HYDROGEN-LIKE ATOMS FROM ULTRARELATIVISTIC NUCLEAR COLLISIONS

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Abstract

The number of hydrogen-like atoms produced when heavy nuclei collide is estimated for central collisions at the Relativistic Heavy Ion Collider using the sudden approximation of Baym et al. As first suggested by Schwartz, a simultaneous measurement of the hydrogen and hadron spectra will allow an inference of the electron or muon spectra at low momentum where a direct experimental measurement is not feasible.

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The production rate of lepton pairs is a rapidly increasing function of temperature and so has long been considered a good probe of the initial high energy density phase of ultrarelativistic nuclear collisions [1]. The experimental detection of such direct leptons is a problem in the sub-GeV range of transverse momentum due to the large number of charged hadrons produced and the need to disentangle direct leptons from those arising from hadron decays. But this is just the kinematic range characterizing a quark-gluon plasma at a temperature of 200 to 500 MeV.

Schwartz [2] proposed to measure the distribution of atoms formed by the binding of a directly produced lepton to one of the charged hadrons emerging from the final state of the nuclear collision. A measurement of the charged hadrons and of the atoms, together with a theoretical calculation relating the distributions of the three particle species, would then imply the spectrum of leptons. The beauty of the idea lies in the fact that nearly all indirectly produced leptons arise from the decay of hadrons, and these decays occur too long after the collision to allow an atom to be formed. Of course, one still cannot tell whether the leptons were produced in quark-gluon plasma or in hadronic matter, but this is another issue.

Five years ago Baym, Friedman, Hughes and Jacak calculated the relationship among the spectra of the atoms and of the charged hadrons and leptons which comprise them [3]. The formula reads:

\[
\frac{dN_{\text{atom}}}{dyd^2p_{\perp,\text{atom}}} = 8\pi^2\zeta(3)\alpha^3 m_{\text{red}}^2 \frac{dN_h}{dyd^2p_{\perp,h}} \frac{dN_l}{dyd^2p_{\perp,l}}.
\]

Here \(\zeta(3) = 1.202\ldots\) and \(m_{\text{red}}\) is the reduced mass of the hadron and lepton making up the atom. Since the binding energy is so small it is an excellent approximation to evaluate the hadron and lepton rapidities at the same rapidity as that of the atom, and to equate their transverse velocities as well: \(p_{\perp,\text{atom}}/m_{\text{atom}} = p_{\perp,h}/m_h = p_{\perp,l}/m_l\). This formula is based on the sudden approximation which simply asks for the overlap of the outgoing wave functions of the hadron and the lepton with their hydrogenic state. The sudden approximation is valid because these particles are formed in a nuclear volume which is extremely small in comparison to the size of the hydrogen atom and over a time interval which is extremely small in comparison to the Bohr period. The specific focus of Baym et al. was on \(\pi-\mu\) atoms. Here we shall be interested in p-e, p-\(\mu\), \(\pi-e\) and \(\pi-\mu\) atoms. Our essential contribution is to estimate \(dN/dyd^2p_{\perp}\) for the leptons, protons and pions in the relevant range of transverse momentum, and from these to estimate the number of hydrogenic atoms to be formed in central Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC).

First we estimate the number of leptons produced in the quark-gluon plasma phase. The reaction rate for the process \(q + \bar{q} \rightarrow l^+ + l^-\) is:

\[
R_q = 12 \int \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_+}{2E_+/(2\pi)^3} \frac{d^3p_-}{2E_-/(2\pi)^3} \times f_{\text{FD}}(E_1)f_{\text{FD}}(E_2) |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_- - p_+) ,
\]
where the 12 arises from three colors and four possible spin states of the colliding quarks, \( f_{\text{FD}} \) is the Fermi-Dirac distribution, and \( \mathcal{M} \) is the matrix element for the reaction. Approximating \( f_{\text{FD}}(E) \approx e^{-E/T} \) the momentum distribution for negatively charged leptons becomes [4]:

\[
E_ - \frac{d^3R}{d^3p_-} = \sum_{q=u,d,s} \frac{e^2 e_q^2}{(2\pi)^6} 2E_ - e^{-E_-/T} \int ds \ln \left(1 + e^{-s/4E_ -} \right) \times \left[1 + 2(m_q^2 + m_l^2)/s + 4m_q^2m_l^2/s^2 \right] \sqrt{(1 - 4m_l^2/s)(1 - 4m_q^2/s)}, \tag{3}
\]

where \( s = (p_1 + p_2)^2 \). The expression for positively charged leptons is the same. If the masses of the leptons and the quarks can be neglected this simplifies to

\[
E_ - \frac{d^3R}{d^3p_-} = \frac{\alpha^2}{3\pi^4} T^2 e^{-E_-/T}. \tag{4}
\]

For the range of transverse momentum of interest in this context the masses of the muon and strange quark cannot be neglected.

To obtain the total number emitted we integrate over the space-time volume according to Bjorken’s model [5]. For central collisions:

\[
\frac{dN}{dyd^2p_T} = \sum_{q=u,d,s} \left(\frac{e_q}{e}\right)^2 \frac{3\alpha^2}{8\pi^3} \left(\tau_0 T_0^3 R_T^2\right)^2 \int_{-\infty}^{\infty} \frac{d\eta}{E} \int_{T_0}^{T_c} \frac{dT}{T^6} e^{-E/T} \int_{s_{\text{min}}}^{\infty} ds \ln \left(1 + e^{-s/4ET}\right) \times \left[1 + 2(m_q^2 + m_l^2)/s + 4m_q^2m_l^2/s^2 \right] \sqrt{(1 - 4m_l^2/s)(1 - 4m_q^2/s)}. \tag{5}
\]

Here \( R_T \) is the nuclear radius, \( y \) is the momentum space rapidity, \( \eta \) is the position space rapidity, \( T_0 \) is the temperature when the plasma is first considered to be thermalized, and \( T_c \) is the critical or phase transition temperature. In the Bjorken model the temperature drops with proper time \( \tau \) according to

\[
T(\tau) = \left(\frac{\tau_0}{\tau}\right)^{1/3} T_0. \tag{6}
\]

Finally \( E = m_\perp \cosh(y - \eta) \) and \( m_\perp = \sqrt{m_l^2 + p_\perp^2} \) where the z-axis is the beam axis.

In general the integrals must be done numerically. However, if the masses can be neglected then the above simplifies to:

\[
\frac{dN}{dyd^2p_T} = \frac{2\alpha^2 R_T^2 (\tau_0 T_0^3)^2}{\pi^3} \left[ (p_T/T_0)^3 K_1(p_T/T_0) + 2(p_T/T_0)^2 K_2(p_T/T_0) - (p_T/T_c)^3 K_1(p_T/T_c) - 2(p_T/T_c)^2 K_2(p_T/T_c) \right]. \tag{7}
\]

If one is interested in hydrogen-like atoms with a transverse momentum of a few GeV/c then the transverse momentum of the lepton must have been

\[
p_\perp, l = \frac{m_l}{m_{\text{atom}}} p_{\perp, \text{atom}}, \tag{8}
\]
which is just a few MeV/c for electrons and a few hundred MeV for muons. For electrons
the above formula simplifies even more since their transverse momentum is always much
less than the temperature.

\[ \frac{dN}{dy d^2p_T} = \frac{\alpha^2 R_T^2}{2\pi^3} \left( \frac{T_0}{T_c} \right)^2 \ln \left( \frac{1.20 T_c}{p_{\perp}} \right) - \ln \left( \frac{1.20 T_0}{p_{\perp}} \right) \] (9)

This invariant distribution diverges logarithmically at small transverse momentum.

Our estimates of the lepton distributions also include those coming from the mixed
phase as the system goes through a first-order phase transition. The contribution from
the quark-gluon plasma phase at the phase transition temperature \( T_c \) is given by the
distribution (3) multiplied by the volume fraction occupied by the plasma, \( f_{\text{plasma}} = (r \tau_c / \tau - 1) / (r - 1) \) plus the corresponding rate in the hadronic phase multiplied by the
hadronic volume fraction \( f_{\text{had}} = 1 - f_{\text{plasma}} \). Here \( r \) is the ratio of the number of degrees
of freedom in the two phases. The hadronic rate is obtained from the annihilation process
\( \pi^+ + \pi^- \rightarrow l^+ + l^- \). These calculations are standard and well-known [4].

The canonical picture of central collisions of gold nuclei at RHIC is that the central
rapidity region will be almost baryon free [5]. However, central collisions of lead nuclei at
the SPS [1] and extrapolations of nucleon collisions such as LEXUS [6] suggest that the
baryon rapidity distribution may be roughly flat. Furthermore, initial temperatures at
RHIC may be as high as 500 MeV [7] and this will be reflected in the final transverse mass
distribution of the outgoing protons since high temperatures eventually get converted to
transverse flow. Indeed, inverse slopes for protons for central lead collisions at the SPS
already reach 300 MeV [1]. Therefore a not unreasonable estimate for protons at RHIC
is to assume a flat rapidity distribution times an exponential falloff in transverse mass.

\[ \frac{dN_p}{dy d^2p_{\perp}} = \frac{Z}{2\pi y_0(m_p + T_p)T_p} \exp \left[ \frac{(m_p - m_{\perp})}{T_p} \right] \] (10)

\( Z \) is the charge of a single nucleus, \( 2y_0 \) is the rapidity gap between projectile and target
nuclei, \( m_{\perp} \) is the proton’s transverse mass, and \( T_p \) is the proton effective temperature
(not really a temperature, just an inverse slope). Similarly, we assume the charged-pion
distribution to be:

\[ \frac{dN_\pi}{dy d^2p_{\perp}} = \frac{Z}{2\pi (m_\pi + T_\pi)T_\pi} \exp \left[ \frac{(m_\pi - m_{\perp})}{T_\pi} \right] , \] (11)

with the pion rapidity distribution \( dN_\pi/dy = 1000 \).

To get a rough idea of the numbers assume: \( T_0 = 3T_c = T_p = T_\pi = 480 \text{ MeV} \), \( Z = 79 \),
\( R_T = 7 \text{ fm} \) for a central collision at the maximum RHIC energy of 100 GeV per nucleon
per beam, \( r = 19/3 \) for three quark flavors and a hadron gas with \( g_h = 7.5 \) effective
degrees of freedom [8].

In figures 1 and 2 we plot the transverse momentum distributions of p-e, \( \pi-e \), p-\( \mu \) and
\( \pi-\mu \) atoms for a central Au+Au collision at RHIC. The trends are easily understood on
the basis of eq. (1). The muonic atoms dominate the electronic ones because they have a greater reduced mass, and the pionic ones dominate the protonic ones because charged pions are much more abundant than protons.

In figures 3 and 4 we plot the number of hydrogen-like atoms expected per unit of rapidity per day for transverse momentum bigger than $p_{\perp,\text{min}}$ for Au+Au collisions with an impact parameter of 1 fm or less. A beam luminosity of $2 \times 10^{26}$/cm$^2$/sec is assumed. For the most abundant species, $\pi$-$\mu$ atoms, we estimate about 1000 per unit of rapidity per day for transverse momenta larger than 1 GeV/c.

In conclusion, we have reinvestigated the rates for the production of hydrogen-like atoms at RHIC. The results are quite promising for their experimental detection. It remains to be seen whether an efficient detector can be designed to observe them.

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**References**


Figure Captions

Figure 1: The transverse momentum distributions for electronic atoms for a central Au+Au collision at RHIC.

Figure 2: The transverse momentum distributions for muonic atoms for a central Au+Au collision at RHIC.

Figure 3: The number of electronic atoms produced with a transverse momentum greater than the indicated value per unit of rapidity per day at RHIC. These assume design luminosity and impact parameters less than 1 fm for Au+Au collisions.

Figure 4: The number of muonic atoms produced with a transverse momentum greater than the indicated value per unit of rapidity per day at RHIC. These assume design luminosity and impact parameters less than 1 fm for Au+Au collisions.
\[ \frac{dN}{dyd^2p_T} \text{ [GeV}^{-2}\text{]} \]
$dN/dy$ [no. per day]

$p_{T,\text{min}}$ [GeV/c]

$\pi$-e

p-e
\[ \frac{dN}{dy} \text{ [no. per day]} \]

\[ p_T,\text{min} \text{ [GeV/c]} \]

- \( p-\mu \)
- \( \pi-\mu \)