Do the E866 Drell Yan data change our picture of the chiral structure of the nucleon ?

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We revisit the evaluation of the pionic mechanism of the $\bar{u} - \bar{d}$-asymmetry in the proton structure function. Our analysis is based on the extended AGK unitarity relation between contributions of different mechanisms to the inclusive particle production and the total photoabsorption cross-section (i.e. the proton structure function). We reanalyze the role of isovector reggeons in inclusive production of nucleons and Delta isobars in hadronic reactions. We find rather large contribution of reggeon-exchange induced production of Delta isobars. This leaves much less room for the pion-exchange induced mechanism of $\Delta$ production and provides a constraint on the $\pi N\Delta$ form factor. The production of leading pions in proton-proton collisions at ISR puts additional constraints on the $\pi NN$ vertex form factors. An extension of the AGK-rules to reggeon exchange suggests a negligible contribution to the proton structure function from DIS off the exchanged $\rho, a_2$ reggeons. All these constraints are used then to estimate the pion content of the nucleon and allow to calculate parameter-free the $x$-dependence of $\bar{d} - \bar{u}$. We discuss the violation of the Gottfried Sum Rule and $\bar{d} - \bar{u}$ asymmetry and compare to the one obtained from the E866 experiment at Fermilab. We estimate the background to the pion structure function being determined by the ZEUS and H1 collaborations at HERA from leading neutron experiments.

I. INTRODUCTION

The violation of the Gottfried Sum Rule discovered by the NMC collaboration at CERN in muon deep inelastic scattering [1] opened the long ongoing discussion on the $\bar{d} - \bar{u}$ asymmetry in the nucleon. Extending and following the early work of Sullivan [4], the asymmetry can be naturally explained within the framework of the isovector meson cloud model of the nucleon (for recent reviews see [2,3] and references therein), which uniquely predicted the asymmetry to be placed at large $x$, in good agreement with the early NA51 Drell-Yan experiment [8]. In the practical evaluation the dominant contribution can be interpreted as due to the admixture of the $\pi N$ and $\pi \Delta$ Fock-states in the physical proton. The model results for the asymmetry depend in essential way on the choice of the parameters for $\pi NN$ and $\pi N\Delta$ vertices, and on the elementary particle vs. Regge treatment of meson exchanges. In most of the calculations the vertex parameters, which at present cannot be calculated from first principles, were simply adjusted to reproduce the observed Gottfried Sum Rule violation. In contrast in Refs. [5,6] a unified approach to hadronic reactions and deep inelastic scattering has been pursued and the vertex parameters were constrained by the experimental data on the leading nucleon and/or leading Delta isobar production in high-energy hadronic reactions to which pion exchange contributes substantially. The numerical evaluations of the $\bar{u} - \bar{d}$- asymmetry were based on the fact that for an elementary particle exchange the contribution to the total virtual photo-absorption cross section equals that to the inclusive cross section for leading baryon production, which is one of the manifestations of the extended AGK unitarity relations [33]. For the pion exchange such a treatment is well justified, because the reggeization effects are negligible.

Besides the pion vertex parameters, the $\bar{u} - \bar{d}$- asymmetry is affected by the contribution of heavier isovector mesons. In Ref. [6] the effect of heavier vector mesons has been evaluated in an extended Fock state decomposition of the nucleon light-cone wave function into meson-baryon, including $N\rho, N\omega, \Delta\rho$ in addition to $\pi N, \pi\Delta$ and $\eta N$ which are known to be important at low and intermediate energy hadron scattering.

A fit to leading baryon experimental data suggests then a rather large $\rho N$ component in the expansion of the nucleon wave function. The relatively heavy $\rho$-meson, carries then a large fraction of the nucleon momentum and with a plausible ansatz for the $\rho NN$ vertex, it introduces the $\bar{d} - \bar{u}$ which extends to rather large $x \gtrsim 0.5$ [6]. The so-constructed model was perfectly consistent with the NA51 experiment at CERN [8,9], which measured the $\bar{u} - \bar{d}$-asymmetry at $x = 0.18$ [8]. Furthermore, after the incorporation of the NA51 result, the global parton model analyses have produced parametrizations which were practically indistinguishable from the results of the meson cloud dynamical calculations in [6].

A recent E866 experiment at Fermilab has reported a first high precision mapping of the $x$-dependence of the $\bar{u} - \bar{d}$ asymmetry from the comparison of the $pp$ and $pd$ Drell-Yan production with the striking finding that the difference of
the \( \bar{u} \) and \( \bar{d} \) distributions seems to vanish at large \( x \gtrsim 0.3 \), and is definitely smaller than the results of the model [6] and of all the early global parton parametrizations based on the NA51 result and the observed GSR violation. There is no consensus yet on understanding this interesting result [11,12]. In the framework of the meson cloud picture, one possible improvement is a Regge treatment of heavy particle exchanges, which seems to be more appropriate in the kinematical region relevant to the \( \bar{u} - \bar{d} \) asymmetry problem. Although, because of the lack of a microscopic QCD picture of reggeon exchange one is forced to use rather a phenomenological approach, one can take advantage of a large body of work on the Regge phenomenology of hadronic two-body exclusive [20,21], and inclusive [34], reactions, which constrains the Regge vertices, and imposes useful constraints between Reggeon contributions to inclusive production of different baryons.

Recently there was some interest in understanding the role of isoscalar reggeons in diffractive lepton deep-inelastic scattering [13,15] and in inclusive production of leading protons at HERA [16]. The role of isovector reggeons was discussed both in the context of leading neutron production in electron DIS [17] and in the context of diffractive DIS scattering with leading neutrons [18]. As a matter of fact, the isovector Reggeon exchanges in neutron and proton production must be much more important than in the evaluations of [17,18] based on the total cross section analyses, because the latter constrain only the spin–n–flip Regge vertices, and the much more dominant spin–flip contribution has not been considered in [17,18]. As will be discussed in the present paper this has important consequences for the inclusive spectra of neutrons from \( p \to n \) transitions both in hadronic and lepton DIS and indirectly also for \( d - \bar{u} \) asymmetry.

In the present paper we discuss the role of isovector reggeon exchange in the production of leading nucleons and \( \Delta \) isobars. The subsequent decay of the \( \Delta \) isobar into \( \pi N \) channel generates additional nucleons both in hadronic as well as in lepton DIS processes. Based on the extended AGK unitarity relation, we shall argue however that in spite of the important role in populating the \( nX \) and \( \Delta X \) inclusive channels, the opening of the \( nX \) and \( \Delta X \) intermediate states via Reggeon exchange leads to a negligible contribution to the hadronic total cross sections and/or the structure functions in DIS. As a consequence the isovector reggeon contribution to the Gottfried Sum Rule violation is rather small. On the other hand the reggeon exchange contributions to the leading baryon spectra leave less room for the pion exchange contributions, which are essential for a quantitative understanding of the Gottfried Sum Rule violation and the \( \bar{u} - \bar{d} \) asymmetry.

Our principal conclusion is that once the wealth of information on leading pions and leading neutrons and \( \Delta \) ‘s is accounted for there emerges a consistent description of the E866 (NuSea) data.

II. INCLUSIVE PRODUCTION OF PIONS, NUCLEONS AND DELTA ISOBARS IN HADRINOIC REACTIONS

A. Recalling the Regge phenomenology

Following Sullivan [4], we expect that the dominant mechanism of leading neutron production in DIS is an absorption of the virtual photon on the pion from the \( \pi^+ n \) Fock state of the nucleon, in which the spectator neutrons are observed as leading neutrons. But the same dynamics is supposed to be at work in the production of leading neutrons in hadronic semi-inclusive reactions, where the virtual photon has been swapped for the hadronic projectile. In the meson-baryon Fock-state picture of the proton’s light-cone wave function one is still left with nonperturbative parameters, the ‘radii’ of the Fock-states or in other words the form factor cut-off parameters. These parameters cannot be obtained from first principles, but one may hope to constrain them in a reasonably reliable way by demanding a consistency with experimental data for hadron production at high energies. This strategy has been taken in the work of the Jülich group [6] and the earlier work by Zoller [5] to extract the parameters of vertex form factors. Some subtle points connected to the distortion of the waves (or absorptive or screening corrections) have been discussed in a recent paper [19], we shall also comment on that issue below.

Let us first collect the pertinent formulae for the following discussions of the experimental data. Following the standard phenomenology of inclusive reactions, we define the Lorentz-invariant cross section (the so-called inclusive structure function) for the \( a + b \to c + X \) reaction (see fig.1)

\[
\Phi^{b \to c}(z, p^2_\perp) = E \frac{d\sigma}{dp^2_\perp} (ab \to cX) = z \frac{d\sigma}{d\bar{z} dp^2_\perp} = \frac{1}{\pi} \frac{d\sigma}{d\bar{z} \, dt} = \frac{s}{\pi} \frac{d\sigma}{dX \, dt}
\]

1The determination of the \( \bar{u} - \bar{d} \) difference in [10] depends on assumptions on the flavour symmetric sea \( \bar{u} + \bar{d} \). Within the error bars of E866 one can regard \( \bar{u} + \bar{d} \) as well constrained by the neutrino experiments, the difference between \( \bar{u} + \bar{d} \) from different gobal parton analyses is marginal.
\[
\frac{1}{(2\pi)^3} \frac{1}{2s} \sum_{i,X} \int d\tau_X \left| A^{a\to cX}_i(s, t, s_X, \tau_X) \right|^2 .
\]

Here \((E, \vec{p})\) is the four-momentum of the outgoing particle \(c, z = \frac{p_{c\perp}}{p_{\text{max}}}\) is the longitudinal momentum fraction (Feynman variable) of \(c\) which for large \(z\) is identical to the lightcone variable, and \(\vec{p}_\perp\) its transverse momentum, \(t = -p_{c\perp}^2/z + t_{\text{min}}\) with \(t_{\text{min}} = -(m_c^2 - m_\pi^2)(1 - z)/z - m_p^2(1 - z)^2/z\). By \(s_X\) we denote the invariant mass squared of the inclusive system \(X\), and \(\tau_X\) is its Lorentz-invariant phase space. Here the Mandelstam variables \(s, t\) refer to the reaction \(ab \to cX\); and the index \(i\) labels the exchange mechanism.\(^2\) In anticipation of large contributions from the region \(1 - z \ll 1\) to the considered cross section we shall use the Regge form throughout the present paper. The large Regge parameter in this case is \(s/s_X = 1/(1 - z)\) and one can write the amplitudes \(A^{a\to cX}_i\) in the form

\[
A^{a\to cX}_i(s, t, s_X, \tau_X) = g^{i\perp}_a(t) \left(\frac{s}{s_X}\right)^{\alpha_i(t)} \eta(\alpha_i(t)) \cdot v^{ib\to X}(\tau_X, s_X) ,
\]

where \(\alpha_i(t)\) is the Regge trajectory, \(\eta(t) = -[1 + \tau \exp(-i\pi \alpha_i(t))]/\sin \pi \alpha_i(t)\) is the signature factor for the trajectory with signature \(\tau\), and \(v^{ib\to X}(\tau_X, s_X)\) is the vertex for the transition \(ib \to X\) that leads to the (operational) definition of the \(ib\)–total cross section via

\[
\sigma^b_{\text{tot}} = \frac{1}{2s_X} \sum_X \int d\tau_X |v^{ib\to X}(\tau_X, s_X)|^2 .
\]

With these definitions we finally obtain for the inclusive cross section

\[
\Phi^{b\to c}(z, p_{\perp}^2) = \frac{1}{(2\pi)^3} \sum_i (g^{i\perp}_a(t))^2 |\eta(\alpha_i(t))|^2 \left(\frac{s}{s_X}\right)^{2\alpha_i(t)-1} \sigma^b_{\text{tot}}(s_X) .
\]

At this point it is convenient to introduce the notion of a flux associated with the exchanged object in the \(t\)--channel:

\[
f_{i/c}(z_i) = \int d^2\vec{p}_\perp \frac{Ed\sigma^{(i)}(ab \to cX)}{\sigma^b_{\text{tot}} d^3\vec{p}}
\]

or more explicitly:

\[
f_{i/c}(z_i) = \int d^2\vec{p}_\perp \frac{(g^{i\perp}_a(t))^2 |\eta(\alpha_i(t))|^2}{16\pi^2} \left(\frac{1}{z_i}\right)^{2\alpha_i(t)-1} .
\]

Here \(z_i = 1 - z = s_X/s\) is the longitudinal momentum fraction flowing through the \(t\)--channel. The function \(f_{i/b}(z_i)\) has an interpretation of a flux of \('i-\)quanta' emitted by the hadron \(a\) in the \(a \to c\) transition. Correspondingly its integral

\[
n_{i/c} = \int_0^1 dz_i f_{i/c}(z_i)
\]

shall be addressed as the number of \('i-\)quanta' in \(b\).

The contribution from the \(t\)--channel pion exchange Born-term to the production of leading neutrons reads

\[
\Phi_{\pi}^{b\to n}(z, p_{\perp}^2) = g_{\pi\pi}^2 \frac{(-t)}{16\pi^3} \left(\frac{t}{t_{\min}}\right)^2 F_{\pi NN}(t)(1 - z)^{1-2\alpha_\pi(t)} \sigma^\pi_{\text{tot}}(s_X) .
\]

Here \(\alpha_\pi(t) = \alpha'_\pi(t - m_\pi^2)\) is the pionic Regge-trajectory, \(\alpha'_\pi \approx 0.7\ \text{GeV}^{-2}\), \(s_X = s(1 - z)\) is the \(a\pi\) cm-energy squared, and \(g_{\pi\pi}^2/4\pi = 27.5\). Furthermore \(t = -p_{c\perp}^2/z + t_{\text{min}}\) with \(t_{\text{min}} = -(1 - z)^2 m_p^2/z\). \(F_{\pi NN}(t)\) is the phenomenological vertex form factor that accounts for the finite size of particles involved and/or off-shell effects. Because of the proximity of the physical pion pole, the reggeization effects, i.e. the departure of \(\alpha_\pi(t)\) from \(J_p = 0\) are marginal and one can use \(\sigma^\pi_{\text{tot}}(s_X)\) for the on-shell mass pion. The analogous formula for the \(\Delta(1232)\) production reads:

\(^2\) Note that in Eq.(1) we have omitted the contributions from interferences of different exchange mechanisms \(i, j\). They are absent in the reactions considered as will be discussed below.
\[ \Phi_{\pi^+\pi^-}(z, p_\perp^2) = \frac{g_{\Delta^+\pi^-}^2}{16\pi^4} \left( \frac{M_\Delta^2 - t}{6m_\Delta(t - m_\Delta^2)} \right)^2 F_{\pi N\Delta}^2(t)(1 - z)^{1 - 2\alpha_c(t)} \cdot \sigma_{\pi N}^{\pi N}(s) , \]

where we introduced \( M_\Delta = m_\Delta \pm m_N \), \( g_{\Delta^+\pi^-}/4\pi = 12.3 \text{ GeV}^{-2} \), and \( t = -p_\perp^2/z + t_{\text{min}} \) with \( t_{\text{min}} = -(m_\Delta^2 - m_N^2)(1 - z)/z - m_N^2(1 - z)^2/z \). Again \( F_{\pi N\Delta}(t) \) is the vertex form factor, a phenomenological quantity, that has a priori no simple relation to its counterpart in Eq. (8). Integrating Eqs. (8,9) over \( p_\perp^2 \) we get

\[ z \frac{d\sigma}{dz}(pp \to nX) = \pi \int dp_\perp^2 \Phi_{\pi^+\pi^-}(z, p_\perp^2) = \frac{2}{3} f_{\pi N}(1 - z) \cdot \sigma_{\pi N}^{\pi N}(s_x) , \]

\[ z \frac{d\sigma}{dz}(pp \to \Delta^{++}X) = \pi \int dp_\perp^2 \Phi_{\pi^+\pi^-}(z, p_\perp^2) = \frac{1}{2} f_{\pi \Delta}(1 - z) \cdot \sigma_{\pi \Delta}^{\pi \Delta}(s_x) . \]  

B. Inclusive spectra of pions

For the moment let us focus on yet another issue, which did not receive due attention in the literature. The Regge phenomenology outlined in section 2.1 allows to constrain the large-\( z \) behaviour of the vertex functions from the experimental data on leading baryons. For smaller \( z \) one must look into another constraint, here the light-cone wave function formalism offers an intimate relationship between the flux of mesons originating from a meson-baryon MB-Fock state and the corresponding flux of baryons. Clearly, if \( f_{M/B}(z_M) \) denotes the probability to find in the MB-Fock state a meson \( M \) carrying the (light-cone) momentum fraction \( z_M \) and if \( f_{B/M}(z_B) \) has the analogous meaning of finding the baryon \( B \), carrying momentum \( z_B \), it must hold true that

\[ f_{M/B}(z) = f_{B/M}(1 - z) . \]  

The probabilistic interpretation is obvious. The symmetry relation (11) allows us to write the contribution from spectator pions to the cross section for the production of leading pions as:

\[ z_\pi \frac{d\sigma}{dz_\pi}(pp \to \pi^0 X) = \frac{1}{3} f_{\pi N}(1 - z_\pi) \cdot \sigma_{\pi N}^{\pi N}(s_x) = \frac{1}{3} f_{\pi \Delta}(z_\pi) \cdot \sigma_{\pi \Delta}^{\pi \Delta}(s_x) , \]

where we focussed solely on the leading pions originating from the \( \pi N \)-Fock state, a contribution from the \( \pi \Delta \)-states does not affect the following conclusions.

In the absence of a theoretical tool to directly calculate the form factor \( F_{\pi N N}(t) \), the parametrization of \( F_{\pi N N}(t) \) can only be judged from its phenomenological success and consistency in a broad spectrum of processes. In the literature is rather bold extrapolations from low energy physics have been habitual and leading pion production data have been seemingly overlooked in previous considerations. As an example in figure 2 we show the differential cross section \( z_\pi d\sigma/dz_\pi \) for production of the \( \pi^0 \)-mesons taken from the NA27 \( pp \) experiment [31]. The spectra of \( \pi^+ \) and \( \pi^- \) from the same experiment show that the contribution from the \( \pi \Delta \) states is much smaller. Shown are several theoretical curves, calculated from Eq.(12) with different functional forms for the \( \pi NN \) form factor. The dotted curve shows the prediction from the model of Holtmann and two of the authors [6]. In this work a simple light-cone parametrization of the form factor had been adopted:

\[ F_{\pi NN}(z, p_\perp^2) = \exp \left[-R_{\text{LC}}^2(M_\perp^2, p_\perp^2) - m_N^2 \right] = \exp \left[ R_{\text{LC}}^2(t - m_N^2) \right] . \]

Here \( M_\perp^2(z, p_\perp^2) = (p_\perp^2 + m_N^2)/z + (p_\perp^2 + m_\pi^2)(1 - z) \) is the invariant mass of the \( \pi N \) Fock state, and \( R_{\text{LC}}^2 \approx 0.5 \text{ GeV}^{-2} \) [6]. In the spirit of the lightcone picture, the formfactor (II B) was meant to describe the low–mass components \( M_\perp^2 \sim (m_\pi + m_\pi)^2 \) and \( 1 - z \sim m_N/m_\pi \). If one stretches this simple parametrization also to large \( z_\pi \) one might run into serious conflict with the experimental data on leading pions. As we shall see below, such an overprediction of the leading pion spectra leads to a related overprediction of the \( \bar{u} - \bar{d} \) asymmetry at large \( x \).

The dipole (monopole) parametrization of the light–cone form factor [12] does not resolve the conflict with the NA27 data. We have neglected here the absorptive corrections, but one would expect them to reduce the Born term by at most a factor of 1.5–2, see for instance [19].

Clearly there is no real conflict between theory and experiment, one simply should not extrapolate to large \( z_\pi \) the functional form of form factors designed for \( z_\pi \ll 1 \). Reversing the attitude, one should look for parametrizations which are consistent with the leading pion data and to explore the resulting constraints from the leading pion data for predictions of the \( \bar{u} - \bar{d} \) asymmetry in DIS. We show the result for two different simple options only, the exponential
parametrization $F_{\pi NN}(t) = \exp(R_E^2(t - m_\pi^2))$ (dashed lines, for values of $R_E^2 = 1.0, 1.5, 2.0$ GeV$^{-2}$) and the 'Gaussian' parametrization $F_{\pi NN}(t) = \exp(-[R_E^2(t - m_\pi^2)]^2)$ (solid lines, for values of $R_E^2 = 1.0, 1.5, 2.0$ GeV$^{-2}$).

We conclude that the forward pion data suggest that pions in the $\pi N$ Fock state of the nucleon wave function are rather "soft", carrying longitudinal momenta of not more than $z \approx 0.5$. This is quite a subtle constraint on the tail of the $\pi N$ two-body wave function and imposes the empirical bounds $R^2_{E,G} > 1$ GeV$^{-2}$.

C. Inclusive spectra of neutrons from $p \to n$

Let us proceed now to the neutron spectra. In what follows we shall use the 'Gaussian' parametrization with a cut-off parameter $R_E^2 = 1.5$ GeV$^{-2}$ which does a reasonable job for the leading pion data (see Fig.(2)). In Figs. (3, 4) we show the relevant data for the forward neutron production. In the analysis of these data we also improved on what has been done before. Absorptive corrections to the $t$–channel pion exchange are incorporated in terms of standard methods of the Reggeon calculus in the quasi-eikonal formulation. The formalism is presented in [21] and there is no need to repeat that here.

For the $\pi N$ total cross section $\sigma^{\pi N}_{tot}(s_\chi)$ we can use the convenient database provided in [29]. It is especially convenient at small $s_\chi$ where the resonance structure of $\sigma^{\pi N}_{tot}(s_\chi)$ is important, as this is the case with the low–energy data in [27] ($\sqrt{s} = 4.93, 6.84$ GeV).

We decompose the inclusive structure function for neutron production in terms of the pion and background contributions:

$$\Phi^{p-n}(z, p^2_\perp) = \Phi^{p-n}_{\pi}(z, p^2_\perp) + \Phi^{p-n}_{a_2}(z, p^2_\perp) + \Phi^{p-n}_{\Delta}(z, p^2_\perp).$$

(14)

We take into account two background contributions: exchange of the $a_2$-trajectories (which are assumed exchange-degenerate following the Regge theory wisdom) and production of neutrons via the two-step processes $p \to \Delta \to n$. The decay of $\Delta$'s had been entirely neglected in all previous analyses except in Ref. [16] where it was shown to play an important role for leading proton production. The contribution of this mechanism to $p \to n$ reaction is suppressed in comparison to $p \to p$ reaction if the dominant mechanism of the $\Delta$ production is an exchange by an isovector object. In the spirit of the Regge phenomenology the contribution of $\rho, a_2$ to (14) is parametrized as

$$\Phi^{p-n}_{\rho,a_2}(z, p^2_\perp) = \sigma_R \left( \frac{p^2_\perp}{4m_p^2} + \chi^2 \right) \frac{1}{(1 - z)^{2\alpha'+t}} \exp \left( -B_R \frac{p^2_\perp}{z} \right).$$

(15)

Please note that the $\rho - a_2$ interference is negligible because of the different G-parity of $\rho$ and $a_2$. The non spin-flip/spin-flip ratio is known to be small: $\chi \approx 0.12$ [25]. In accord with the Regge phenomenology we take $\alpha' = 1$ GeV$^{-2}$.

There are no unique extrapolations from the $\rho, a_2$ particle pole to the Reggeon exchange and $\sigma_R$ remains a free parameter. Our analysis of the neutron data in the large $z$ region suggests $\sigma_R \approx 250$ mb GeV$^{-2}$ (for the slope a typical value of $B_R \approx 6.5$ GeV$^{-2}$ was taken), which is similar to earlier estimates [23].

In evaluation of the background coming from the decay of fast $\Delta$ resonances, one can either model the spectra of $\Delta$ isobars, as done in Ref. [16], or take any convenient parametrization, which reproduces the experimental data [31]. The decay of $\Delta$'s lead to a significant shift in $z$ distributions, compared to initial $\Delta$ distributions and only moderate modification in perpendicular momentum distributions. We have found that the spectrum of neutrons from $\Delta$-decays can be parametrized to a good approximation by the following simple form

$$\Phi^{p-n}_{\Delta}(z, p^2_\perp) = \sigma_0 z^3 (1 - z)^3 \exp(-B_\Delta p^2_\perp).$$

(16)

with $\sigma_0 = 48$ mb GeV$^{-2}$, $B_\Delta = 2$ GeV$^{-2}$.

We find that there is substantial correlation between the parameter of the form factor and the strength of the absorptive corrections. The choice of the form factor parameter made above ($R_E^2 = 1.5$ GeV$^{-2}$) results in rather weak absorptive corrections which has only a weak effect of a $10 - 20\%$ reduction of the pion-exchange Born-term, mildly depending on $z$ and $p^2_\perp$.

The description of the experimental data is excellent in general. The pion exchange is clearly seen to be the dominant mechanism in the region of $z = 0.7 - 0.9$ and $p^2_\perp < 0.2 - 0.3$ GeV$^2$. Because $\alpha_{\rho,a_2}(t) > \alpha_\pi(t)$, the Regge theory uniquely predicts the dominance of the Reggeon exchange at $z \to 1$ and the experimental data confirm that. We improve upon an oversimplified treatment of the background in [21], where its contribution was modelled scaling up the pion exchange contribution, which underestimates the $\rho, a_2$ background at large $z$. In the earlier work of two of the authors [6], the $\rho$ (particle-) exchange had been included within the light-cone formalism with the form factors parametrized in the form following eq. (11B) which also underestimates the $\rho$ contribution at large $z$.  

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Notice that especially in the region of large \( z \) the inclusion of \( \rho, a_2 \) Reggeons leads to much improved description as compared to the previous calculations \([6,19]\) where only the pion exchange did contribute at large \( z \). The fluctuations in the theoretical curves in Fig.3 are due to the use of experimental values of the cross sections. Indeed the region of large \( z \) corresponds to rather small \( s_X \) of the inclusive system \( X \), for example the peak at \( z \) close to unity corresponds to the \( \Delta(1232) \)-resonance in pion-nucleon scattering.

D. Inclusive spectra of \( \Delta^{++} \) from the \( p \to \Delta^{++} \) reaction

By far less ideal is the situation with to the production of forward \( \Delta \)'s. Experimental data is scarce, and if available often of rather poor quality. Many sets of data suffer from substantial ambiguities from the nonresonant \( \pi N \) background subtraction (a factor \( \approx 2 \) uncertainty is typical, see for instance \([32]\)) and cannot be used in the analysis. We do not have at hand measurements of double differential cross sections, that would allow us a meaningful determination of the \( p_\perp^2 \)-dependence of the individual mechanisms. In particular the data is not sufficient to make any quantitative statement on the role of absorptive corrections. Nonetheless the experimental data provide valuable bounds on the \( \pi N \Delta \) form factor which translate into substantial constraints on the \( \pi \Delta \) contribution to the \( \bar{d} - \bar{u} \) asymmetry.

In Fig.5 we show the \( p_\perp^2 \)-integrated cross section \( d\sigma/dz(pp \to \Delta^{++}X) \) from two CERN experiments obtained by the ABCDHW \([30]\) and LEBC-EHS \([31]\) collaborations. The upper bound to the pion exchange can be obtained by adjusting the parameter of the form factor in Eq.(9) to the CERN data \([30,31]\). Based on the Regge factorization, we evaluate the background from the \( \rho \) and \( a_2 \) reggeon exchanges as follows. The experimental data on the two-body reaction \( \pi^+p \to \pi^0\Delta^{++} \) exhibit a deep minimum of the differential cross section at \( t = 0 \), which is consistent with a pure \( M1 \)-transition \([22,24]\). The related data on the \( \pi^+p \to \eta^0 \Delta^{++} \) charge exchange reaction show a similar dominance of the \( M1 \) transition in the \( a_2 N \Delta \) vertex.

Therefore we take the following parametrization for the \( \rho, a_2 \) contribution:

\[
\Phi_{\rho,a_2}^{\pi \to \Delta^{++}+}(z,p_\perp^2) = \lambda_0 \sigma_R \frac{p_\perp^2}{4m_p^2} (1-z)^{-2\alpha_R} \exp \left(-B_R \frac{p_\perp^2}{z} \right).
\]  

A careful inspection of two-body charge-exchange reactions \( \pi^-p \to \pi^0n \) and \( \pi^+p \to \pi^0\Delta^{++} \) in a broad range of energy suggests (assuming Regge factorization) that the \( \rho \Delta^{++} \)-coupling strength should be about a factor 1.5 larger than that for the \( ppp \) case. Correspondingly, \( \lambda_0 = 1.5 \). Consequently, the Regge phenomenology offers a parameter–free evaluation of the \( \rho, a_2 \) exchange in production of leading \( \Delta \)'s in terms of the \( \rho, a_2 \) exchange background to the production of leading neutrons. To the best of our knowledge, this relationship has not been used before. Consistently with the two-body reactions we take here the same slope parameter \( B_R \) as for the neutron production. The contribution of the reggeon exchange is shown in Fig.5 as the dotted line. As can be seen from the figure it exhausts a significant fraction of the spectra leaving less room for the pion exchange contribution.

Let us return to the estimate of the pion exchange contribution which we take in the form according to Eq.(9) with an exponential form factor \( F_{\pi N \Delta}(t) = \exp(-R_{\Delta}^2 |t - m_\Delta^2|) \). Unfortunately the quality of the data \([30,31]\) does not allow for a fit of \( R_{\Delta} \). The difficulties with the inclusive \( \Delta \) spectrum have already been observed in the early work on the topic \([26]\), and without the constraints from the two-body reactions, one might be tempted to neglect the \( \rho \)-contribution at all, we quote from Gotsman \([26]\):“In an attempt to improve the fits we added the exchange of a \( \rho \) trajectory, but to no avail [...].” Only a lower limit on \( R_{\Delta} \) can be obtained by comparing the sum of the reggeon exchange and pion exchange contributions

\[
\frac{d\sigma}{dz}(pp \to \Delta^{++}X) = \frac{\pi}{z} \int dp_\perp^2 \left( \Phi_{\rho}^{\pi \to \Delta^{++}+}(z,p_\perp^2) + \Phi_{\rho,a_2}^{\pi \to \Delta^{++}+}(z,p_\perp^2) \right)
\]

to the experimental data \([30,31]\). We consider \( R_{\Delta}^2 = 2 \text{ GeV}^{-2} \) to be the lower limit on the parameter. The corresponding upper limit of the pion exchange contribution is shown by the dashed line in Fig.5. The pion exchange contribution found here is even smaller than in Ref. \([6]\) and excludes the scenario with large \( \pi \Delta \) component discussed in \([12]\) as a possible explanation of the restoration of \( \bar{u} - \bar{d} \) symmetry at intermediate Bjorken-\( x \) observed in \([10]\). This will have important consequences for the Gottfried Sum Rule violation and \( \bar{d} - \bar{u} \) asymmetry.

A digression into unitarity relations and/or the extended AGK rules is in order before we proceed with implications of the above analyses of inclusive reactions for the partonic structure of protons as seen in DIS.
III. FROM INCLUSIVE HADRONIC CROSS SECTION TO THE TOTAL CROSS SECTION: THE EXTENDED AGK RULES

As has been demonstrated in the previous section the reggeon exchanges may contribute significantly to individual inclusive channels and definitely influence the production of leading baryons. The impact of specific inclusive channels on the total cross section (total photoabsorption cross section for DIS) is controlled by unitarity. The well known example is the so-called AGK rules for diffractive scattering (Pomeron exchange) [33], by which the opening of diffractive channels gives rise to the shadowing correction and reduces the proton structure function [35–37]. The gross features of this relationship can be understood as follows. In order to isolate how opening of the individual inclusive channels ab → cX via specific exchange mechanisms affects the total ab cross section, let us consider the discontinuity of the ab-forward scattering amplitude associated with the c + X intermediate state (see fig.1). The optical theorem relates its contribution to the total cross section

\[ \Delta_i^{(2)} \sigma_{tot} = \frac{1}{s} \text{Im} T_i^{(2)}(s, t = 0). \]  

(19)

For the \( i \)-type–Reggeon exchange mechanism of \( c + X \) production it can be calculated as \(^3\)

\[ T_i^{(2)}(s, t = 0) = \frac{1}{2i} \int \frac{d^4 k}{(2\pi)^4} \frac{[g_{ac}(k^2)]^2 (p-k)^2 - m^2 + i\epsilon}{g_{ab}(k^2) + m^2} \eta_{i}(k^2)^2 \left( \frac{s}{s_X} \right)^{2\alpha_i(k^2)} T(i b \to i b). \]  

(20)

Where the Reggeon-particle scattering amplitude \( T(i b \to i b) \) is defined such that it fulfills the generalized unitarity condition

\[ \text{Im} T(i b \to i b) = \frac{1}{2} \int d\tau_X |v_{b\to X}(\tau_X, s_X)|^2 = s_X \sigma_{tot}^b. \]  

(21)

Utilizing this relation after integrating over the propagator pole of particle \( c \) and a suitable change of integration variables, we obtain

\[ T_i^{(2)}(s, t = 0) = i \int \frac{d^2 k_{\perp}}{2(2\pi)^3} \frac{[g_{ac}(k^2)]^2}{g_{ab}(k^2)} \eta_{i}(k^2)^2 \left( \frac{s}{s_X} \right)^{2\alpha_i(k^2) - 1} \sigma_{tot}^b(s_X). \]  

(22)

Now we observe that \( \eta_{i}(k^2)^2 = \tau \exp(-i\pi \alpha_i(k^2))|\eta_i(k^2)|^2 \) and approximate the phase factor by its value at \( k^2 = 0 \). We can now easily establish (cf. eq. (4)) the connection between the double scattering contribution to the total cross section \( \Delta_i^{(2)} \sigma \equiv \text{Im} T_i^{(2)}/s \) and the inclusive cross section:

\[ \Delta_i^{(2)} \sigma_{tot}^b = \text{Im} \left( i e^{-i\pi \alpha_i(0)} \right) \int \frac{d^2 k_{\perp}}{\pi} \frac{d\sigma_{(ab \to cX)}}{ds_X dt} \equiv \xi_i \sigma_{incl}^i. \]  

(23)

where we defined

\[ \sigma_{incl}^i = \int \frac{d^2 k_{\perp}}{\pi} \frac{d\sigma_{(ab \to cX)}}{ds_X dt} \approx \int dt \int \frac{d\sigma_{(ab \to cX)}}{ds_X dt} \]  

(24)

Let us illustrate the implications of the extended AGK-rule (23) by considering some examples:

- for the pomeron, \( \alpha_{P}(0) \approx 1, \tau = +1 \) and thus \( \xi_P = -1 \). This is a well established fact, which has become known as one of the AGK cutting rules, meaning that opening of diffractive channels leads to a reduction of, or the absorptive correction to, the total cross section.
- for the pion, \( \alpha_{\pi}(0) \approx 0, \tau = +1 \), hence \( \xi_{\pi} = +1 \). This implies that inelastic interaction with pions in the target hadron enhances the total cross section by an amount \( n_{\pi}/a \sigma_{tot}^b \).
- for the Reggeon with \( \alpha_{R}(0) \approx 0.5 \), \( \rho, a_2, f_0, \omega \), \( \xi_R = 0 \); which means that the contribution from inelastic interaction with Reggeons in the hadron \( a \) to the total cross section vanishes.

The latter two results concerning the contributions from the \( \pi^- \)-trajectory and the secondary Reggeons are also known in a different context for some time: in calculations of nuclear shadowing in hadron-deuteron collisions it was observed that the inelastic intermediate states excited by \( \pi^- \)-exchange contribute to an anti-shadowing, while the contributions from excitation of intermediate states by other secondary Regge trajectories are negligible [38,39].

\(^3\)see for instance ref. [34].
IV. PION CONTENT OF THE NUCLEON AND $\bar{D} - \bar{U}$ ASYMMETRY

Having set the parameters of the pion flux factors in section 2 we can proceed to the calculation of the corresponding quark distributions. Those can be calculated as a simple convolution of the flux factor and quark distributions in the pion

$$\Delta(\pi)q_f(x, Q^2) = \int_0^x \frac{d\pi}{z\pi} f_\pi(z\pi) \cdot q_f^2(x, Q^2),$$

(25)

where $f_\pi = f_{\pi/N}, f_{\pi/\Delta}$ as introduced in section 2. In our calculation we take the GRV-pion structure function [42] at the average value of the E866 experiment $Q^2 \approx 50$ GeV$^2$. We focus on the difference $\bar{d}(x, Q^2) - \bar{u}(x, Q^2)$. Due to flavour symmetries only valence quark distributions in the pion contribute to this quantity. The less known sea contribution cancels in $\bar{d}(x, Q^2) - \bar{u}(x, Q^2)$ [40]. While the quark distributions in the pion can be verified in the valence region (intermediate and large $x$), the Drell-Yan processes do not allow to determine them in the sea region (low $x$).

As discussed in the previous section the contribution of DIS off exchanged $\rho, a_2$-Reggeons to the proton structure function is negligible and will be omitted in the following.

In Fig.7 we display $\bar{d}(x) - \bar{u}(x)$ which is due to pionic contributions as obtained in the present analysis and compare it to the recent result of the NuSea collaboration [10]. The $\pi\Delta$ contribution (dashed line) becomes important only at rather small values of Bjorken-$x$ which is due to rather soft form factor as suggested by the analysis of leading $\Delta$ isobars. The analysis above clearly demonstrates that it is possible to construct the pion flux factor consistent with both hadronic and Drell-Yan data, provided the background processes in hadronic reactions are taken carefully into account.

Let us finally comment on the violation of the Gottfried sum rule. We obtain a number of pions in the proton in the $\pi N$ Fock state of $n_{\pi N} = 0.21$ for the cutoff parameter $R_G^2 = 1.5$ GeV$^{-2}$ and $n_{\pi N} = 0.28$ for $R_G^2 = 1$ GeV$^{-2}$. For the $\pi\Delta$ Fock state, a cutoff $R_{\Delta}^2 = 2$ GeV$^{-2}$ yields $n_{\pi\Delta} = 0.03$. The latter value is considerably smaller (by a factor 2 - 8) than those obtained in all previous analysis. These pionic multiplicities translate into the integrated value of $\bar{d} - \bar{u}$ asymmetry by means of

$$I_A = \int_0^1 [\bar{d}(x) - \bar{u}(x)] \, dx = \frac{2}{3} n_{\pi N} - \frac{1}{3} n_{\pi\Delta},$$

(26)

yielding values of $I_A = 0.129$ for $R_G^2 = 1.5$ GeV$^{-2}$ and $I_A = 0.177$ for $R_G^2 = 1$ GeV$^{-2}$.

For the Gottfried integral

$$S_G = \frac{1}{3} - \frac{2}{3} \int_0^1 [\bar{d}(x) - \bar{u}(x)] \, dx$$

(27)

we obtain correspondingly $S_G = 0.247$ for $R_G^2 = 1.5$ GeV$^{-2}$ and $S_G = 0.215$ for $R_G^2 = 1$ GeV$^{-2}$. The value reported by NMC is $S_G = 0.235 \pm 0.026$ [1] which is perfectly consistent with our result $0.21 \lesssim S_G \lesssim 0.25$.

V. NON-PIONIC BACKGROUND IN THE PRODUCTION OF LEADING NEUTRONS AT HERA

There is still another interesting spin off of the present analysis. As discussed in section 2 the reggeon exchange contribution and decays of the $\Delta$ resonances constitute a sizeable background to the pionic contribution especially at large transverse momenta of leading nucleons. The same peripheral processes are at work in the production of leading neutrons in electron deep inelastic scattering at HERA, where leading neutron tagged DIS is presently studied experimentally as the method of determination of the pion structure function [41,43,44]. The quality of the neutron tagging can be judged from the pion purity factor shown in fig.8, here we present the ratio $R(z, t)$ defined as

$$R(z, t) = \frac{I^\pi(z, t)}{[I^\pi(z, t) + I^{bg}(z, t)]},$$

(28)

where

$$I^{\pi, bg}(z, t) = \int_{t_{\text{min}}}^{t_{\text{max}}} d\Phi^\pi_{\pi, bg}(z, p_t^2),$$

(29)

and the index $bg$ denotes the background (reggeon exchange + $\Delta$ decays in our case). The calculations for fig. 8 were performed for the hadronic reaction $pp \rightarrow nX$, they can, however, also be used as an estimate for the deep inelastic reaction $\gamma p \rightarrow nX$. One should be aware of the fact, that this amounts to an implicit assumption of
\[
\frac{F_2^R(x)}{F_2^L(x)} = \frac{\sigma_{RH}}{\sigma_{\pi L}}. 
\]

Certainly this relation is approximate and can be used as a guiding principle only. A better approximation is however beyond the present, rather phenomenological, understanding of reggeon exchanges.

Our phenomenological analysis suggests that in order to extract the pion structure function one should limit rather to low \( t \) and/or low transverse momenta of neutrons. This is the region where the leading proton spectrometers of both H1 and ZEUS collaborations have the largest sensitivity. In order to identify the "unwanted" background one could make the analysis of the experimental data for different cuts in transverse momentum.

VI. CONCLUSION

The NuSea E866 experiment provided the first detailed mapping of the \( x \)-dependence of the \( \bar{d} - \bar{u} \) asymmetry in the proton sea. While confirming the global features of the asymmetry predicted by the meson cloud picture, the large-\( x \) behaviour of these data called for revisiting the model adding more constraints from applications of the same model to hadronic inclusive reactions. Specifically, in this paper we demonstrated that the experimental data on fragmentation of protons into leading pions impose useful constraints onto the pion induced \( \bar{d} - \bar{u} \) asymmetry such that the asymmetry must be negligibly small at \( x \gtrsim 0.3 \) in perfect consistency with the NuSea findings.

Our reanalysis has been based on a unified treatment of inclusive production of leading nucleons and \( \Delta \)'s in hadronic collisions and we paid special attention to the background to pion exchange from isovector \( \rho, a_2 \) exchanges. Then, based on the extended AGK unitarity rules, we related the contributions of different exchange mechanisms to inclusive \( ab \to cX \) cross section to the contribution of inelastic interaction (DIS) on the exchanged objects to the total \( ab \) cross section (DIS structure function of the target hadron). We observe that the extended AGK rules suggest a negligible contribution of DIS off the exchanged \( \rho, a_2 \) to the \( \bar{d} - \bar{u} \) asymmetry, leaving pions as the dominant source of the asymmetry. In numerical evaluation of the pion contribution to inclusive cross section, and eventually to the asymmetry, we made an extensive use of the Regge factorization which allows to relate the \( \rho, a_2 \) exchange contribution to the leading neutron and \( \Delta \) production based on the Regge phenomenology of two-body charge exchange reactions. One of the results is that the isovector \( \rho, a_2 \) reggeon exchanges exhaust a large fraction of inclusive leading \( \Delta \) production. The implication for the asymmetry is a substantial reduction of the contribution from the \( \pi \Delta \) Fock states which. As a result a reduction of the \( \bar{d} - \bar{u} \) asymmetry by the contribution from the \( \pi \Delta \) Fock states in the proton turns out much weaker than evaluated before. We re-evaluated the GSR with the result \( \int_1^0 dx [\bar{d} - \bar{u}] \simeq 0.13 - 0.18 \) and \( S_G \simeq 0.21 - 0.25 \).

As a spin off of our evaluation of isovector reggeon exchange to leading neutron production, we estimated which fraction of the inclusive leading neutrons in DIS can be considered as due to DIS off pions in the proton. In substantial part of the phase space covered by the Forward/Leading Neutron Calorimeters installed at HERA by H1 and ZEUS [43,44], the purity of neutron tagging of DIS off pions can be estimated as 70-80 per cent. We feel that the corresponding uncertainties with the determination of the pion structure function do not preclude useful test of its \( x, Q^2 \) evolution. To this effect we recall that judging from experimental data on the proton structure function, the pion structure function is expected to vary by the factor \( \sim 4 \) from \( x = 10^{-1} \) to \( x = 10^{-4} \) and by the factor \( \sim 2 \) from \( Q^2 = 5 \) to \( Q^2 = 100 \text{ GeV}^2 \) at \( x = 3 \cdot 10^{-3} \).

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FIG. 1. The inclusive production of particles \(c\) in the reaction \(ab \rightarrow cX\).
FIG. 2. Differential cross section $zd\sigma/dz(p p \rightarrow \pi^0 X)$ at $p_{LAB} = 400$ GeV/c. The data are taken from [31]. The dotted curve shows a prediction of the model [6]. The solid curves were calculated with a 'Gaussian' form factor $F_{\pi NN}(t) = \exp(- [R_G^2(t - m_{\pi}^2)]^2)$. The curves in the figure correspond to $R_G^2 = 1.0, 1.5, and 2.0$ GeV$^{-2}$ (from top to bottom). The dashed curves were calculated with an exponential form factor $F_{\pi NN}(t) = \exp(R_E^2(t - m_{\pi}^2))$. The curves in the figure are for $R_E^2 = 1.0, 1.5, and 2.0$ GeV$^{-2}$ (from top to bottom).
FIG. 3. Invariant cross section for the reaction $pp \rightarrow nX$ at $p_{\text{LAB}} = 24 \text{ GeV/c}$. The experimental data are taken from [27]. The long dashed curve shows the contribution from the pion exchange; the dotted curve is the $\rho, a_2$-exchange contribution, and the dashed curve shows the contribution from the two step process $p \rightarrow \Delta \rightarrow n$. In addition, we present the sum of the two background contributions as the dot-dashed line. Finally, the solid curve represents the sum of all components.
FIG. 4. Invariant cross section for the reaction $pp \rightarrow nX$ as a function of $z$ for $p_T^2 = 0$. The long dashed curve shows the contribution from the pion exchange; the dotted curve is the $\rho$, $a_2$-exchange contribution, and the dashed curve shows the contribution from the two-step process $p \rightarrow \Delta \rightarrow n$. In addition we present the sum of the two background contributions as the dot-dashed line. The solid curve represents the sum of all components. The experimental data are taken from [28].
FIG. 5. Differential cross section $d\sigma/dz$ for the reaction $pp \rightarrow \Delta^{++}X$ at $p_{LAB} = 400$ GeV/c. The dashed curve is the contribution from pion-exchange; the dotted curve shows the $\rho, a_2$ contribution. Shown by the solid curve is the sum of the two. Experimental data are taken from [31] (open circles), and [30] (filled circles).
FIG. 6. The amplitude for the double reggeon exchange.
FIG. 7. Flavour asymmetry $\bar{d}(x) - \bar{u}(x)$ at $Q^2 = 5.4 \text{GeV}^2$. Experimental data are from E866 [10]. The solid curves show the contribution from the $\pi N$-Fock state and were calculated for Gaussian form-factors; the upper curve belongs to $R_G^2 = 1 \text{GeV}^{-2}$, the lower one to $R_G^2 = 1.5 \text{GeV}^{-2}$ The dashed line shows the contribution of the $\pi\Delta$-Fock state, calculated for an exponential form factor with $R_\Delta^2 = 2 \text{GeV}^{-2}$. 
FIG. 8. The signal to background ratio $R(z,t)$ as defined in Eq.(28) shown as a function of $t$ for several values of $z$. Note that the pion contribution peaks between $z = 0.8 - 0.9$. 