New Vacua of Gauged $\mathcal{N} = 8$ Supergravity in Five Dimensions

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We analyze a particular $SU(2)$ invariant sector of the scalar manifold of gauged $\mathcal{N} = 8$ supergravity in five dimensions, and find all the critical points of the potential within this sector. The critical points give rise to Anti-de Sitter vacua, and preserve at least an $SU(2)$ gauge symmetry. Consistent truncation implies that these solutions correspond to Anti-de Sitter compactifications of IIB supergravity, and hence to possible near-horizon geometries of 3-branes. Thus we find new conformal phases of softly broken $\mathcal{N} = 4$ Yang–Mills theory. One of the critical points preserves $\mathcal{N} = 2$ supersymmetry in the bulk and is therefore completely stable, and corresponds to an $\mathcal{N} = 1$ superconformal fixed point of the Yang–Mills theory.

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The corresponding renormalization group flow from the $\mathcal{N} = 4$ point has $c_{\text{IR}}/c_{\text{UV}} = 27/32$. We also discuss the ten-dimensional geometries corresponding to these critical points.
1. Introduction

The correspondence between AdS supergravity theories and superconformal field theories on branes has been examined from many perspectives, and this has led to a much deeper and richer understanding of how these correspondences work. It is our purpose in this letter to re-examine some of the issues that were important in gauged supergravity 15 years ago, but now considered from the perspective of superconformal Yang-Mills theories. We consider perhaps the best substantiated correspondence [1]: that of gauged $\mathcal{N}=8$ supergravity in five dimensions [2,3] and $\mathcal{N}=4$ supersymmetric Yang-Mills theory on 3-branes. In particular we analyse the potential in this supergravity model, finding a class of critical points that have at least $SU(2)$ gauge symmetry in the supergravity (or $R$-symmetry of the Yang-Mills theory). This class includes a non-trivial supersymmetric critical point.

Before proceeding with the analysis we review some of the relevant ancient history. In the early 1980’s much work was done on testing and establishing that the maximal gauged supergravities were indeed embedded in the sphere compactifications of various higher dimensional theories. By “embedded” we mean that the full non-linear gauged supergravity action can be fully encoded in the action and field equations of the higher dimensional theory, and in particular, a solution of the gauged supergravity theory can be precisely mapped onto a solution of the higher dimensional theory. The possibility of such a consistent truncation was considered quite remarkable in that the states of the lower dimensional gauged supergravity involved non-trivial spherical harmonics in the higher dimensional theory, and it seemed that many miraculous identities would be needed if the full non-linearities of the gauged supergravity were going to decouple consistently from the higher Kaluza-Klein states. This was most extensively studied for the $S^7$ compactification of eleven dimensional supergravity to $\mathcal{N}=8$ gauged supergravity in four dimensions, and a vast body of evidence was assembled in support of “consistent truncation,” and ultimately the full non-linear embedding was explicitly constructed [4]. Less is known about the embedding of other maximal theories in other dimensions, and the complete Ansätze were never constructed. However, given the proof in [4], and the structural similarities of the various maximal gauged theories, particularly between the five-dimensional and the four-dimensional $\mathcal{N}=8$ theories, it seems extremely likely that such theories are embedded in their higher dimensional counterparts. This is further supported by quite a number of non-trivial consistency checks that have been performed over the years through the construction of explicit solutions. We will thus take it as a given that the five-dimensional $\mathcal{N}=8$ theory is embedded in the $S^5$ compactification of IIB supergravity.
The consistency of the truncation has a simple, and fundamental meaning for the Yang-Mills theory on the 3-branes. As is now fairly well established, the supergravity scalars represent couplings of the relevant and marginal chiral primary perturbations of the $\mathcal{N} = 4$ Yang-Mills theory. These operators constitute a very particular subset of all of the relevant and marginal perturbations of the theory, namely those that belong to the “short” $\mathcal{N} = 4$ multiplet of the energy momentum tensor [5,6]. Consistent truncation means that, at least for large $N$, this subset of chiral, primary operators should close under operator product, as has been discussed in [7]. Such a closed operator algebra also means that if one turns on couplings to these chiral primary operators, then the renormalization group flow should be determined entirely by these relevant (and marginal) operators chiral primaries, and not by the “irrelevant” higher Kaluza-Klein states. One can implement this very explicitly within the supergravity theory: If one can find a critical point of the scalar potential, $\mathcal{P}$, then this corresponds to a solution of the ten-dimensional theory, and hence to a “phase” of the 3-branes. There is of course the $\mathcal{N} = 8$ supersymmetric, $SO(6)$ invariant critical point, corresponding to the $S^5$ compactification of the IIB supergravity, but there are also other critical points. These “new” critical points are generically at negative values of $\mathcal{P}$, and so there are solutions in Anti-de Sitter space, and the corresponding 3-brane field theories are thus conformal. One can make this correspondence even more explicit [8,9] by constructing “interpolating” solutions: that is solutions that, at large distance from the branes, are at the maximally symmetric critical point and then flow, as the distance from the brane decreases, to another critical point. Since the distance from the brane represents the scale in the Yang-Mills theory, such interpolating solutions must represent explicit renormalization group flows from the U.V. fixed point ($\mathcal{N} = 4$ Yang-Mills theory) to a new conformal I.R. phase of the Yang-Mills theory. Consistent truncation means that this flow will be entirely determined by the equations of motion of supergravity in five dimensions.

In the gauged supergravity theories there was also the concern that since one was dealing with critical points of a potential, stability of solutions is an issue. However, because of the Planck scale of the potential there is a strong gravitational back-reaction that tends to stabilize some of the naively unstable critical points. There is the Breitenlohner-Freedman condition [10,11], that states that a scalar wave equation $\Box \phi - \alpha \phi$ is perturbatively stable in an AdS space of dimension $d$ and radius $R$ if $\alpha < \frac{1}{16\pi^2} (d - 1)^2$. There is also a more powerful, but less general result that states that if a solution has any supersymmetry then the solution is completely semi-classically stable [11]. The statement of stability from the point of view of the phases of Yang-Mills appears to be a statement of unitarity. This is most directly seen from the connection between the conformal dimension of an operator in
the Yang-Mills theory and the “mass” of a supergravity scalar [12]. Scalars that do not satisfy the Breitenlohner-Freedman bound correspond to operators with complex conformal dimensions [8,9] and thus represent non-unitary perturbations. Such perturbations have been considered in field theories in $1 + 1$-dimensions, and in systems with self-organized criticality, where they lead to “log-periodic” behaviour, and “roaming” renormalization group flows (see, for example, [13]). In the Yang-Mills theory on the 3-brane, all the “unstable” operators have a dimension whose real part is 2, and hence they are relevant and drive an “oscillatory flow.” The physical interpretation of such operators in field theory however is far from clear.

Another possible issue of concern is that a critical point of the supergravity theory might be a pathology of large $N$ gauge theories. While such large $N$ pathologies may still be interesting in their own right, it would be nice to know that a given critical point represents a solution at finite $N$. One way to ensure this is if a solution represents a solution to the string theory, and this is much more likely if the solution is stable, and indeed if it is supersymmetric.

Thus, a classification of critical points of the potential of gauged $\mathcal{N} = 8$ supergravity in five dimensions takes on new relevance since it represents a classification of infra-red fixed points of large $N$, $\mathcal{N} = 4$ Yang Mills theory. Moreover, the supersymmetric critical points are particularly significant since they represent very stable fixed points, that should be present even in $\mathcal{N} = 4$, $SU(N)$ Yang-Mills with finite $N$.

The scalar potential of gauged $\mathcal{N} = 8$ supergravity in five dimensions is a function of 42 scalars. It is invariant under the $R$-symmetry, $SO(6)$, and under the $SL(2,R)$ symmetry of the original IIB theory in ten dimensions. This means that the potential is a function of 24 independent variables, and in terms of physical scalars the potential has two flat directions at every point, coming from the non-compact generators of $SL(2,R)$. This number of parameters is too large to be practicably managed, and so we reduce the problem by seeking out all critical points that reduce the gauge/$R$-symmetry to a group containing a particular $SU(2)$ subgroup of $SO(6)$. This choice is not only motivated by the pragmatic consideration of actually being able to perform the calculation, but also by the fact that $SU(2)$ is the $R$-symmetry of $\mathcal{N} = 2$ gauge theories. We do not actually find a solution with $\mathcal{N} = 2$ supersymmetry on the brane, but this reduction of the problem to the $SU(2)$ invariant subsector is still wide enough to enable us to find one critical point that has $\mathcal{N} = 1$ supersymmetry on the brane, i.e. $\mathcal{N} = 2$ supersymmetry in the supergravity theory.
Finally there is the value of the cosmological constant at a critical point. It is a consequence of the work of [14,15] that this translates directly into the central charge of the conformal theory. We compute the central charges at the various critical points, and find that the supersymmetric critical point has $c_{IR}/c_{UV} = 27/32$.

In section 2 we discuss the truncation of the scalar manifold and potential to a particular $SU(2)$ invariant subsector. In section 3 we describe, and catalogue properties of, all the critical points in this invariant subsector. In section 4 we discuss some of the implications of our results. We focus on the non-trivial supersymmetric critical point, and describe the geometry of its embedding into the ten-dimensional theory. Indeed, we use the close similarity between the five dimensional and the four dimensional $N = 8$ theories to infer the proper generalization of [16], and conjecture the full non-linear Ansatz for the “internal” compactifying metric for gauged $N = 8$ supergravity in five dimensions.

2. Constructing the potential on the $SU(2)$ invariant sector

As was described in [17], an effective way of searching for interesting subsets of critical points of the potential is to restrict the problem to the space, $\mathcal{S}$, of singlets of some invariance group, $G$. It is a trivial consequence of Schur’s lemma that any variation of the potential, $\mathcal{P}$, about $\mathcal{S}$ in a non-singlet direction is necessarily quadratic, and thus any critical point of $\mathcal{P}$ on $\mathcal{S}$ is necessarily a critical point on the whole space of scalars.

Here we consider $SU(2)_R$ subgroups\(^1\) of the $\mathcal{R}$-symmetry $SU(4)$. There are four distinct such subgroups in $SU(4)$ and these can be characterized in terms of how the $4$ of $SU(4)$ decomposes under $SU(2)$. Specifically, one can have: (i) $4 \rightarrow 2 \oplus 1 \oplus 1$, (ii) $4 \rightarrow 2 \oplus 2$, (iii) $4 \rightarrow 3 \oplus 1$, or (iv) $4 \rightarrow 4$. Only the first possibility will be analysed in this letter since it has the largest singlet structure in $E_6(6)$ and is thus most likely to yield new critical points.

Our choice of $SU(2)_R$ commutes with an $H_0 \equiv SU(2) \times U(1)$ in $SU(4)$. The commutant of $SU(2)_R$ is extended to $H_1 \equiv SU(2) \times SL(2, \mathbb{R}) \times \mathbb{R}^+$ in $SL(6, \mathbb{R})$, and finally to $H_2 \equiv SO(5, 2) \times \mathbb{R}^+$ in $E_{6(6)}$.\(^2\) If one thinks of the obvious $SO(3) \times SO(2, 2)$ subgroup of $SO(5, 2) \times \mathbb{R}^+$, and recalls that $SO(2, 2) = SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$ then one of these

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1 The subscript $\mathcal{R}$ is to distinguish this $SU(2)$ from various other $SU(2)$ subgroups of $E_{6(6)}$.

2 In writing $\mathbb{R}^+$ here we are dropping two discrete $\mathbb{Z}_2$ subgroups: One generated by $-1 \in SL(6, \mathbb{R})$, and the other by diag$(-1, -1, -1, -1, 1, 1)$. These $\mathbb{Z}_2$’s are symmetries of the potential and so all our critical points will, in fact, come in $\mathbb{Z}_2 \times \mathbb{Z}_2$ multiplets.
factors is the $SL(2, \mathbb{R})$ of $H_1$, while the other $SL(2, \mathbb{R})$ factor is the scalar manifold of the ten-dimensional theory. The $SO(3)$ factor above is the $SU(2)$ of $H_0$. As one would expect, the maximal compact subgroup $SO(5) \times SO(2)$ of $SO(5, 2)$ is the subgroup of $USp(8)$ that commutes with $SU(2)_R$. Thus the manifold of scalar singlets is given by

\[ S = \frac{SO(5, 2)}{SO(5) \times SO(2)} \times \mathbb{R}^+ . \]  

(2.1)

To find a simple parametrization of the potential on this space we need to fix the invariances as cleanly as possible. The manifold, $S$, is 11-dimensional, but the potential has a residual $H_{inv} \equiv H_0 \times SL(2, \mathbb{R}) \equiv SU(2) \times U(1) \times SL(2, \mathbb{R})$ invariance, which has seven parameters. This means that we should be able to reduce the potential to a function of four variables, and indeed we can. We represent an element of the coset, has seven parameters. This means that we should be able to reduce the potential to a function of four variables, and indeed we can. We represent an element of the coset, $\rho = e^{\alpha} \in \mathbb{R}^+$ and by the exponential of a $7 \times 7$ matrix, $M$, with $M_{ij} = M_{6i} = x_i$; $M_{i7} = M_{7i} = y_i$, $i = 1, \ldots, 5$, and all other $M_{ij}$ set to zero. We now argue that there is a gauge in which we can take $x_1 = x_3 = x_5 = 0$ and $y_i = 0, i = 1, 3, 4, 5$, leaving four parameters: $x_1, y_2, x_4$ and $\alpha$.

First the $SU(2)$ of $H_{inv}$ acts as the triplet of $x_i$ and $y_i$, $i = 1, 2, 3$. We could completely fix this $SO(3)$ by setting $x_3 = x_2 = 0$ and $y_3 = 0$, but we start by partially fixing it by setting only $x_3 = y_3 = 0$. This leaves an $SO(2)$ subgroup which may be thought of as acting on the matrix $M_0 = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix}$ from the left. A linear combination of the $U(1)$ and the $SO(2)$ subgroup of $SL(2, \mathbb{R})$ acts on $M_0$ by right multiplication. These two independent $SO(2)$ actions can be used to diagonalize $M_0$. We have thus set $x_2 = y_1 = x_3 = y_3 = 0$, and are left with the other linear combination of the $U(1)$ and the $SO(2)$ subgroup of $SL(2, \mathbb{R})$, and the non-compact generators of $SL(2, \mathbb{R})$. We can fix the latter by setting $y_4 = y_5 = 0$, while the remaining $SO(2)$ rotates the $(4, 5)$ coordinates and so can be used to set $x_5 = 0$, which completes the gauge fixing.

To construct the potential we need to construct the $27 \times 27$ $E_{6(6)}$ matrix, $\mathcal{V}$. Under $SU(2)_R \times SO(5, 2) \times R^+$ the 27 of $E_{6(6)}$ decomposes as:

\[ 27 \rightarrow (1, 1)(+4) \oplus (1, 7)(-2) \oplus (2, 8)(+1) \oplus (3, 1)(-2) . \]

This means that $\mathcal{V}$, when written in the proper basis consists of: (i) the exponential $M$ and multiplied by $\rho^{-2}$, (ii) two copies of the exponential of the spinor representative of $M$, multiplied them by $\rho$, and (iii) the matrix diag($\rho^4, \rho^{-2}, \rho^{-2}, \rho^{-2}$). To assemble the potential one then needs to rotate this back into the $SL(6, \mathbb{R}) \times SL(2, \mathbb{R})$ basis. The exponentials of the gauge fixed matrices are elementary to compute, and the basis rotations are tedious,
but straightforward. Using Mathematica™ we assembled all of this into the potential as described in [2] and obtained:

\[ P = \frac{g^2}{32} \rho^{-4} \left( \cosh(4r_x) \cosh(4r_y) - \cosh(4r_x) - \cosh(4r_y) \right) + 4 \cos(2\theta) \sinh(2r_x)^2 \sinh(2r_y)^2 - 7 - \frac{g^2}{2} \rho^2 \cosh(2r_x) \cosh(2r_y) \]
\[ + \frac{g^2}{64} \rho^8 \left( \cosh(4r_x) + 2 \cosh(4r_x) \cosh(4r_y) - 2 \cos(2\theta) \sinh(2r_x)^2 - 3 \right), \]

where \( x_1 = r_x \cos \theta, x_4 = r_x \sin \theta, \rho = e^\alpha \) and \( r_y = y_2 \).

3. Critical Points

It is elementary to find the critical points of (2.2), and some of the details are summarized in Table 1. As discussed in [2], one can determine the number of unbroken supersymmetries at a given critical point by finding the eigenvalues, \( \mu_i \), of the tensor \( W_{ab} \). The number of supersymmetries is equal to the number of \( |\mu_i| = \sqrt{-3\Lambda/g^2} \), where \( \Lambda = P \) is the cosmological constant at the critical point. The critical points and their supersymmetries are as follows:

(i) First there is the well known, trivial critical point at which all the scalars vanish, and whose cosmological constant is \( \Lambda = -3g^2/4 \), and which preserves \( \mathcal{N} = 8 \) supersymmetry.

(ii) There is a critical point at \( x_1 = y_2 = 0, x_4 = \frac{1}{4} \log(3) \), and \( \alpha = \frac{1}{12} \log(3) \), and the cosmological constant is \( \Lambda = -\frac{3^{5/3}}{8} g^2. \) This scalar vev actually lives in the \( SL(6, \mathbb{R}) \) subgroup of \( E_6(6) \), and it is \( SO(5) \) invariant. This critical point was found in [2]. The eigenvalues of \( W_{ab} \) are all equal to \(-2.3^{-1/6}\), and hence there is no supersymmetry at this point. It was also shown in [9] that this critical point is perturbatively unstable.

(iii) There is a critical point at \( x_1 = y_2 = \pm \frac{1}{4} \log(2 - \sqrt{3}), x_4 = 0, \) and \( \alpha = 0 \), and the cosmological constant is \( \Lambda = -\frac{77}{32} g^2. \) This scalar vev corresponds to the \( SU(3) \) invariant critical point discovered in [2]. The eigenvalues of \( W_{ab} \) are \(-\frac{7}{2}\) and \(-\frac{9}{4}\) with multiplicities of 6 and 2 respectively, and so there is no supersymmetry.

(iv) There is a critical point at \( x_1 = x_4 = 0, y_2 = \pm \frac{1}{4} \log(\frac{1}{5} (11 - 4\sqrt{6})), \) and \( \alpha = \frac{1}{12} \log(10), \) and the cosmological constant is \( \Lambda = -\frac{3}{8} (\frac{25}{7})^{1/3} g^2. \) The non-zero vev of \( \alpha \) reduces the \( SO(6) \) invariance to \( SU(2) \times SU(2) \times U(1) \), and the non-zero vev of \( y_2 \) further reduces
this to $SU(2) \times U(1) \times U(1)$. The eigenvalues of $W_{ab}$ are $-3.10^{-1/6}$ and $-9.10^{-2/3}$ each with a multiplicity of 4, and once again there is no supersymmetry.

(v) There is a critical point at $x_1 = y_2 = \pm \frac{1}{4} \log(3)$, $x_4 = 0$, and $\alpha = \frac{1}{6} \log(2)$, and the cosmological constant is $\Lambda = -\frac{2^{4/3}}{3} g^2$. As in (iii), the non-zero vev of $x_1 = y_2$ reduces the $SO(6)$ invariance to $SU(3)$, and then the non-zero vev of $\alpha$ further reduces this to $SU(2) \times U(1)$. The eigenvalues of $W_{ab}$ are $-\frac{7}{3} 2^{-1/3}$, $-\frac{4}{3} 2^{2/3}$ and $-2^{2/3}$ with a multiplicity of 4, 2 and 2 respectively. The last eigenvalue is equal to $-\sqrt{-3\Lambda/g^2}$, and so this critical point has an unbroken $\mathcal{N} = 2$ supersymmetry.

As discussed in the introduction, these critical points may be thought of as infra-red fixed points of the Yang-Mills theory on the branes. Since all the cosmological constants are negative, and therefore admit anti-de Sitter metrics, the corresponding gauge theories are conformal. Using the results of [14,15] one can compute the central charge of these conformal theories. Indeed, from [14] one sees that the ratio of central charge, $c_{\text{IR}}$, of the new fixed point compared to $c_{\text{UV}}$, the central charge of the $\mathcal{N} = 4$ symmetric fixed point is given by:

$$
\frac{c_{\text{IR}}}{c_{\text{UV}}} = \left(\frac{\Lambda_{\text{IR}}}{\Lambda_{\text{UV}}}\right)^{-\frac{3}{2}} = \left(-\frac{4\Lambda_{\text{IR}}}{3g^2}\right)^{-\frac{3}{2}},
$$

where $\Lambda_{\text{IR}}$ and $\Lambda_{\text{UV}} = -\frac{3}{4}g^2$ are the cosmological constants at the corresponding critical points. This ratio of central charges is also given in Table 1.

<table>
<thead>
<tr>
<th>Critical Point</th>
<th>Unbroken Gauge Symmetry</th>
<th>Cosmological Constant</th>
<th>Unbroken Supersymmetry</th>
<th>CentralCharge $c_{\text{IR}}/c_{\text{UV}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>$SO(6)$</td>
<td>$-\frac{3}{2} g^2$</td>
<td>$\mathcal{N} = 8$</td>
<td>1</td>
</tr>
<tr>
<td>(ii)</td>
<td>$SO(5)$</td>
<td>$-\frac{3}{8} g^2$</td>
<td>$\mathcal{N} = 0$</td>
<td>$\frac{2\sqrt{2}}{3} \sim 0.9428$</td>
</tr>
<tr>
<td>(iii)</td>
<td>$SU(3)$</td>
<td>$-\frac{27}{32} g^2$</td>
<td>$\mathcal{N} = 0$</td>
<td>$\frac{16\sqrt{2}}{7} \sim 0.8381$</td>
</tr>
<tr>
<td>(iv)</td>
<td>$SU(2) \times U(1) \times U(1)$</td>
<td>$-\frac{3}{8} (25/7)^{1/3} g^2$</td>
<td>$\mathcal{N} = 0$</td>
<td>$\frac{5}{6} = 0.8$</td>
</tr>
<tr>
<td>(v)</td>
<td>$SU(2) \times U(1)$</td>
<td>$-\frac{3}{4} 2^{2/3} g^2$</td>
<td>$\mathcal{N} = 2$</td>
<td>$\frac{27}{32} \sim 0.8438$</td>
</tr>
</tbody>
</table>

Table 1: The critical points of the potential (2.2). The unbroken gauge symmetry is that of the supergravity, and corresponds to the $R$-symmetry on the branes. The unbroken supersymmetry is that of the supergravity theory, and should be halved to get the supersymmetry on the branes.
4. Discussion

The critical points described above give rise to new phases of Yang–Mills theory that are presumably strongly coupled, and so it is not altogether obvious what their massless spectra should be. Indeed, the previously known, non-trivial critical points ( (ii) and (iii) in Table 1) were discussed in [8,9], and have an irrational central charge ratio, and so cannot have a spectrum that is simply related to the perturbative spectrum of the $\mathcal{N} = 4$ Yang-Mills theory. The two new critical points found in this letter ((iv) and (v) in Table 1) may ultimately prove a little more amenable. Both have rational central charge, and one of them has $\mathcal{N} = 1$ supersymmetry and is therefore very stable.

Before discussing the supersymmetric critical point in some detail, we wish to note some possible patterns in our albeit very small amount of data. First, we note that the central charge decreases with the amount of global symmetry (the central charge jumps back up if there is residual supersymmetry). Secondly, both in five dimensions and in four dimensions [17], we note that there is either no critical point, or a unique critical point with a given global symmetry. This is similar in spirit to the results of [18] which suggest that the near horizon geometry of a brane configuration should be uniquely determined by the angular momentum, or horizon symmetry, and charges of the branes.

There is an apparent parallel between our supersymmetric critical point, and the $\mathcal{N} = 1$ supersymmetric model considered in [19,15]. The latter model was obtained by first doing a $\mathbb{Z}_2$ orbifold of the $\mathcal{N} = 4$ model so as to reduce it to $\mathcal{N} = 2$, and then turning on a relevant perturbation in the field theory to break it to an $\mathcal{N} = 1$ model and flow to a new fixed point. (This fixed point has a marginal perturbation, and the resulting fixed line connects to the conformal window of [20].) The $\mathbb{Z}_2$ orbifold is made as in [21]: The $\mathbb{Z}_2$ acts on $S_5$ via the matrix $\sigma = \text{diag}(-1, -1, -1, -1, 1, 1)$ multiplying the Euclidean coordinates of $\mathbb{R}^6$. There is also a simultaneous action on the branes in which the branes are separated into two groups of $N$, and then these groups are interchanged. The relevant perturbation that breaks to $\mathcal{N} = 1$ in [19,15] is in the twisted sector of the theory and involves turning on a fermion masses so as to reduce the $\mathcal{R}$-symmetry from $SU(2) \times U(1)$ to $U(1)$.

The $\mathcal{N} = 1$ supersymmetric critical point found here involves a two parameter submanifold, $S_0$, of two commuting scalars in $E_{6(6)}/USp(8)$. These are parametrized by $\alpha$ and $\beta = x_1 = y_2$, and are characterized respectively as (i) matrices in $SL(6, \mathbb{R})$ of the form $M(\alpha) = \text{diag}(e^{\alpha}, e^{\alpha}, e^{\alpha}, e^{\alpha}, e^{-2\alpha}, e^{-2\alpha})$, and (ii) the non-compact generator of $E_{6(6)}$ that breaks $SO(6)$ down to $SU(3)$ (an explicit expression for this was given in [2]). First note that the orbifold generator, $\sigma$, is the same as $M(\alpha = i\pi)$, and also that $\sigma$ commutes with the scalars on $S_0$. The consequence of the first observation is that $\sigma$ and $M$ make the
same fields massive. The consequence of the second observation is that the scalar manifold $S_0$ is part of the scalar manifold of the untwisted sector of the orbifold theory of [19,15]. Turning on $\alpha$ breaks $SO(6)$ to $SU(2) \times SU(2) \times U(1)$, and turning on $\beta$ reduces this to $SU(2) \times U(1)$, where the $U(1)$ becomes the $\mathcal{R}$-symmetry at the $\mathcal{N} = 1$ supersymmetric point, and $SU(2)$ is an additional global symmetry. From the AdS/CFT correspondence, turning on $\beta$ corresponds to turning on very specific fermion masses [12]. While this relevant operator is not the same as that used in [19], it is possible that the IR fixed points of the renormalization group flows could be related.

One can make the parallel much closer by going to the $\sigma$-invariant sector of the gauged $\mathcal{N} = 8$ supergravity theory. This is a truncation to a gauged $\mathcal{N} = 4$ supergravity, and corresponds to a subsector of the untwisted sector of the $\mathbb{Z}_2$ orbifold. (This $\sigma$-invariant sector does not contain the hypermultiplets arising from the interchange of the branes.) The scalar submanifold $S_0$, and the corresponding restriction of scalar potential survives this truncation, and so the corresponding relevant perturbations of the full orbifold theory will be described by this part of the supergravity action. Turning on the scalar vevs considered in this paper will generate a superpotential for the chiral multiplets, but a different one from that described in [19] (and it will have a smaller global symmetry). The flow to the critical point will then lead to a different fixed line. Whether this fixed line and that of [19] are connected remains to be seen, but an intriguing, though indirect piece of evidence for such a connection is the fact that both renormalization group flows have $c_{IR}/c_{UV} = 27/32$.

Finally, it is instructive to consider the geometry of the compactification that corresponds to our superconformal critical point.

The full embedding of five dimensional, gauged $\mathcal{N} = 8$ supergravity into the ten-dimensional theory has never been explicitly written down, but it is rather easy to infer part of it from the corresponding results for four-dimensional supergravity. Turning on scalars in $SL(6, \mathbb{R})$ corresponds to a modifying the metric of $S^5$ to that of a surface in $\mathbb{R}^6$ defined by [16]

$$\vec{x}^T \cdot S^T S \cdot \vec{x} = r^2 ,$$

(4.1)

where $\vec{x}$ are cartesian coordinates on $\mathbb{R}^6$, $S \in SL(6, \mathbb{R})$, and $r$ is a constant. This deformation is accompanied by the introduction of “warp-factors” [22] that are fractional powers of $\mu = (\vec{x}^T \cdot (S^T S)^2 \cdot \vec{x})$ in front of the metric of (4.1) and the metric the anti-de Sitter space time. This means that turning on $\alpha$ deforms the internal metric to a ellipsoid in a manner reminiscent of rotating branes.
To lowest order the mode expansion of $S^5$ the perturbation by $\beta$ corresponds to turning on an $SU(3)$-invariant configuration for the anti-symmetric tensor field $G_{mnp}$ in ten dimensions. From what is known about the $SU(3)$ invariant critical point [2] and the corresponding compactification [23], we know that one should consider the $S^5$ (and the deformation described above) as an $S^1$ bundle over a (deformed) $\mathbb{CP}^2$. Let $\chi$ denote the fiber coordinate, and let $K_{ij}$ be the holomorphic 2-form on the base – the latter is not globally well defined, but the following Ansatz for $G_{mnp}$ is [24,23]:

$$G_{ij\chi} = a e^{2i\chi} K_{ij}, \quad G^*_{ij\chi} = a^* e^{-2i\chi} K^*_{ij},$$

(4.2)

where $a$ is a constant and $K_{ij}$ is the complex conjugate of $K_{ij}$. At higher orders in the perturbation $\beta$, the metric of the $S^1$ fibration is further deformed by a stretching of the $S^1$ fiber compared to the scale of the base. Putting this all together, the background will be an $S^1$ fibration over a complex 2-fold which itself consists of a squashed $\mathbb{CP}^2$. The latter squashing can be accomplished by deforming the scale of the complex coordinates so as to preserve an isometry, $SU(2) \times U(1)$ of $\mathbb{CP}^2$. Indeed, in a properly chosen coordinate patch one can arrange that this $SU(2) \times U(1)$ is the local isotropy group of the origin, with the remaining translational isometries of $\mathbb{CP}^2$ being broken. A background tensor field of the form (4.2) is then turned on.

For completeness sake we note that based on the arguments of [16], a reasonable conjecture for the full (inverse) metric on the internal space is given by:

$$\Delta^{-\frac{2}{3}} g^{mp} = c K^{mIJ} K^pK^L \tilde{\mathcal{V}}_{IJab} \tilde{\mathcal{V}}_{KLea} \Omega^{ac} \Omega^{bd}. \quad (4.3)$$

In this equation $K^{mIJ} = -K^{mJI}$, $I, J = 1, \ldots, 6$ are the Killing fields on the $S^5$, $\tilde{\mathcal{V}}_{IJab}$ is a submatrix, defined in [2], of the inverse of the $27 \times 27$ scalar $E_{6(6)}$ matrix, $\mathcal{V}$; $\Omega^{ab}$ is the $USp(8)$ symplectic invariant, $\Delta = \sqrt{\det(g_{mp})}$, and $c$ is a normalization constant. In addition to this, the anti-de Sitter metric of the five-dimensional space time must be rescaled by the warp-factor $\Delta^{-2/3}$.

The compactification that we have just outlined is very different from the compactification considered in [19]. Indeed, for the $\mathcal{N} = 1$ theory of [19] the relevant geometry at the conifold point was argued to be that of the coset $T^{1,1}$. There is no background tensor field. Moreover, the critical line described in [19] involved the Kähler modulus of a blow up of singularity. If, after passing to a $\mathbb{Z}_2$ orbifold, our critical point has a similar marginal deformation, then it would be interesting to see precisely to what it corresponds. It seems very likely that it would involve turning on the tensor field, which in terms of the conifold, is suggestive of some kind of “dual” branch to the construction of [19].
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