INTRODUCTION

In this paper we examine the fundamental question of whether the dark matter halo of a galaxy can be formed by the dynamical friction of stars, and how the halo properties are determined by the relative mass and velocity distribution of the stars. We consider the case of a galaxy with a flat rotation curve, and a typical halo that is formed by the gravitational interaction of stars and dark matter.

We show that the dark matter halo in a galaxy is formed by the dynamical friction of stars, and that the halo properties are determined by the relative mass and velocity distribution of the stars. We also show that the dynamical friction of stars can be used to determine the mass and velocity distribution of the stars in a galaxy, and that the dark matter halo can be used to trace the mass and velocity distribution of the stars in a galaxy.
ter a bar formed in the disc. Hernquist & Weinberg (1992) simulated systems containing a non-rotating halo and a rigid bar (with no disc) and concluded that the gravitational coupling was 'sufficiently strong to remove all of the bar’s angular momentum on a time-scale much shorter than a Hubble time'. Little & Carlgren (1993a, b) followed the long-term evolution of barred disc galaxies with live halos using a two-dimensional code, which greatly improves spatial resolution but requires a somewhat unrealistic flat halo. They found that \( R \) increased from 1 to 2 over \( \approx 30 \) initial bar rotation periods. The most recent and comprehensive simulations of this process are by Debattista & Sellwood (1998), who followed self-consistent \( N \)-body models of a barred disc and initially spherical halo. In the simulation with the most massive halo, they found that the bar pattern speed dropped by a factor of five (to \( \mathcal{R} \approx 2.3 \)) within 50 initial bar rotation periods. Only in models with low-density halos did the bar pattern speed remain high.

Thus a wide range of analytic arguments and numerical simulations concur that high-density halos strongly decelerate bars. On the other hand, observations show that most bars are rapidly rotating. Bar pattern speeds can be estimated in several ways: (i) By comparing the shocks in models of gas flow with dust lanes in barred spiral galaxies. Using this approach Athanassoula (1992) estimated \( \mathcal{R} = 1.2 \pm 0.2 \). (ii) From the rotation curve of the old disc population and the equation of continuity. This method has been applied to the SB0 galaxy NGC 936 and yields \( \mathcal{R} \approx 1.4 \pm 0.3 \) (Kent & Gnedin 1989; Merrifield & Kn{	extsuperscript{i}}jken 1995). (iii) By identifying photometric and kinematic properties of stars and gas with Lindblad or corotation resonance (Debattista & Sellwood 1998). A recent review by Elmegreen (1996) concludes that \( \mathcal{R} = 1.2 \pm 0.2 \) in early-type galaxies; bars in late-type galaxies may rotate more slowly but so far there are very few observations.

Thus, there is a significant and growing contradiction between theory (\( \mathcal{R} \gg 1 \)) and observations (\( \mathcal{R} \sim 1 \)). Debattista & Sellwood (1998) argue that the absence of slowly rotating bars can only be explained if the halo contributes less than \( \sim 25\% \) of the radial force in the region where bars are found (around 2 disc scale lengths); this in turn requires that the disc provides almost 90\% of the total circular speed (\( \gamma \approx 90\% \)). However, some of the observational difficulties may be reinterpreted as indicating a more complex reality (Bottema 1997; Broeils & Courteau 1997; Courteau & Rix 1997; Sackett 1997; Fuchs 1998; Sellwood 1998), but it seems unlikely that the very high value of \( \gamma \) required by Debattista and Sellwood is present in most disc galaxies. For example, a careful recent discussion of mass models for the Galaxy is given by Olling & Merrifield (1998); their models they describe with various values of the solar circular speed and radius have \( \gamma \) in the range 0.66-0.83 and their favored model has \( \gamma = 0.72 \). Even ‘maximal disc’ models of external galaxies—which have the largest possible disc mass consistent with the observed rotation curve and an isothermal dark halo—have a median \( \gamma \) of only 0.85 (Sackett 1997), which is slightly lower than Debattista and Sellwood require.

A rapidly rotating inner halo could resolve this apparent contradiction, by reducing or even reversing the drag on a rotating bar.\(^*\)

## 2 Disc-Halo Gravitational Torques

Disc-halo angular momentum transfer can result from structure in either the disc or halo. We first examine the effects of disc structure (§2.1) and find that no known disc features transfer angular momentum rapidly enough to spin up the inner halo. Halo structures (§2.2) are more efficient.

### 2.1 Structure in the Disc

The discs of spiral galaxies exhibit a wide variety of structure, including molecular clouds, spiral arms and central bars. All of these can transfer angular momentum to the halo.

To estimate the transfer rate we adopt a simple model in which the circular speed of the disc, \( v_c \), is independent of radius, and a fraction \( f_\text{d} \) of the gravitational force on a disc particle is due to a spherical halo; the remaining fraction comes from the self-gravity of the disc. In this model the halo density and disc surface density are (Binney & Tremaine 1987)

\[
\rho_\text{d}(r) = f_\text{d} \frac{v_c^2}{4\pi G r^2}, \quad \Sigma_\text{d}(r) = f_\text{d} \frac{v_c^2}{2\pi G r}.
\]

If the torque per unit disc area exerted on the halo is \( N \), then the disc material in the radius range \( [r, r + dr] \) loses angular momentum at the rate \( 2\pi r N \, dr \); if we assume that this angular momentum is gained by the halo material in the same radius range, with mass \( 4\pi r^2 \rho_\text{d}(r) \, dr \) and characteristic specific angular momentum \( \sim r v_c \), then the halo is expected to flatten on a characteristic time

\[
\tau \approx \frac{2\pi}{r v_c} \frac{N}{f_\text{d}} = \frac{f_\text{d} v_c^3}{2\pi GM N}.
\]

First consider the effects of discrete masses in the disc. We suppose that a fraction \( f_\text{M} \) of the disc mass is in objects of mass \( M \) and radius \( R \). Each such object loses specific angular momentum to the halo by dynamical friction, at a rate (Binney & Tremaine 1987, eq. 7-24)

\[
\dot{L} = -0.43 f_\text{M} GM \frac{v_c}{r} \ln \Lambda,
\]

where \( \Lambda \approx r / R \). The torque per unit area on the halo is then

\[
N = f_\text{M} \Sigma_\text{d} \dot{L},
\]

and the halo flattening time is

\[
\tau = \frac{2.3}{f_\text{M} \Sigma_\text{d} GM \ln \Lambda}.
\]

\(^*\) Debattista & Sellwood (1998) find that halo rotation is not sufficient to suppress bar formation, but their rotating halos are constructed by reversing the angular momenta of stars in a spherical halo, and thus do not rotate as rapidly as a flattened halo.
\[
= 1 \times 10^2 \text{yr} \left( \frac{0.1 \text{ km s}^{-1}}{f_{sfr}} \right) \left( \frac{v}{300 \text{ km s}^{-1}} \right)^2 \left( \frac{r}{3 \text{ kpc}} \right)^2 \times \frac{10^6 M_{\odot}}{M} \left( \frac{10}{\ln \Lambda} \right) \tag{4}
\]

The largest disc objects are giant molecular clouds (GMCs). These have a wide range of masses but their contribution to dynamical friction depends on the weighted average \(\langle M^2 \rangle / \langle M \rangle\), which is dominated by the largest clouds at mass roughly \(10^6 M_{\odot}\) (e.g., Combes 1991). The largest GMCs do not transfer significant angular momentum to the bulk of the inner halo, though individual stars at radial \(r \approx 5 \text{kpc}\) may spiral through the inner halo, losing mass than with the outer gas streams (Combes 1991).

Next we examine angular momentum transfer by spiral structure in the disc. A tightly wrapped spiral pattern with azimuthal wavenumber \(m\), pattern speed \(\Omega_\phi\), radial wavenumber \(k\) and fractional surface density amplitude \(\Sigma_1 / \Sigma_0\) exerts a torque per unit area on the halo (Mark 1976)

\[
N = \frac{2 \pi^2 m^2 M^2 \Omega_\phi \rho_0 \sigma^2}{k^2 \sigma^2} \left( \frac{\Sigma_1}{\Sigma_0} \right)^2, \tag{5}
\]

where \(\sigma\) is the one-dimensional velocity dispersion in the halo and the halo distribution function is assumed to be Maxwellian. In our model \(\sigma = v_c / \sqrt{2}\), and using equations (1) and (2) the halo flattening time is

\[
\tau = \frac{3 \pi^2 m^2}{4 \Omega_\phi f_s} \left( \cot \phi \right) \left( \frac{\Sigma_1}{\Sigma_0} \right)^2 . \tag{6}
\]

where \(i = \tan^{-1}(m/k)\) is the pitch angle of the pattern. An optimistic set of parameters is \(m = 2\), \(i = 20^\circ\), \(f_s \approx 0.7\), \(\Omega_\phi \approx 5 \times 10^7\) yr, and \(\Sigma_1 / \Sigma_0 \approx 0.2\), which yields \(\tau \approx 10^7\) yr, too long to be of interest. Usually strong and open spiral patterns could transfer angular momentum more rapidly, but these are normally transitory, induced for example by an encounter with a nearby galaxy.

Angular momentum transfer by dynamical friction on a central bar has been discussed in the introduction. Although the frictional forces are strong enough to despin the bar, it is much harder to spin up the halo, because the reservoir of angular momentum in the bar is too small. Bars typically contain \(\lesssim 30\%\) of the disc luminosity and extend only to \(\sim 30\%\) of the disc radius, so their moment of inertia is only a few percent of the disc’s; thus transferring all of the bar’s angular momentum to the halo as noted, a rapid process—would still impart negligible halo rotation.

We conclude that the known disc structures cannot spin up the inner halo.

### 2.2 Structure in the halo

The dark halo is believed to form by gravitational instability of small density fluctuations in the early Universe. In standard hierarchical models, dense, low-mass halos form first, and then merge to build up successively more massive but less dense objects. The process terminates at a redshift \(z\) given roughly by \(1 + z \approx \Omega_m^{-1}\), where \(\Omega_m\) is the density parameter. When a small halo merges with a larger one, it loses energy by dynamical friction and spirals towards the centre until it is tidally stripped or completely disrupted, at a radius \(r \sim r_x(M/M_x)^{1/3}\), where \(r_x\) and \(M_x\) are the radius and mass of the small halo and \(M\) is the mass of the larger halo interior to \(r\). The disrupted halo is shocked into a tidal stream; the length of the stream after a time \(t\) is roughly \(\Delta \phi \sim (t/P)(r_x/r) \sim (t/P)(M_x/M)^{1/3}\), where \(P\) is the orbital period (Tremaine 1993; Johnston 1998). Such streams can survive as distinct—though unbound—structures in phase space for a Hubble time or longer; since the halo material is collisionless the streams never overlap in phase space but simply become longer as phase mixing proceeds, forming a kind of phase-space spaghetti. Structure resulting from complete phase mixing is already well-known in several other contexts, notably groups in local disc stars (Eggen 1965; Dehnen 1995; de Zeeuw et al. 1990; Quinn & Goodman 1987; Quinn & Gwinn 1988; Malm 1998), clumps in the arm patterns of the metal-weak halo (Majewski, Munn & Hawley 1996), and the disrupted Sagittarius dwarf galaxy (Ibata et al. 1997).

A rotating disc exerts dynamical friction on a tidal streamer. To estimate the frictional force, consider the idealized case in which a single streamer of length \(L \ll r\) pierces the disc at right angles. At distances \(\ll L\), we can approximate the streamer as an infinite, straight wire, and we can approximate the wire as stationary since the velocity vector of its material is approximately aligned with the streamer. We can approximate the disk as a uniform, infinite, zero-thickness sheet moving past the streamer at velocity \(v_c\). If the linear density in the streamer is \(\lambda \approx M_c / L\), the frictional force exerted on it from disc material with impact parameter \(b \ll L\) is

\[
F_{\text{disc}} = \frac{4\pi^2 G \lambda^2 \Sigma_0 b_1}{v_c^2}, \tag{7}
\]

where \(b_1\) is the maximum impact parameter considered. This drag force is exerted only while the streamer intersects the disc. If the streamer follows a circular orbit of radius \(r\), then it intersects the disc for a fraction \(L/(2\pi r)\) of its orbit, so the average drag force is

\[
\langle F_{\text{disc}} \rangle = \frac{2\pi G \lambda^2 \Sigma_0 b_1}{v_c^2} = \frac{2\pi G M_c^2 \Sigma_0 b_1}{v_c^2 L^2}. \tag{8}
\]

The maximum impact parameter is \(b_1 \approx L\). Disc material with impact parameter \(b \gg L\) sees the streamer as a point mass, and the time-average of the frictional force component from this material is

\[
\langle F_{\text{disc}} \rangle = \frac{2\pi G M_c^2 \Sigma_0 \Delta \phi}{v_c^2 r} = \frac{2\pi G M_c^2 \Sigma_0 L^2 \Delta \phi}{v_c^2 L^2}. \tag{9}
\]

where \(\Delta \phi = b_2 / b_1\), and \(b_2\) and \(b_1\) are the maximum and minimum impact parameters considered. Equations (8) and (9) are similar except for the Coulomb logarithm in the latter, but the ratio \(b_2 / b_1\) is not usually large, so material with \(b \ll L\) and \(b \gg L\) contributes comparable drag forces.

Estimating the frictional force is more delicate for long streamers (\(\Delta \phi \gg 2\pi\)). A long streamer intersects the disc...
hese inter-


dures of mass $M/L$ (of black holes). Then the orange on the halo

can be described by $\frac{M}{L} = \frac{\Delta L}{\Delta M}$, where $\Delta L$ is the change in mass and $\Delta M$ is the change in luminosity. This equation helps in understanding the relationship between mass and luminosity of black holes.

The equation also suggests that the ratio of mass to luminosity is a constant, which implies that the growth of the halo is proportional to the growth of the mass.

In conclusion, the equation $\frac{M}{L} = \frac{\Delta L}{\Delta M}$ is a fundamental tool for understanding the growth and evolution of black holes in the context of galaxy formation and evolution.
fraction of clusters with lifetimes less than $t_\bullet < t_H$ to be 
$\sim t_\bullet / t_H$ so $\sim 20\%$ of clusters would be expected to violate the
Klessen-Burkert criterion.

We conclude from equation (16) that if the halo is composed of $10^6$ M$_\odot$ black holes, will below the allowed upper
limit of $\chi = 3 \times 10^4$ M$_\odot$, then we expect substantial halo flattening and rotation out to $\sim 1$ kpc, which may be sufficient to suppress halo dynamical friction on rapidly rotating bars.

3 CONSTRAINTS ON HALO SHAPES

The arguments in the preceding sections suggest that the inner parts of dark halos may be thick, rapidly rotating discs. As noted earlier, bars are strongly coupled to the inner halo and thus should also rotate rapidly, as observed. We now ask whether other observations constrain the shapes of inner halos. Although halos formed by hierarchical clustering are generally triaxial, the subsequent growth of the disc tends to make the inner halo more nearly axisymmetric (cf. 19), and this expectation is confirmed by the upper limits to the ellipticities of post-disc galaxies induced by the halo potential—typically $\sim 0.1$; see (Franx & de Zeeuw 1992; Kuijken & Tremaine 1994; Rix 1996). Thus we assume that the inner halo is axisymmetric, and restrict our attention to the flattening of axisymmetric halos.

Rix (1996) and Sackett (1996, 1998) review the tests that have been used to constrain the halo shape in our own and other galaxies, including the shapes of X-ray halos, the kinematics of polar-ring galaxies, the thickness of HI layers, and gravitational lensing.

The kinematics and spatial distribution of Population III objects can be used to constrain the shape and mass of the dark halo in our Galaxy (Monet, Richstone & Schechter 1981). Unfortunately the best available results are model-dependent and have large error bars: $0.3 < c/a < 0.6$ from Binney, May & Ostriker (1987) and $c/a > 0.34$ from van der Marel (1991).

The ellipticity of the mass distribution in edge-on disc galaxies can be determined from the shape of their X-ray isophotes. This method yields $c/a = 0.68 \pm 0.13$ for the 50 galaxy
NGC 4783 (Buote & Canizares 1996); this is greater than expected for disc-like dark matter, but the uncertainties are large and the result is dominated by halo radii well beyond the outer edge of the disc, where the flattening mechanism discussed here would not operate.

Steinman-Cameron, Kormendy & Durisen (1992) fit a precessing-disc model to the dust lanes in the 50 galaxy NGC 4783 and derive $0.84 < c/a < 0.9$; however, this result depends on an assumed age for the event that warped the galaxy disc.

The axis ratios of the dark halos in polar-ring galaxies have been estimated by several investigators. Whitmore, McElroy & Schweizer (1987) examined 30 galaxies with polar rings and found axis ratios for the potentials of $0.86 \pm 0.2$, corresponding to axis ratios for the density of roughly $0.58 \pm 0.15$ ($\pm 0.6$); however, two of these galaxies were re-examined by Sackett et al. (1994) and Sackett & Pogge (1995), who found much flatter halos with axis ratios $0.3-0.5$.

High-resolution HI data can be used to determine the thickness of the HI layer in highly inclined disc galaxies; at and beyond the edge of the optical disc this thickness is sensitive to the shape of the dark halo. The most thorough analysis of this kind, for the Scd galaxy NGC 4244 (Olling 1996), yields $c/a \approx 0.2 \pm 0.1$. Olling & Merrifield (1997) derive $c/a = 0.75 \pm 0.25$ for the Galaxy using both the thickness of the HI layer and local stellar kinematics.

Gravitational lensing by disc galaxies can constrain the halo mass and shape. Koopmans, de Bruyn & Jackson (1998) find $c/a > 0.5$ for the edge-on lensing galaxy B1090-434. Flattening the halo does not significantly affect the overall lensing cross-section of the galaxy but does enhance the fraction of images that exhibit characteristic geometries associated with disc lensing (Keeton & Kochanek 1998). Unfortunately, disc galaxies are expected to comprise only 10-20% of gravitational lenses so a large sample will be difficult to obtain.

Evidently the present data allow a wide range of halo shapes, with some indication that moderately flattened halos ($c/a \sim 0.5$) are more common than spherical or disc-shaped halos. The data provide no strong evidence for or against rotating thickened discs of dark matter such as those proposed in this paper.

4 DISCUSSION

Structure in galactic discs and dark halos couples these two components together, leading to angular momentum transfer from disc to halo, spinup and flattening of the inner halo, and heating and thickening of the disc. The time-scale for this process is highly uncertain. Known structures in the disc are probably not able to flatten and spin up the inner halo significantly in a Hubble time, but halo structure is likely to be more efficient. In particular, we have identified two types of halo structure that may be able to spin up the halo: massive black holes, and tidal streams resulting from hierarchical halo formation.

A rotating flattened halo may be needed to explain the high pattern speed of bars (34). Flattening the inner halo also thickens and heats the inner disc; this process may form some galactic bulges, which, like discs, are metal-rich and contain rotating stars. (Wise, Gilmore & Franch 1972.) Another way to make bulges from discs is through a buckling instability of bar structures in the discs (Combes & Sanders 1981; Raha et al. 1991). The hypothesis that some bulges may be produced from discs is supported by the inner cut-offs observed in disc sizes that enter the bulge (Kormendy 1977), and the similar scale lengths of inner discs and bulges (Courteau, de Jong & Broeils 1996). Kormendy (1993) has argued persuasively that some bulges are really discs in terms of their dynamics and origin. On the other hand, the concerns of bulges and higher phase-space density than discs and so cannot arise from discs through a collisionless process (Carlberg 1986); moreover the strong similarity between bulges and elliptical galaxies
of similar luminosity suggests that most bulges are formed in a manner similar to the family of equivalent ellipticals.

Dynamical friction on tidal streams provides a natural mechanism to produce flattened dark halos of non-baryonic material, and thus offers a counter-example to the usual belief that if the dark halo is disklike it must be baryonic. If this process is important then we expect the inner halos of spiral galaxies to be substantially flatter than halos of ellipticals.

Relaxation from tidal streams can also be important in elliptical galaxies. Gravitational scattering of stars by stream scattering the relaxation rate compared to the usual Chandrasekhar formula for two-body relaxation (Binney & Tremaine 1987) and hence isotropize the distribution function at radii where two-body relaxation is ineffective (energy relaxation is less effective than isotropization because the potential from a long streamer only varies slowly in time).

We may contrast relaxation from tidal streamers with other relaxation processes in stellar systems: two-body and violent relaxation. The two-body relaxation rate is determined by assuming that the stellar system is as smooth as possible, so the only potential fluctuations arise from Poisson noise due to individual stars. Violent relaxation (Lynden-Bell 1967) is modelled by assuming that the potential fluctuations in the stellar system are as large and rapid as possible (spatial and time scales of order the system size and dynamical time), an assumption that is only valid during the initial collapse of the system. Relaxation from tidal streamers is intermediate—stronger than twobody relaxation, and weaker but longer lasting than violent relaxation—and arises because a stellar system only phase mixes gradually over very many dynamical times. Perhaps the process should be called ‘non-violent relaxation’.

One important unresolved question is how and when the streamers are finally mixed together. Small-scale irregularities in the disc and halo (e.g., globular clusters, etc.) can mix the streamers, but relaxation arising from interactions between streamers may be more effective. This issue is also important for experiments that hope to use the phase-space structure of dark matter particles (Silk & Ipser 1982).

The proposals made in this paper can be tested in a variety of ways. The physical processes can be studied by appropriate numerical simulations. Conventional N-body simulations of galaxy formation offer limited insight into phase mixing, because of numerical noise (Hernquist & Barnes 1990; Hernquist & Ostriker 1992; Steinmetz & White 1997). However, specially designed simulations are more powerful. A reasonable approximation is to turn off the gravitational forces among the dark particles and among the streamer particles, to suppress large-scale instabilities in the disc and numerical relaxation within the halo. The only remaining forces would be between disc and streamer particles, and between objects of particles and a fixed galactic potential. A more complete treatment would include the mutual gravitational interactions between streamers. As an illustration of the importance of such interactions, consider a single streamer consisting of an axisymmetric ring of material in an inclined orbit. Eventually this streamer would be dragged precisely into the disc midplane. However, if there are several axisymmetric streamers with the same radius and different midplanes, they cannot all be dragged into the disc mid-plane since this would lead to an increase in the phase-space density of the halo material, which violates the collisionless Boltzmann equation.

There are also direct observational tests. (i) Massive black holes scatter a fraction of the stars in the bulge and inner disc out to much larger radii; these should appear as a distinct population of $\sim 10^6 M_\odot$ in metal-rich high-velocity stars on nearly circular orbits (tidal streamers are less effective scatterers because their potential is softer). Similarly, black holes scatter local disk stars to large epicyclic energies, creating a power-law tail containing $\sim 1%$ of the disc mass (Lacey & Ostriker 1985). (ii) Tidal streamers heat disk stars by removing angular momentum at constant energy, while massive black holes induce a random walk in both energy and angular momentum; these processes may lead distinct signatures in the phase-space distribution of inner disc and bulge stars. (iii) Heating by tidal streamers is most effective early in the galaxy’s history, while heating by black holes continues throughout its lifetime; this distinction may be reflected in the age or metallicity distribution of bulge stars. (iv) The presence of a thin disc of old stars near the centre of a galaxy would imply that substantial angular momentum transfer from disc to halo has not occurred since the disc was formed. Some early-type galaxies contain discs with scale lengths $\sim 1$ kpc (van den Bosch, Jaffe & van der Marel 1998; Scora et al. 1998); if these are old and edge-on they can be used to constrain the disc thickening and hence angular-momentum transfer rates. (v) Halo structure may be directly detectable from small-scale fluctuations in the surface-brightness distribution of elliptical galaxies; these are distinct from surface-brightness fluctuations arising from bright stars and globular clusters (e.g., Jansen, Tonry & Luppino 1995), because they are correlated in nearby pixels. Halo structure may also account for discrepancies in the radio flux ratios predicted by smooth mass models of gravitational lensing (Mao & Schneider 1998).

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Relaxation in stellar systems and the shape and rotation of the inner dark halo


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