Likewise, the
“best value” for $r_{\text{eff}}$ of 6 kpc ($H_0=65$) proves to be in good agreement with those typically
measured in ellipticals (Kormendy 1977).

Figure 3 shows that the KS tests provide a means of assessing the goodness of fit for
each simulation and determining the best values of $\alpha$ and $B_{lim}$. The value of $M_{B_{1.1}}$ (and
thus $H_0$) remains undetermined from this test, however. To illustrate the agreement of
this simple model with observations, we have run a simulation with an absolute magnitude
consistent with that of Paper VI and the corresponding “best-values” $B_{lim}=18.3$ and $\alpha=0.15$
kpc$^{-1}$. Figure 5 shows the recession velocity, apparent magnitude, angular separation, and
radial distance distributions of 50,000 detected SNe, along with the observed histograms.
The match to the observations is satisfactory with KS probabilities of 0.74 and 0.91 for the
recession velocity and angular separation distributions, respectively. The KS probabilities
for the apparent magnitude and radial distance distributions (0.46 and 0.20), on the other
hand, are significantly lower, but this is not unexpected since the “best values” for the
fitting parameters correspond only to those that maximize the KS probabilities for recession
velocity and angular separation. For the time being we consider this to be a satisfactory
solution that will allow us to estimate biases in the CT survey in the following section.

4. Evaluation of Biases in the Calán/Tololo Sample

We have shown that our simple model for the selection effects can satisfactorily
reproduce the angular separation and recession velocity distributions of the CT SN sample.
In this section we employ the “best-values” for the model parameters ($M_{B_{1.1}}=-19.258$,
$B_{lim}=18.3$, $\alpha=0.15$ kpc$^{-1}$) in order to quantify biases in the observed $M/\Delta m_{15}$ and
luminosity functions, and to look for bias in the galaxy type-redshift relation.

4.1. The absolute magnitude–decline rate relation

We have assumed thus far that the parent population of SNe obeys the $M/\Delta m_{15}$
relation given by equation (1) and have ignored possible biases in the observed zero-point
and slope of this relation due to selection effects. In order to examine the effect that the
selection criteria have on constraining the parent population of SNe, we first analyze an idealized situation where we suppress the peculiar velocities of the host galaxies ($\sigma_V = 0$), the scatter in limiting magnitude ($\sigma_{B_{lim}} = 0$), and any scatter about the $M/\Delta m_{15}$ relation ($\sigma_M = 0$). With these (over)simplifications we consider the following four cases:

- **Case I (Malmquist bias):** We assume that the generated events have constant luminosities (independent of time) and we select them solely on the basis that their apparent magnitudes are brighter than $B_{lim}$. The left panel of Figure 6 (top) shows the absolute magnitudes of 1,000 detected SNe as a function of recession velocity. This is the classical Malmquist bias of a flux limited survey, in which the faintest objects of the luminosity distribution are lost near the detection limit. The diagonal dashed line is the relation $M^B_{MAX} = B_{lim} - 5\log(v_{MAX}/H_0) - 25$, and is drawn to indicate the maximum recession velocity to which each SN type can be found. The horizontal solid line indicates the average absolute magnitude of the parent population ($M^B_{MAX} = -19.023$) and the horizontal dashed line corresponds to the average magnitude of the detected population ($M^B_{MAX} = -19.091$); their difference is the bias introduced in the zero-point of the $M/\Delta m_{15}$ relation. In this case the bias amounts to only 0.068 mag. In general, its magnitude is an approximately quadratic function of the spread of the absolute magnitudes of the parent population. The corresponding $V/V_{MAX}$ distribution for the detected SNe is shown in the right panel of this figure (top). The histogram is flat, with an average value of 0.5, the expected distribution of a flux-limited sample drawn from a spatially uniform parent distribution.

- **Case II (Malmquist bias and Shaw effect):** Here, objects are selected not only by demanding that their apparent magnitudes are brighter than $B_{lim}$, but also that their angular separations from the nucleus of the host galaxy are sufficiently great (the Shaw effect). Although it proves difficult to see any additional effect with respect to Case I in the left panel of Figure 6 (second plot from top to bottom), it is evident that the $V/V_{MAX}$ histogram (right panel) becomes somewhat skewed toward lower values, a clear indication that more distant objects are more difficult to discover. This comes about because the minimum angular separation of 4 arcsec required for detection maps into increasingly larger radii in the more distant galaxies. Fewer of the SNe generated in distant galaxies are detected because a larger fraction of the stellar mass lies within the “exclusion” region. The Shaw effect reduces the detection efficiency by $\sim 24\%$ with respect to Case I.

- **Case III (Malmquist bias and “light-curve” bias):** We now include the fact that the luminosity of SNe Ia evolves on a timescale comparable to the interval between observations, but ignore the effect of proximity to the host galaxy nucleus. We generate the lightcurves of the supernovae from the templates as described above. For successful detection we demand that the apparent magnitude at the time of observation is brighter than the detection
threshold. In this case the most distant objects prove more difficult to detect because they remain above the detection threshold for a shorter interval in time. The $V/V_{MAX}$ distribution for the detected SNe is now heavily skewed toward lower values (right panel; third plot of Figure 6, from top to bottom). This is also evident in the left panel where the concentration of objects near the detection limit (diagonal line) is significantly lower than in Case I. Note also that this effect is much more severe for the dimmer events; the more rapid evolution of the less luminous objects decreases their probability of discovery. The combined effect of these two selection effects is to increase the average absolute magnitude of the detected SNe by 0.094 mag with respect to the value of the parent distribution. The “light-curve” effect reduces the detection efficiency by 85.5% with respect to Case I for the particular time sampling of the CT survey. For a survey which examined each field once each day, on the other hand, this effect would be insignificant.

- Case IV (Malmquist bias, Shaw effect, and “light-curve” bias): Finally, we consider the combined effect of proximity, correlation between luminosity and duration, and limiting magnitude. As expected, the $V/V_{MAX}$ histogram becomes even more skewed toward lower values (right panel; bottom plot of Figure 6). The bias in absolute magnitude is now 0.096 mag and the detection efficiency is reduced by 88.6% with respect to Case I.

These exercises show that the mean absolute magnitude of the detected sample (the zero-point of the Hubble diagram) could be substantially biased because the detected sample contains a relatively larger fraction of luminous events (with broader light curves) than that of the parent population. However, the existence of the $M/\Delta m_{15}$ relation allows us to correct for the luminosity excess of these SNe. In the limiting cases considered above, where the scatter about this relation is zero, there is a one-to-one mapping between absolute magnitude and decline rate, so that the mean corrected absolute magnitude has identically zero bias. In other words, both the slope and the zero-point of the parent function remain unaffected by the selection effects.

In reality, of course, there is significant intrinsic scatter in the peak magnitudes and we expect a bias in the mean corrected absolute magnitude because the selection effects will tend to pick the brightest events from the corrected peak luminosity distribution. We now estimate the magnitude of this bias under the assumptions that the parent $M/\Delta m_{15}$ function has a dispersion due to the peculiar motion of the host galaxies ($\sigma_V=600$ km sec$^{-1}$) and an additional scatter in absolute magnitude independent of the SN lightcurve shape ($\sigma_M=0.15$). To make the simulation somewhat more realistic, we also assume that the limiting magnitude for SN detection has a fluctuation of $\sigma_{BLim}=0.3$.

Figure 7 (top panel) presents the results of this simulation in the form of absolute magnitudes of the detected SNe plotted versus $\Delta m_{15}$(B). We note first that the density
of points in this diagram is significantly lower for dimmer events since, as expected, these SNe are more difficult to discover. The dashed line is the least-squares fit (taking into account errors in absolute magnitudes) to the detected SNe. This should be compared with the solid line (almost indistinguishable) that corresponds to the parent relation (with coefficients $a_0=-19.258$ and $b_0=0.784$). The fit to these 1,000 SNe gives a zero-point of $a=-19.30 \pm 0.01$ and a slope of $b=0.76 \pm 0.02$. Selection effects can thus introduce a systematic bias of 0.04 mag in the zero-point of the observed relation, less than half the 0.10 mag bias in the uncorrected absolute magnitudes. This is because the detected sample suffers only from the bias left after correcting for the decline rate. When the scatter of this relation is increased to $\sigma_M=0.50$ (a value greatly in excess of that observed), we obtain $a=-19.59 \pm 0.02$ and $b=0.78 \pm 0.06$ (second panel of Figure 7), and the much larger bias of 0.33 mag in the zero-point. It is interesting to note that the slope of the parent function is almost unaffected even by such a large scatter about the relation. The magnitude of the bias depends only on the dispersion of the absolute magnitudes and not on their values. Because we assume the same dispersion for all lightcurve widths, the magnitude of the bias is the same for all widths.

In summary, this test shows that a dispersion about the parent $M/\Delta m_{15}$ function results only in a bias of the zero-point of the observed relation; the slope of the observed relation remains unaffected. More generally, the shape of the observed relation will be the same as that of the parent function, no matter what the shape of the parent function, under the assumption that the scatter around the function is independent of lightcurve width. To illustrate this, we show the results of a simulation with a quadratic function for the $M/\Delta m_{15}$ relation, with coefficients $a_0=-19.258$, $b_0=0.457$, and $c_0=1.293$ corresponding to the preliminary parameters found by Phillips et al. 1998. To enhance the biases from selection effects we once again adopt the excessively large scatter of $\sigma_M=0.50$. A quadratic fit to the detected SNe yields $a=-19.59 \pm 0.02$, $b=0.49 \pm 0.12$, and $c=1.15 \pm 0.23$. As anticipated, the linear and quadratic coefficients remain unchanged (within the error bars) and the bias of 0.33 mag in the zero-point is identical to that of the previous case. The third panel of Figure 7 shows very clearly that the shape of the parent function is unchanged.

A different result obtains when the dispersion about the $M/\Delta m_{15}$ function is correlated with lightcurve width. If we assume that the scatter ranges from $\sigma_M=0.15$ at the smallest decline rate to $\sigma_M=0.50$ at the largest, the magnitude of the bias increases with decline rate, resulting in a change in the slope of the $M/\Delta m_{15}$ relation as seen in the bottom panel of Figure 7. The fit to the detected events yields $a=-19.34 \pm 0.01$ and $b=0.51 \pm 0.04$. Both

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3An examination of the residuals of the 1,000 fake SNe about the least-squares fit shows, in fact, that they have a Gaussian distribution.
the zero-point and the shape of the parent function might thus be substantially biased under these circumstances.

A related question is whether the observed zero-point and slope could be substantially in error purely by chance due to the small number of events (27) used in their determination. To answer this question we estimate the uncertainties in the a and b coefficients by running a large number of simulations for the case $\sigma_M=0.15$, always fixing the number of detections to 27. From a run of 200 simulations we get $a=-19.29 \pm 0.04$ and $b=0.77 \pm 0.14$ where the quoted uncertainties (standard deviations) reflect the fluctuations of these parameters due to statistical errors. From a single experiment with 27 detected SNe the observed zero-point and slope are expected to have random errors of only 0.04 mag and $\sim20\%$, respectively; these are almost identical to those quoted in Paper V.

We can assess the reality of a $M/\Delta m_{15}$ relation by a similar experiment, asking “What is the probability that the CT sample could yield, by chance, the observed slope given the hypothesis of a null slope?” From a run of 200 simulations and the initial assumption that $b_0=0$ and $\sigma_M=0.15$ (again fixing the number of detections to 27), we obtain a nearly Gaussian distribution of slopes with $b=-0.009 \pm 0.12$. The value of of $b=0.784 \pm 0.182$ derived in Paper V thus lies $3.6 \sigma$ away from the assumption of a null slope and the data of the CT survey show this hypothesis can be rejected with 99.97% confidence.

In Paper V concern was expressed about the disagreement in the slopes of the $M/\Delta m_{15}$ relation inferred from the CT SNe and from Phillips’ original sample. The disagreement can be seen very clearly in Figure 2 of Paper V and, also, by performing a linear fit to the data (given in Table 2 of Paper V) of the nearby sample of eight SNe with $0.87 \leq \Delta m_{15}(B) \leq 1.73$ (the same range of decline rates used to determine equation (1)), which yields $a=-19.18 \pm 0.08$ and $b=1.89 \pm 0.34$. The likelihood that this result is consistent with the CT slope can be evaluated from Monte Carlo simulations. Although the SNe of Phillips’ sample are not part of a systematic survey like the CT search, for the sake of simplicity we will assume in the following test the same selection criteria. From a run of 200 simulations (fixing the number of observations to that of the nearby sample) and the initial assumptions that $b_0=0.784$ and $\sigma_M=0.15$ (the CT parameters), we obtain $b=0.764 \pm 0.31$. The slope of the nearby sample thus differs from the CT slope by $2.4 \sigma$, i.e, the likelihood of obtaining the slope of the nearby sample by pure chance from a parent population like the CT sample is less than 2%.

As a possible explanation for this evident discrepancy, in Paper V it was speculated

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4This sample was selected by having 1) precise optical photometry, 2) well-sampled light curves, and 3) accurate relative SBF and PNLF distances (at the distances of the Virgo and Fornax clusters or closer).
that this could be due to biases in the CT search. As discussed above, one possibility of reconciling the shallower slope from the CT sample with that obtained from the nearby sample is that the dispersion about the parent M/Δm\textsubscript{15} relation increases toward large decline rates. To examine this possibility in more detail we use the zero-point and the slope obtained from the nearby sample as parent coefficients to ask how much scatter in the parent function is required in order to get the slope of the CT sample. We find that to match this slope one must increase the scatter about the relation from 0.15 mag for the slowest-declining events to 0.95 mag for events with Δm\textsubscript{15}(B) \sim 1.7. An unsatisfactory consequence of this large dispersion in the parent distribution, however, is that the sample of detected events also displays an exceedingly large dispersion at low decline rates, a feature which contrasts with the relatively small and constant scatter observed about the CT M/Δm\textsubscript{15} relation. Thus, a strongly-increasing scatter around the actual relation in the range 0.87 \leq Δm\textsubscript{15}(B) \leq 1.73 does not appear to be a viable explanation. As was mentioned in Paper V, another possible source of discrepancy between the two data samples is the uncertainty in the SBF/PNLF distances employed in the nearby sample. It is also possible that corrections for extinction from dust could lead to a better agreement between the two samples.

In summary, the results of this section and the small scatter observed in the CT M/Δm\textsubscript{15} relation in the range 0.87 \leq Δm\textsubscript{15}(B) \leq 1.70 suggest that the bias in the observed slope is not large and the observed zero-point is expected to be biased by no more than \sim 0.04 mag. Additional tests performed with different values of M\textsubscript{\text{B}}\textsubscript{1} show that these findings are insensitive to the choice of the absolute magnitude for the SNe. This result is further insensitive to the shape of the SN Ia luminosity function. This supports our initial assumptions regarding equation (1) in the range 0.87 \leq Δm\textsubscript{15}(B) \leq 1.70. As we mentioned above, however, it is troubling that the relation overestimates the absolute magnitude of SN 1992K (Δm\textsubscript{15}(B)=1.93) by 2σ. Further evidence for a departure of fast-declining events from the linear approximation is provided by the SN 92K-like event in the nearby sample (SN 1991bg) and by the recently discovered SN 1997cn (Turatto et al. 1998). In Section 5 we consider a M/Δm\textsubscript{15} relation that accounts for these facts.

4.2. The luminosity function of SNe Ia

In this section we study possible biases in determining the luminosity function of SNe Ia. Figure 8 (left panel) shows with solid circles the resulting decline rate distribution of the 50,000 SNe detected in a simulation performed with the standard set of parameters [M\textsubscript{\text{B}}\textsubscript{1}=-19.258 (H\textsubscript{0}=65), B\textsubscript{lim}=18.3, α=0.15 kpc\textsuperscript{-1}]. Also shown in this figure is the
histogram of the CT sample. The comparison reveals a relatively good agreement between
the simulation and the data (with a KS probability of 0.51), which suggests that the
initial assumption of a flat decline rate distribution in the simulation is not a bad guess.
Since, by construction, the parent population in the simulation has equal numbers of SNe
per bin in decline rate, the solid circles represent a measure of the detection efficiency.
This curve reveals an increasing incompleteness toward higher decline rates which is a
clear signature of the M/Δm_{15} relation (equation (1)); events with faster decline rates are
intrinsically fainter and more transient and, hence, more difficult to detect. The shape of
this function proves to be independent of the adopted value for M_{15} and we can employ
it in principle to correct the observed histogram for incompleteness. Figure 9 shows the
“corrected” luminosity function obtained by dividing the observed histogram of decline
rates by the efficiency curve (properly normalized to preserve the total number of objects)
and transforming the decline rates of the abscissa to absolute magnitudes through equation
(1). As anticipated, the “corrected” distribution is remarkably flat over the whole range
of decline rates with, perhaps, an increase in frequency for the intrinsically faint events as
indicated by the straight line (an approximate visual fit to the “corrected” distribution). It
is important to note that this result is very sensitive to the M/Δm_{15} relation adopted and,
hence, is somewhat uncertain. For example, if the actual slope of this relation was steeper
than that assumed in the simulation, the efficiency curve would become steeper and the
“corrected” distribution would display an even larger number of faint SNe.

Another important caution regarding the “corrected” distribution is the possible
non-linearity of the M/Δm_{15} relation toward high decline rates. With M_{MAX} \sim
17.7+5\log(H_0/65), SN 1992K is \sim0.9 mag fainter than the value obtained from the adopted
relation (equation (1)) for a decline rate of Δm_{15}(B)=1.93. Consequently, SN 1992K-like
events are not produced in the simulation. This can be seen in the right panel of Figure 8
which compares the absolute magnitude distribution of the 50,000 simulated SNe with that
of the CT sample. Although there is overall agreement between the simulation and the
data, the lack of simulated events like the subluminous SN 1992K is evident. To account
for these subluminous SNe it would be necessary to modify the parent M/Δm_{15} function
by adopting a steeper slope toward the highest decline rates. In such a case, the actual
efficiency for detection of the subluminous events in the simulation would be much lower
than what we estimate here and, hence, the actual frequency of the fast-declining SNe
would be substantially higher than that shown in the “corrected” histogram of Figure 9.
In summary, the “corrected” decline-rate distribution should be much more reliable in the
range 0.87 \leq Δm_{15}(B)\leq1.7 where the M/Δm_{15} relation is well represented by equation (1)
(see Paper V).
4.3. The galaxy type-redshift relation

One of intriguing features of the CT sample is that the ratio of elliptical to spiral hosts (hereafter E/S) increases dramatically with redshift (see Figure 4 of Paper VI). Clearly a selection bias must be at work here because it is hard to imagine that evolutionary effects could be this important at redshifts below than 0.1. Within the framework of our simple model the only agent that can explain this feature is the Shaw effect.

At zero redshift, the minimum angular separation maps into a vanishingly small region of the host so that in principle nearly all SNe can be discovered in nearby galaxies. At larger redshifts, the same separation maps into a larger radius of the parent galaxy so that an increasingly larger volume of the host remains unsurveyed; only a fraction of the SNe are potentially discoverable. The E/S ratio can change with redshift since the mass of this “exclusion” region varies with radius differently for different morphological types. This variation can be determined analytically, assuming that the SN radial distribution follows the light distribution, by integrating the light profile from an inner radius out to infinity. The inner radius defines the outer bound of the “exclusion” region and is linearly related to the distance (or redshift) of each host. The top panel of Figure 10 illustrates this effect. The solid lines show the fraction of the total host light that is sampled in the SN survey as a function of redshift ($H_0=65$) for an elliptical with $r_{\text{eff}}=6$ kpc and a spiral with $\alpha=0.15$ kpc$^{-1}$ ($r_0=6.7$ kpc) (for the latter this curve is calculated from the projected light distribution on the plane of the sky, averaged over all disk inclinations). Since the fraction of sampled light for spirals differs at any given redshift from that of ellipticals, the E/S ratio varies with redshift. The dashed line shows the E/S ratio, arbitrarily normalized to unity at zero redshift. The E/S ratio increases strongly at high redshifts as the light from an $r^{1/4}$ law declines more slowly with radius than that from an exponential disk. This effect can be seen in the simulations as well. The bottom panel of Figure 10 shows the comparison between the analytic E/S curve (solid line) and simulations (solid points) performed with the same parameters ($r_{\text{eff}}=6$ kpc, $r_0=6.7$ kpc) based on $10^6$ detected events.

Although the magnitude of this effect in our simple model is not sufficient to account for the CT E/S ratio (which increases from $0.14\pm0.15$ in the bin $v<10,000$ km sec$^{-1}$ to $1.33\pm0.72$ in the bin $v>10,000$ km sec$^{-1}$), the E/S ratio is very sensitive to the values of $r_{\text{eff}}$ and $r_0$. The dashed line in the figure is determined from $r_{\text{eff}}=6$ kpc and $r_0=4$ kpc. Note the sudden rise of the E/S ratio at lower redshifts. This exercise merely demonstrates that while there may be other selection effects responsible for this feature in the CT sample, the Shaw effect can modify the E/S ratio in the same direction as that observed.
5. A More Speculative Model

Figure 11 shows the individual apparent magnitudes, absolute magnitudes, and decline rates of a random subsample of 1,000 SNe detected in the simulation performed with the simple model described above and the standard set of parameters, as functions of recession velocity. The large circles show the CT SNe. While the match to the observations is encouraging, a few deficiencies are obvious. In the middle panel it is clear that the simulation lacks faint events like SN 1992K at $M_{M_{AX}}^B \sim -17.7 + 5 \log (H_0/65)$. As pointed out in Paper VI, it is also clear that the CT sample lacks bright events in the bin of distant SNe ($\log(v) \geq 4$) (an anti-Malmquist bias?).

Another problem is posed by the two most distant SNe in the observed sample which appear to have been detected well beyond the nominal detection limit of the simulation (corresponding to $B_{lim}=18.3$) represented by the dashed diagonal line in the middle panel. Although the detection of events beyond this limit can occur in the simulation (since we allow for fluctuations of the limiting magnitude), statistically this is insufficient to explain the presence of these two outliers.

In Figure 12 we show the individual apparent magnitudes, absolute magnitudes, and decline rates of the same subsample of 1,000 SNe, as functions of the projected distance between the SN and the center of the host galaxy. The diagonal line in the top panel represents the boundary of the central 4 arcsec where no SNe were discovered. Not surprisingly, the simulations match this feature. In the other panels the agreement is generally good, although it is again evident that the simulations lack dim SNe (central panel).

Having estimated biases in some of the observed quantities, we can use this information to review our assumptions and attempt to resolve some of these problems. We introduce the following modifications to our initial assumptions about the parent $M/\Delta m_{15}$ relation and the SN luminosity function.

- The lack of intrinsically faint SNe in the simulation suggests we modify the assumed $M/\Delta m_{15}$ function. Because the tests performed in Section 4.1 suggest that the observed relation is not severely biased by the selection effects, we will still adopt the relation given by equation (1) and the coefficients $a_0 = -19.258$ and $b_0 = 0.784$ for the range $0.87 \leq \Delta m_{15}(B) \leq 1.7$. For the range $1.7 \leq \Delta m_{15}(B) \leq 1.93$, however, we will assume a steeper slope which describes the absolute magnitude of SN 1992K and ensures the continuity of the parent function. With these constraints the zero-point and slope of this function in the range $1.7 \leq \Delta m_{15}(B) \leq 1.93$ are -21.573 and 4.642, respectively. Note that this “two-slope” model is not much different than the quadratic form recently used by Phillips et al. 1998.
for the $M/\Delta m_{15}$ relation after applying corrections for host galaxy extinction. Although it would be preferable to employ corrected absolute magnitudes in our simulations, these data are not yet available.

- Our examination of bias in the observed luminosity function suggests that the frequency of the parent SN population might increase toward fast decline rates. In the following we will assume a parent luminosity function with the linear form shown in Figure 9, i.e,

$$N = k \times \Delta m_{15}(B), \quad (3)$$

where $k$ is a normalization. With this approximation, the fastest-declining observed events occur with approximately twice the frequency of the slowest-declining. This is reasonable as we found above that the corrected frequency of the fastest-declining events would have been higher than that shown in Figure 9 had we employed a steeper $M/\Delta m_{15}$ relation for these events.

Using these revisions in our description of the parent population, we perform a parameter search in the range $-19.558 \leq M_{B1} \leq -18.958$, $17.3 \leq B_{lim} \leq 18.7$, $0.05 \leq \alpha \leq 0.40$ kpc$^{-1}$. We have also relaxed the constraint of a fixed $\sigma_{Blim}$ and vary this parameter in the range 0.3-1.2 mag, allowing detection of events well beyond the nominal detection limit of the simulation. We then look for the set of parameters that simultaneously yields the highest KS probabilities for the apparent magnitude, recession velocity, angular separation, and projected radial distance between the SN and the host galaxy nucleus. Note that in this case the solutions are much more severely constrained than for the simple model (where we searched for parameters which fit only the recession velocity and the angular separation distributions).

With these requirements we find that the best solution in the $(\alpha, B_{lim}, \sigma_{Blim})$ parameter space changes little within the range of absolute magnitudes considered. For the specific case of $M_{B1}=-19.258$ ($H_0=65$), the best solution is obtained for $\alpha=0.175$ kpc$^{-1}$, $B_{lim}=17.9$, and $\sigma_{Blim}=0.7$. This last value is significantly larger than our initial assumption of $\sigma_{Blim}=0.3$. Figure 13 shows the recession velocity, apparent magnitude, angular separation, and radial distance distributions of 50,000 detected SNe in a simulation performed with these parameters along with the observed supernovae. The match to the observations is remarkably good with KS probabilities 0.99, 0.76, 0.96, and 0.46, respectively, and evidently better than that shown in Figure 5 for the simpler model. Figure 14 shows the decline rate and absolute magnitude distributions, with KS probabilities of 0.81 and 0.86, respectively. Again, the agreement with the observations is remarkably good and certainly much better than that obtained with the simple model (cf. Figure 8). One interesting feature is the long tail of the absolute magnitude distribution toward intrinsically faint SNe, a consequence of the steeper slope adopted for the $M/\Delta m_{15}$ relation. The decline rate distribution of the
simulation has a shape that closely follows the observed histogram, suggesting that relaxing the assumption of a flat luminosity function is a real improvement.

While these modifications to our simple model improve agreement with observation, they are admittedly somewhat *ad hoc* and one might question whether some other change would prove similarly attractive. While there is no way to systematically examine the space of possible models, as an example we examine the consequences of another possibility.

We adopt instead the hypothesis that a flat luminosity function should be extended to fainter absolute magnitudes (ELF). Events with with $\Delta m_{15}(B)>1.93$ have never been observed, and we must guess at the lightcurve shape of such events. We have extrapolated the set of six known templates to $\Delta m_{15}(B)=3$ and extended the “two-slope” $M/\Delta m_{15}$ relation to higher decline rates. Adopting $M^B_1=-19.258$, an examination of the parameter space yields a best solution for $\alpha=0.2 \, \text{kpc}^{-1}$, $B_{lim}=17.7$, and $\sigma_{Blim}=0.9$, results very similar to the previous case. For these parameters the KS probabilities for recession velocity, apparent magnitude, angular separation, and radial distance distributions of 50,000 detected SNe are 0.97, 0.88, 0.58, and 0.61, respectively, and are comparable to those obtained for the non-flat luminosity function.

The decline rate and absolute magnitude distributions, on the other hand, give KS probabilities of 0.54 and 0.42, significantly lower than those obtained with the non-flat luminosity function (0.81 and 0.86). This drop in the KS probabilities characterizes the whole parameter space explored. For 600 parameter combinations, the average KS probability for $\Delta m_{15}(B)$ is 0.77 in the non-flat case and 0.54 in the ELF case. For absolute magnitudes, the probabilities are 0.86 and 0.41, respectively. This exercise shows that the Monte Carlo technique and the KS test have some power to distinguish between different models despite the limitations imposed by the low number of detected events. In particular, we can safely conclude that an ELF is less consistent with the data than the non-flat luminosity function.

Returning to the favored model, Figure 15 shows the individual apparent magnitudes, absolute magnitudes, and decline rates of a random subsample of 1,000 SNe detected in the simulation performed with the best parameters as functions of recession velocity. The large circles show the CT SNe which are in very good agreement with the simulations. A noticeable difference with the simple model (Figure 11) is the large scatter displayed by the absolute magnitudes in the nearby bin and the presence of SN 1992K-like objects as in the CT sample. This larger scatter is due in part to the new population of intrinsically faint SNe which cannot be detected at very large distances and do not appear in the distant bin ($\log(v)\geq4$). Another effect that contributes to the larger dispersion in the nearby bin is the peculiar motion of the host galaxies (600 km sec$^{-1}$) which constitutes a larger fraction
of the recession velocity of the nearby events. In the distant bin, on the other hand, the discovered SNe appear to have a lower dispersion than the simulated sample. Although these objects cluster precisely in the region of the diagram with the highest density of simulated SNe, it is possible that the lack of intrinsically bright events in the distant bin is a signature of a significant difference between the simulations and the data.

Another improvement with respect to Figure 11 is that the two most distant CT SNe appear to be much closer to the bulk of synthetic SNe, a result of the large fluctuation of 0.7 mag in the limiting magnitude of the simulation. Even better agreement would have been obtained if these two distant SNe had brighter intrinsic luminosities (or, equivalently, smaller decline rates), since these are the events which are most likely found near the detection limit of the simulation. In any case, it is hard to ascertain with such few objects whether or not this is a significant difference. The distribution of decline rates (bottom panel) is similar to that of absolute magnitudes as a consequence of the $M/\Delta m_{15}$ relation. The sharp cutoffs at small and large $\Delta m_{15}(B)$ are there by construction, as explained in section 3.1.

Figure 16 shows the individual apparent magnitudes, absolute magnitudes, and decline rates of the same subsample of 1,000 SNe, as functions of the projected distance between the SN and the host galaxy nucleus. A comparison with Figure 12 shows a noticeable improvement over the simple model. Motivated by the claim by Wang et al. 1997 that the dispersion in absolute magnitude among the CT SNe decreases with galactocentric distance, we ask whether this feature is present in the sample of simulated SNe as a consequence of the Shaw effect. Figure 17 shows the absolute magnitude distributions from the sample of 50,000 synthetic SNe separated into two bins: events with projected radial distances larger than 20 kpc (solid line) and those with distances smaller than 10 kpc (dashed line). Despite a small (but real) difference due to the Shaw effect, the scatter in absolute magnitude does not change dramatically from one bin to the other. This may mean that the model is unable to reproduce this feature, but it is equally likely that the effect seen by Wang et al. 1997 is merely a statistical fluctuation. In fact, most of the dispersion in absolute magnitude among the CT events is caused by one object, SN 1992K. It remains to be seen if this correlation will persist as the diagram is populated with more SNe.

Finally, Figure 18 (top panel) shows the age at discovery of the 50,000 SNe plotted as a function of recession velocity. This illustrates very clearly the effect that distant events can be found only during a short period of time around maximum light. For comparison, the bottom panel of Figure 18 shows the corresponding plot for the CT SNe, revealing a satisfactory agreement with the simulation.
6. The SNe Ia Rate

The actual rate of SNe Ia (in units of SNe/volume/time), \( \dot{n} \), can in principle be calculated from the number of discovered SNe in the CT survey, \( N_{\text{dis}} \), and the detection efficiency of the search (in units of volume \( \times \) time), \( \text{eff} \), which we define as

\[
eff = \frac{N_{\text{dis}}}{\dot{n}}.
\] (4)

In the following we assume that the actual efficiency of the search corresponds to that of the simulation performed with the parameters that best fit the CT data. The efficiency of the simulation can be easily calculated from the number of detected SNe, \( N_{\text{det}} \), and the input rate which can be computed from the number of generated SNe, \( N_{\text{gen}} \), the volume and the period of time over which the events are distributed in the simulation. Since we generate SNe in a spherical volume of radius \( R \) (in units of Mpc) over a period of 4 years, the efficiency of detection (in units of Mpc\(^3\) \( \times \) century) appropriate for the CT survey is,

\[
eff = \frac{N_{\text{det}}}{N_{\text{gen}}} \times \frac{4\pi R^3}{3} \times \frac{4}{100} \times \frac{60 \times 26.129}{41,253}.
\] (5)

The 4/100 factor converts the rate from units of 4 years to 100 years. The last factor in this equation corresponds to the fraction of the celestial sphere (41,253 square degrees) surveyed by the CT project (a total of 60 fields each providing a sky coverage of 26.129 square degrees), which is appropriate since the simulations do not constrain the detection of SNe by their location in the sky. From equations (4) and (5), it is possible to solve directly for \( \dot{n} \), the actual rate of SNe Ia in units of SNe Mpc\(^{-3}\) century\(^{-1}\). However, SN rates are usually expressed in units of “SNu”: SNe per 10\(^{10}\) L\(_\odot\) per century (where L\(_\odot\) corresponds to the B solar luminosity). We denote the rate in units of SNu by \( \nu \) (Capellaro et al. 1997); its numerical value can be obtained by dividing \( \dot{n} \) by L\(_B\), the blue luminosity density of galaxies (in units of 10\(^{10}\) L\(_\odot\) Mpc\(^{-3}\)) of the local Universe. Marzke (private communication) has kindly provided us with a recent determination of the blue luminosity density of

\[
L_B = 1.65(\pm0.42) \times 10^{-2} h \left[ 10^{10} L_\odot Mpc^{-3} \right],
\] (6)

where \( h=H_0/100 \). This value of L\(_B\) has been calculated from asymptotic magnitudes B\(_T\) which are defined to account for the whole integrated light of the galaxies of his sample (see de Vaucouleurs et al. 1991 for the definition of B\(_T\)). Combining equations (4),(5), and (6), the SN Ia rate in SNu is,

\[
\nu = 31 \times \frac{N_{\text{gen}}}{N_{\text{det}}} \times \frac{3}{4\pi R^3} \times \frac{100}{4} \times \frac{60 \times 26.129}{41,253} \times \frac{1}{1.65 \times 10^{-2} \times h}.
\] (7)
where we have replaced \( N_{\text{dis}} \) by the total number (31) of SNe Ia events discovered in the course of the CT survey.

Table 4 gives the parameters \( \alpha, B_{\text{lim}}, \) and \( \sigma_{B_{\text{lim}}} \) that best fit the CT observations for \( M_{11}^B = -19.258 \), as described in Section 5, along with the values of \( N_{\text{gen}}, N_{\text{det}} \), the radius \( R \) of the simulation run, and the corresponding rate computed with equation (7). A large value of \( N_{\text{det}} \) was chosen to minimize the statistical error in the determination of the detection efficiency. The uncertainty quoted for the calculated rates is the sum in quadrature of the following errors, which we assume to be uncorrelated:

- As in all counting experiments, we expect statistical fluctuations in \( N_{\text{dis}} \) which we estimate by running many simulations, always fixing \( N_{\text{det}} \) to 31 and computing the standard deviation of the rate. The error due to the statistical fluctuations amounts to \( \sim 16\% \) (which proves to be comparable to \( \sim \sqrt{N/N} \sim 18\% \)).
- The uncertainty in \( \alpha \), which we assume to be \( \pm 0.05 \text{ kpc}^{-1} \). This error amounts to \( \sim 13\% \).
- The uncertainty in \( B_{\text{lim}} \), which we assume to be equal to \( \sigma_{B_{\text{lim}}} \) (\( \pm 0.7 \text{ mag} \)). The error bar in the resulting SN rate is not symmetric and amounts to \( \sim 50-150\% \).
- The uncertainty in the luminosity density, which amounts to 25\% (equation (6)) in the SN rate.

Clearly (and somewhat remarkably), the largest source of uncertainty in the calculated rates is the lack of a precise value for \( B_{\text{lim}} \) and not the small number of SNe found in the survey.

Recently, Capellaro et al. 1997 employed a sample of 7,773 galaxies from the Third Reference Catalogue of Bright Galaxies (de Vaucouleurs et al. 1991, hereafter RC3) and the combined sample of SNe from five searches to compute rates of SNe using the “control time method”. Although the CT SNe were included in this calculation, these objects constitute only 10\% of the total sample so that the two estimates are quite independent of each other. Also, while we have searched a fixed volume of space for supernovæ in a systematic manner, Capellaro et al. 1997 use a fixed set of galaxies and take the results of a heterogeneous set of searches. Capellaro et al. 1997 report an average rate of SNe Ia of \( \nu = 0.20 \) (\( H_0/75 \))^2 SNe. Since the normalization to galaxy luminosity of their work is based on the extinction-corrected \( B_T \) magnitudes of the RC3 catalogue, their value cannot be directly compared to our rate which is based on uncorrected \( B_T \) magnitudes. To allow a proper comparison between the two calculations, Capellaro (private communication) has kindly provided an alternative rate for SNe Ia of 0.22 (\( H_0/75 \))^2 SNe, duly normalized to the galaxy luminosity uncorrected for internal extinction. Given the uncertainties,
Capellaro’s rate of $0.161 \pm 0.051$ (for $H_0=65$) agrees surprisingly well with our estimate of $0.21^{+0.30}_{-0.13}$.

7. Discussion and Conclusions

We have shown that a simple model of the selection effects of the CT photographic survey, coupled with the assumptions of a linear $M/\Delta m_{15}$ relation and a flat luminosity function for the parent population of SNe Ia accounts for the basic properties of the observed SN sample. The model accounts for biases due to the flux limited nature of the survey, the different light curve morphologies displayed by different SNe Ia, and the difficulty of finding SNe projected near the central parts of the host galaxies (Shaw 1979).

Using the simple model we have estimated the bias in the zero-point and the slope of the $M/\Delta m_{15}$ relation. For an assumed scatter of 0.15 mag about this relation, the selection effects of the search (as modeled) decrease the zero-point by 0.04 mag (and, hence, decrease the actual value of $H_0$ by 2%). The estimated bias would thus have a negligible effect in the determination of cosmological parameters from high redshift SNe. If the $M/\Delta m_{15}$ relation is ignored, the bias in the zero-point of the Hubble diagram proves to be at least 0.10 mag. The slope of the relation, however, does not appear to be substantially affected by selection effects in the range $0.87 \leq \Delta m_{15}(B) \leq 1.7$. It appears, then, that the shape and zero-point of the $M/\Delta m_{15}$ relation determined from the CT SN sample are quite reliable.

We have estimated the degree of incompleteness of the survey as a function of decline rate and used this efficiency curve to produce a corrected luminosity function for SNe Ia in which the frequency of SNe increases with decline rate. This result is somewhat uncertain as it is very sensitive to the adopted $M/\Delta m_{15}$ function.

With the simple model and radial light profiles for the host galaxies assuming $r^{1/4}$ and exponential laws for elliptical and spiral galaxies, respectively, we have shown that the Shaw effect will affect the observed ratio of elliptical to spiral hosts as a function of redshift, an effect present in the CT sample.

Based on these results we have proposed a new model for the parent population of SNe which better accounts for the properties of the observed SN sample. This assumes a non-linear $M/\Delta m_{15}$ relation and a skewed luminosity function. Not surprisingly, with the larger number of degrees of freedom this model allows, the simulations provide a much better match to the observations than that obtained with the simple model.

Since our goal was to propose a model of the selection effects of the CT survey
that entails the least possible number of assumptions and parameters, we have made no
consideration of an additional potential bias due to extinction by dust in the host galaxy
(the inclusion of dust in the model would require at least three additional parameters to
describe the geometry and magnitude of the extinction). Although the success of this
model may provide some indication that these additional selection biases are small, at least
in surveys performed with photographic techniques, it is fair to ask how important dust
extinction may be among SNe Ia.

Capellaro et al. 1997 find that dust absorption in the host galaxy causes SNe Ia to be
detected $1.8\pm0.5$ times more frequently in spirals with low inclinations. van den Bergh &
McClure 1990 find that SNe Ia do not exhibit such an inclination effect. For the specific
sample of CT SNe, the spectroscopic data of these objects revealed only moderate amounts
of extinction by dust (Paper VI), in agreement with the conclusion of van den Bergh
& McClure 1990. This is confirmed by the color excesses calculated from the observed
($B_{MAX}$-$V_{MAX}$) colors of the CT sample and a preliminary ($B_{MAX}$-$V_{MAX}$) intrinsic color
versus decline rate relation kindly provided to us before publication (Phillips et al. 1998).
Definitive reddening and corrected absolute magnitudes will be soon available. Figure 19
shows the color excesses of the CT SNe Ia due to extinction in the host galaxy as a function
of the projected radial distance. As expected, the extinction increases toward the center
of the spiral galaxies (solid dots) where the dust density may be expected to be higher.
The effect of dust on the ensemble of CT SNe Ia, however, is not very large (the average
$E(B-V)$ is less than 0.1). This is consistent with the success of the model presented in this
paper. In a future paper we will include a reddening model in the simulations, once these
reddening-corrected absolute magnitudes for the CT SNe become available.

In a recent paper Hatano et al. 1998 proposed a model of the spatial distribution
of dust and SN progenitors which produces a large dispersion in the absolute magnitudes
of the SNe Ia projected on the nuclear regions of the host galaxies (see their Figure 3).
An obvious problem with this model is that this scatter proves to be three times larger
than the dispersion implied by the color excesses of the CT SNe (shown in Figure 19). An
additional problem with this model is their choice of the luminosity function for SNe Ia,
$M^{B}_{MAX}$=-19.2$\pm$0.2, which does not account for the population of intrinsically faint SNe.
The inclusion of faint SNe would increase the scatter in luminosity still further, making the
comparison with observation even worse. It appears likely that while dust plays a role in
explaining some of the luminosity dispersion of the SNe projected near the central parts of
the host galaxies, it is not as important as Hatano et al. 1998 have claimed.

We have assumed that SNe Ia occur in idealized host galaxies consisting of smooth
exponential disks, with the radial distribution of SNe following that of the host galaxy light.
Since the Shaw effect can modify the observed radial distribution of SNe, we included this selection effect in our simulation. This showed that the effect is too small to account for the observed increase in scatter in absolute magnitudes at small projected radial distances, first noticed by Wang et al. 1997 among the CT SNe. A probable cause for this disagreement is simply small number statistics. Most of the dispersion among the CT SNe is caused by one object, SN 1992K. The most likely alternative is dust absorption. From Figure 19 it is clear that the highest extinction occurs at the smallest galactocentric distances, so some of the dispersion could be due to uncorrected dust absorption. Part of the observed fluctuation is due to over-luminous events, however, which cannot be the result of obscuration. We have postponed any further consideration of the effects of dust until corrected magnitudes become available (Phillips et al. 1998). We note as well that since elliptical galaxies dominate the hosts at large separations (due to the extended distribution of light from elliptical galaxies and the Shaw effect, as suggested in section 4.3), the observed decrease in the absolute magnitude scatter may just represent the more uniform stellar population found in early type galaxies.

Wang et al. 1997 has also called noted that no SN Ia has ever been found in the inner 1 kpc of the center of any spiral galaxy, and suggested again that this may be a consequence of the strong dust extinction. In our model the lack of objects in the inner 1 kpc is not unexpected for SNe arising from a disk population, since the probability of detection in this region is, indeed, very small (see Figure 13). In our simulations this is a consequence of the decrease of the disk area element \(2\pi r dr\) toward the galactic center as well as the Shaw effect.

Finally, we have computed the integrated detection efficiency of the simulation in order to infer the rate of SNe Ia from the 31 events detected during the course of the CT survey. For a value of \(H_0=65\) (\(M_1=19.258\), that obtained in Paper VI) we obtain a rate of \(0.21_{-0.13}^{+0.30}\) SNu in good agreement with the value \(0.16\pm0.05\) SNu recently published by Capellaro et al. 1997. While this is not a sensitive test of selections effects, it nonetheless lends credence to our assumptions regarding selection effects in the CT survey.

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Figure Captions

Fig. 1.— The complete time sampling for the 60 fields of the CT survey. A total of 1,019 photographic plates were obtained.

Fig. 2.— The distribution of recession velocities, apparent magnitudes, decline rates, absolute magnitudes, projected angular separations and radial distances between the 26 “best-observed” CT SNe and the host galaxy nuclei. The dashed histograms show the corresponding distributions for the subsample of SNe hosted by spiral galaxies.

Fig. 3.— The thin solid lines show the contour levels of the KS probability for the angular separation distribution, for four different values of $M_{B,1}$. Overplotted with thick lines are shown the contour levels of the KS probability for the recession velocity. The contour levels correspond to KS probabilities of 40%, 50%, 60%, and 70%. In each panel the $\oplus$ symbol indicates the values of $\alpha$ and $B_{lim}$ that simultaneously yield the highest KS probabilities for angular separation and recession velocity.

Fig. 4.— (Top panels) The distribution of host galaxy distances and angular separations between the nuclei of the host galaxies and 3,000 SNe detected in a Monte Carlo simulation, assuming $M_{B,1}=-19.758$, $B_{lim}=18.3$, and $\alpha=0.15$ kpc$^{-1}$. (Bottom panels) Same as above but for intrinsically less luminous SNe with $M_{B,1}=-18.558$.

Fig. 5.— The distribution of recession velocities, apparent magnitudes, projected angular and radial separations between the nucleus of the host galaxies and the 26 “best-observed” CT SNe. With solid dots are shown the corresponding distributions (properly scaled) of 50,000 SNe detected in a simulation performed with parameters $M_{B,1}=-19.258$, $B_{lim}=18.3$, and $\alpha=0.15$ kpc$^{-1}$, a flat luminosity function, and a linear $M/\Delta m_{15}$ relation.

Fig. 6.— (left) The absolute magnitudes of 1,000 SNe detected in Monte Carlo simulations assuming $-19.438 \leq M_{B,MAX} \leq -18.607$ ($M_{B,1}=-19.258$) and four different selection criteria (see text), plotted as a function of recession velocity. The dashed diagonal line indicates the maximum recession velocity up to which each SN type can be found, the horizontal solid line indicates the average magnitude of the parent population ($<M_{B,MAX}>=-19.023$), and the horizontal dashed line corresponds to the average magnitude of the detected SNe. In all these simulations we assumed $B_{lim}=18.3$, $\alpha=0.15$ kpc$^{-1}$, $\sigma_V=0$, $\sigma_{B_{lim}}=0$, $\sigma_M=0$. (right) The corresponding $V/V_{MAX}$ distributions of the 1,000 detected SNe in the four cases.

Fig. 7.— (top) The absolute magnitudes of 1,000 SNe detected in a Monte Carlo simulation assuming $M_{B,1}=-19.258$ ($H_0=65$), $B_{lim}=18.3$, $\alpha=0.15$ kpc$^{-1}$, $\sigma_V=600$ km sec$^{-1}$, $\sigma_{B_{lim}}=0.3$, and $\sigma_M=0.15$, plotted as a function of $\Delta m_{15}(B)$. The dashed line corresponds to a linear least-
squares fit to the SNe, and the solid line corresponds to the parent relation. (second panel from top to bottom) Same as above but for $\sigma_M=0.50$. (third panel from top to bottom) Same as above but for a quadratic parent $M/\Delta m_{15}$ relation. (bottom panel) Same as above but for a linear $M/\Delta m_{15}$ relation with variable scatter ranging between $\sigma_M=0.15-0.50$, increasing toward larger decline rates.

Fig. 8.— The decline rate and absolute magnitude distributions of the 26 “best-observed” CT SNe. With solid dots are shown the corresponding distributions (properly scaled) of 50,000 SNe detected in a simulation performed with parameters $M_{1,1}^B=-19.258$, $B_{lim}=18.3$, $\alpha=0.15$ kpc$^{-1}$, a flat luminosity function, and a linear $M/\Delta m_{15}$ relation.

Fig. 9.— The luminosity function of the 26 “best-observed” CT SNe (dotted lines) and the corresponding distribution corrected for selection biases (solid line). The straight line is an approximate visual fit to the corrected distribution. Error bars reflect the uncertainties due to the number of objects per bin.

Fig. 10.— (top) The solid lines show the fraction of detectable events in an elliptical and a spiral galaxy due to the Shaw effect, as a function of redshift ($H_0=65$). These are analytical calculations where we assume that the SN radial distribution follows the light distribution of each galaxy. The dashed line shows how the ratio of elliptical to spiral hosts varies with redshift. (bottom) The ratio of elliptical to spiral hosts as a function of redshift obtained from a Monte Carlo simulation (points) and the analytical calculation (solid line) for hosts with $r_0=6.7$ kpc and $r_{eff}=6$ kpc. The solid line shows the same calculation, with $r_0=4$ kpc and $r_{eff}=6$ kpc.

Fig. 11.— The apparent magnitudes, absolute magnitudes, and decline rates of 1,000 SNe detected in a simulation performed with parameters $M_{1,1}^B=-19.258$, $B_{lim}=18.3$, $\alpha=0.15$ kpc$^{-1}$, a flat luminosity function, and a linear $M/\Delta m_{15}$ relation, plotted as a function of recession velocity. The large circles represent the CT SNe. The vertical line at log(v)=4 (10,000 km sec$^{-1}$) illustrates the separation of the sample into two bins. For reference, in the middle panel we have drawn the diagonal dashed line from Figure 6, that defines the detection limit in an idealized simulation with $\sigma_M=0$, $\sigma_V=0$, and $\sigma_{B_{lim}}=0$.

Fig. 12.— The apparent magnitudes, absolute magnitudes, and decline rates of 1,000 SNe detected in a simulation performed with parameters $M_{1,1}^B=-19.258$, $B_{lim}=18.3$, $\alpha=0.15$ kpc$^{-1}$, a flat luminosity function, and a linear $M/\Delta m_{15}$ relation, plotted as a function of projected radial distance between the SN and the host galaxy nucleus. The large circles represent the CT SNe. The diagonal line in the top panel represents the boundary of the central 4 arcsec overexposed region of the host galaxies where no SNe were discovered.
Fig. 13.— The distribution of recession velocities, apparent magnitudes, projected angular and radial separations between the nucleus of the host galaxies and the 26 “best-observed” CT SNe. With solid dots are shown the corresponding distributions (properly scaled) of 50,000 SNe detected in a simulation performed with parameters $M_B^{1.1}=-19.258$, $B_{lim}=17.9$, $\alpha=0.175$ kpc$^{-1}$, $\sigma_{Blim}=0.7$, a non-flat luminosity function and a non-linear $M/\Delta m_{15}$ function.

Fig. 14.— Same as for figure 13, but for the decline rate and absolute magnitude distributions.

Fig. 15.— The apparent magnitudes, absolute magnitudes, and decline rates of 1,000 SNe detected in a simulation performed with parameters $M_B^{1.1}=-19.258$, $B_{lim}=17.9$, $\alpha=0.175$ kpc$^{-1}$, $\sigma_{Blim}=0.7$, a non-flat luminosity function and a non-linear $M/\Delta m_{15}$ function, plotted as a function of recession velocity. The large circles represent the CT SNe. The vertical line at log(v)=4 (10,000 km sec$^{-1}$) illustrates the separation of the sample into two bins. Typical error bars for the observed quantities are shown in each panel at log(v)=4.8.

Fig. 16.— The apparent magnitudes, absolute magnitudes, and decline rates of 1,000 SNe detected in a simulation performed with parameters $M_B^{1.1}=-19.258$, $B_{lim}=17.9$, $\alpha=0.175$ kpc$^{-1}$, $\sigma_{Blim}=0.7$, a non-flat luminosity function, and a non-linear $M/\Delta m_{15}$ function, plotted as a function of projected radial distance between the SN and the host galaxy nucleus. The large circles represent the CT SNe. The diagonal line in the top panel represents the boundary of the central 4 arcsec overexposed region of the host galaxies where no SNe were discovered. Typical error bars for the observed quantities are shown in each panel at a radial distance of 50 kpc.

Fig. 17.— The absolute magnitude distribution for the synthetic SNe with projected distances from the host galaxy nucleus larger than 20 kpc (solid lines) and smaller than 10 kpc (dashed line).

Fig. 18.— (top) The discovery age of 50,000 SNe detected in a simulation performed with parameters $M_B^{1.1}=-19.258$, $B_{lim}=17.9$, $\alpha=0.175$ kpc$^{-1}$, and $\sigma_{Blim}=0.7$, a non-flat luminosity function, and a non-linear $M/\Delta m_{15}$ function, plotted as a function of the recession velocity of the host galaxies. (bottom) Same as above but for the 26 “best-observed” CT SNe.

Fig. 19.— The color excess of the CT SNe Ia due to extinction in the host galaxy as a function of the projected radial distance. The region bounded by the two horizontal lines corresponds to the location of the unextinguished SNe.
Table 1. Calán/Tololo Type Ia Supernovae

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$^a$Projected Galactocentric Distance
Table 2. B Templates for Type Ia Supernovae

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<td>1.02:</td>
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<td>11.01:</td>
<td>12.21:</td>
<td>15.25:</td>
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Note. — Values with : denote extrapolations to the data.

Note. — The template values between days 0 and +80 can be found in Table 1 of Paper VIII.
Table 3. Parameters of the Monte Carlo simulations

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<th>Parameter</th>
<th>Default Value</th>
<th>Description</th>
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<tr>
<td>$M^B_{1.1}$</td>
<td>-19.280</td>
<td>Absolute B magnitude of a SN with $\Delta m_{15}(B) = 1.1$</td>
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<tr>
<td>$a_0$</td>
<td>-19.258</td>
<td>Zero point of the absolute magnitude-decline rate relation$^a$</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.784</td>
<td>Slope of the absolute magnitude-decline rate relation$^a$</td>
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<tr>
<td>$\sigma_M$</td>
<td>0.15 mag</td>
<td>Scatter of the absolute magnitude-decline rate relation$^a$</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>600 km sec$^{-1}$</td>
<td>Peculiar velocity of host galaxies</td>
</tr>
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<td>$B_{lim}$</td>
<td>18.30</td>
<td>B limiting magnitude for SN discoveries</td>
</tr>
<tr>
<td>$\sigma_{B_{lim}}$</td>
<td>0.30</td>
<td>Scatter in the B limiting magnitude</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.15 kpc$^{-1}$</td>
<td>Inverse of scale length of galaxy disks</td>
</tr>
<tr>
<td>separation</td>
<td>4.0 arcsec</td>
<td>Minimum separation between SN and galaxy nucleus for detection</td>
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$^aM^B_{MAX} - 5 \log(H_0/65) = a_0 + b_0 [\Delta m_{15}(B) - 1.1]$

Table 4. Rate of Type Ia Supernovae

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<thead>
<tr>
<th>$N_{gen}$</th>
<th>$N_{det}$</th>
<th>$M^B_{1.1}$</th>
<th>$H_0$</th>
<th>$B_{lim}$</th>
<th>$\sigma_{B_{lim}}$</th>
<th>$\alpha$</th>
<th>Radius [Mpc]</th>
<th>SN Rate</th>
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<td>9000</td>
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<td>65.0</td>
<td>17.90</td>
<td>0.70</td>
<td>0.175</td>
<td>1404.96</td>
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