From the Cosmological Microwave Background to Large-Scale Structure

Joseph Silk and Eric Gawiser
Departments of Astronomy and Physics, and Center for Particle Astrophysics,
University of California, Berkeley, CA 94720

The shape of the primordial fluctuation spectrum is probed by cosmic microwave background fluctuations which measure density fluctuations at $z \sim 1000$ on scales of hundreds of Mpc and from galaxy redshift surveys, which measure structure at low redshift out to several hundred Mpc. The currently acceptable library of cosmological models is inadequate to account for the current data, and more exotic models must be sought. New data sets such as SDSS and 2DF are urgently needed to verify whether the shape discrepancies in $P(k)$ will persist.

1 Introduction

Our understanding of primordial fluctuations in the early universe was revolutionized first with inflation and then by the actual detection of temperature fluctuations in the cosmic microwave background (CMB). Inflation gave the spectrum of fluctuations, but not the normalization. The COBE-DMR experiment measured the amplitude of temperature fluctuations on an angular scale that was acausal at last scattering, and hence directly probed inflationary fluctuations, and in particular the fluctuation strength. More than twenty subsequent experiments have plugged the causal gap, measuring fluctuations over angular scales less than or of order the acoustic peak at $\lambda = 220 \Omega_{\Lambda}^{1/2}$ that corresponds to the maximum sound horizon in the early universe.

The shape of the fluctuation spectrum is now being probed. The CMB fluctuations measure density fluctuations at $z \sim 1000$ on scales of hundreds of Mpc. At large redshifts one degree subtends a comoving scale of 100 Mpc. A complementary measure arises from galaxy redshift surveys. These measure variations in the luminous matter density out to $\sim 300$ Mpc at the present epoch. One can combine, after choosing a model, the CMB and large-scale structure (LSS) measures of the fluctuation spectrum. Here we will describe the current status of our understanding of the shape of the primordial fluctuation spectrum.

It is customary to use a two-parameter fit to the LSS power: $\sigma_8$, the normalization at $8 \, h^{-1}$ Mpc, and $\Gamma$, measure of shape relative to CDM and nearly equal to $\Omega_0 h$ for CDM. Given the several data sets, each with a number of independent data points, this may be an unnecessarily restrictive approach. Of course any data set is imperfect, with possible systematic errors, and the
data sets have different selection biases. However, provided the bias is scale-independent, one can renormalize the data sets and examine detailed shape constraints. One has to decide whether to compare a nonlinear power spectrum with the data or whether to correct the data for nonlinearity and compare the data with linear theory. We will employ the latter approach here.

2 CMB: Status of the Theory

Inflation-generated curvature fluctuations provide the paradigm for interpreting the cosmic microwave background temperature anisotropies. There are three components to $\delta T/T$, schematically summarized as

$$\frac{\delta T}{T} = |\delta \phi + \delta \rho_{\gamma} + \delta v|.$$  

These are the gravitational potential, intrinsic and Doppler contributions from the last scattering surface. The combined effect of the first two terms results in the Sachs-Wolfe effect $\delta T/T = \frac{1}{3} |\delta \phi|$ which represents the only superhorizon contributions to $\delta T/T$. Inflationary initial conditions then require that the Fourier component $|\delta T/T|_k \propto |\cos(kv_{ts}t_{ls})|$ at the last scattering epoch $t_{ls}$, where $k$ is the wavenumber and $v_{ts}$ is the sound speed, must be constant as $k \to 0$. Inflation of course specifies the phases of density fluctuations that decompose to sound waves of wavelength less than that of the maximum sound horizon, $v_{ts}t_{ls}$. Longer wavelengths correspond to power-law growing modes at horizon crossing. The wave just entering the horizon at last scattering has a peak at wavenumber $n\pi/v_{ts}t_{ls}$, $n = 1$, and a succession of waves crest at $n = 2, 3, \ldots$ before damping sets in as the photon mean free path increases relative to the wavelength. The first acoustic peak projects to $\delta T/T$ on angular scales $\sim \Omega^{1/2}$ degree, and is a robust measure of the curvature of the universe. Doppler peaks are $90^\circ$ out of phase and of lower amplitude, so they fill in the troughs of the acoustic oscillations as measured by the radiation power spectrum. Peak heights are determined in large part by choice of $\Omega_B$ and $\Omega_\Lambda$. An increase in $\Omega_B$ enhances the wave compression and reduces the rarefaction phases. An increase in $\Omega_\Lambda$ enhances the ratio of radiation to matter in a flat model, and thereby boosts the peak potential decay and the low $\ell$ power via the early integrated Sachs-Wolfe effect. Of course increasing the spectral index $n$ also raises the peak height. Peak heights are lowered by reionization and secondary scattering. Not all of these degeneracies are removed by examining the higher peaks. For example, combination of $\Omega_B$ and $\Omega_\Lambda$ at specified $\Omega$ is nearly degenerate in peak height and peak location since the angular size-redshift relation depends only on $\Omega$. 

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Lensing by nonlinear and quasilinear foreground structure redistributes the peak power towards very high $\ell$ in a way that breaks the degeneracies in the CMB. One can search for this effect with interferometer experiments at $\ell \gtrsim 1000$ or else by correlating the CMB fluctuations with large-scale power from either large redshift surveys such as the SDSS or via weak lensing distortions of the CMB or LSS.

3 Reconstruction of the Primordial Power Spectrum from the CMB

The inflationary, approximately scale-free, power spectrum $P(k) = A k^n$, $n \approx 1$, is modified by the transition from radiation to matter domination, since in matter-dominated epochs different growth occurs on subhorizon scales: in the radiation era, there is no subhorizon growth of fluctuations. This modifies $P(k)$ to $P(k) \propto k^{-3}$ on larger scales. The transition occurs at the horizon during matter-radiation equality, namely $12(\Omega_h^2)^{-1}$ Mpc. The Boltzmann equation can be solved for temperature fluctuations, mode by mode, and the solutions for $\delta T/T$ are scaled to agree with the quoted errors for each CMB experiment. For each specified cosmological model, we then infer the power spectrum amplitude over scales that correspond to the deprojection on the sky of the experimental window function. We confirm that standard CDM ($\Omega_{\text{CDM}} = 1$, $h = 0.5$) fits the CMB data rather poorly, with best fit renormalization that corresponds to a high value of $\sigma_8$. The $\Lambda$CDM model ($\Omega = 1$, $\Omega_m = 0.4$, $h = 0.6$) gives a reasonably good fit with an acceptable value of $\sigma_8$. Much stronger constraints however come when these fits are combined with LSS data.

4 Reconstruction of $P(k)$ from Large-Scale Structure Data

There are several large-scale structure data sets that one may use to reconstruct $P(k)$. Galaxy redshift surveys include the Las Campanas Survey of 25,000 galaxies, the PSCz survey of 1,500 galaxies, and the SSRS2/CfA2 survey of 7,000 galaxies. There is also the APM cluster survey which probes to $\sim 300 \, h^{-1}$ Mpc and the real space inversion of the 2D APM galaxy survey. The local mass function of clusters has been used to measure $\sigma_8\Omega^{0.6}$, and the high redshift cluster abundance has been used to break the degeneracy between $\sigma_8$ and $\Omega$. Peculiar velocities and large scale bulk flows also yield $\sigma_8\Omega^{0.6}$ in a completely bias-independent approach, although systematic uncertainties remain large.

The redshift space surveys can be corrected in a straightforward way, for peculiar velocities on small scales and bulk flows on large scales, to derive the
real space $P(k)$ by assuming a cosmological model\cite{15}. Correction of data in the nonlinear regime is best done by numerical simulation, but can be performed using an empirical formulation calibrated to numerical simulations based on a smooth interpolation from spherical collapse by a factor of 2 in radius on cluster scales\cite{16}. Renormalization of the various measures of $P(k)$ is effected by assuming that all measurements are subject to a scale-independent bias, allowed to be independent for each probe of $P(k)$.

5 Confrontation of $P(k)$ with CMB and LSS

Model fitting to LSS alone results in the following conclusions. Of course the standard COBE-normalized CDM model fails completely. Without a large scale-dependent bias factor on $10 - 100$ Mpc scales, peculiar velocities and the galaxy cluster abundance are greatly overpredicted. Low density models circumvent these problems. The cluster abundance, evolution and baryon fraction are all in satisfactory agreement with observations\cite{13}.

However the combined CMB/LSS fits to $P(k)$ lead\cite{6} to a surprising conclusion. The surprise is that almost all models, while occasionally faring better than sCDM, still provide unacceptable fits to all of the data. Consider for example ΛCDM, currently favored by the SN Ia Hubble diagram. The reduced $\chi^2$ is 2.1 for 70 degrees of freedom. Data set by data set, one still has a problem. For example the values of $\chi^2$(d.o.f.) are APM clusters: 25(8); LCRS 17(5); APM 44(9); IRAS 16(9). The acceptable data sets are CfA for 2 d.o.f., cluster abundances/peculiar velocities for 3 d.o.f. and CMB for 34 d.o.f.

6 Neutrinos and LSS

The only mildly acceptable model (reduced $\chi^2 = 1.2$) is CHDM, hot and cold dark matter with $\Omega = 1$. This model overpredicts current cluster abundances and underpredicts the small number of high redshift, luminous x-ray clusters (2 at $z = 0.5$, 1 at $z = 0.8$). However the cluster evolution constraint is disputed\cite{17}, and the local normalization is not necessarily robust. The cluster baryon fraction provides an independent and powerful constraint that favors $\Omega_m \approx 0.3$. Of course this rests on the reasonably plausible assumption that clusters provide a fair sample of the baryon fraction of the universe. This need not necessarily be true if gas has had a complex history prior to cluster formation: e.g. the gas may have been preheated as is suggested by recent considerations of the entropy of intracluster gas\cite{18}. This would reduce the baryon fraction, but one can equally well imagine scenarios for cluster formation in dense sheets or filaments where the baryon fraction was already enhanced.
Consider the model preferred by the combination of SNIa and cluster constraints, namely ΛCDM. Figure 1 shows the ΛCDM power spectrum compared with observations of Large-Scale Structure and CMB anisotropy. One can pose the following question: does adding a hot component improve the marginally acceptable LSS fit? We find that as an admixture of HDM is added to the dominant CDM, the combined fit to CMB and LSS deteriorates (Figures 2,3). The reason is that low density CDM models have a $P(k)$ peak that is longward of the apparent peak in the APM data. Adding HDM only exacerbates the mismatch.

Neutrino masses imprint a distinct signature on $P(k)$. This will eventually be a measurable probe of the neutrino mass, from LSS as well as from CMB. Indeed the LSS probe may potentially be more powerful. There is more dynamical range available in probing $P(k)$ with LSS on the neutrino free streaming scale, where the primary signature should be present. Even present data is sensitive to a neutrino mass of around an eV: for example we find that the fit changes significantly between 0.1 and 1 eV. If ΛCDM is in fact the right model, our analysis yields an indirect upper limit on the mass of the most massive neutrino species of $m_\nu \leq 2\text{eV}$. While there are considerable systematic uncertainties in this approach, it is promising as a complement to the direct evidence for mass difference between neutrino species from SuperKamiokande and the solar neutrino problem, and is already beginning to conflict with results from LSND that require a large mass difference.

One class of exotic models is the following. Take a model that fits all constraints except for the shape. The best contender for such a model is ΛCDM. Inspection of the LSS constraints reveals that there is a deficiency of large-scale power near 100 Mpc. One can add an ad hoc feature on this scale from considering inflationary models with multiple scalar fields (see Figure 4). This could be generated for example by incomplete coagulation of bubbles of new phase in a universe that already has been homogenized by a previous episode of inflation. One can tune the bubble size distribution to be sharply peaked at any preferred scale. This results in nongaussian features and excess power where needed. The non-gaussianity provides a distinguishing characteristic.

Other suggestions that fit both CMB and LSS data appeal to an inflationary relic of excess power from broken scale invariance, arising from double inflation in a ΛCDM model, which results in a gaussian feature that is essentially a step in $P(k)$ at the desired wavenumber. This improves the fit in much the same way as adding a hot component to CDM improves the empirical fit. While such ad hoc fits may seem unattractive, one could argue that other aspects of cosmological model building are equally ad hoc, such as postulating a universe in which Λ is only becoming dynamically important at the present
epoch. Clearly one has to accommodate such arguments in order to fit the
data, if the data is indeed accepted at face value. Moreover there are positive
side effects that arise from the tuned void approach. The bubble-driven shells
provide a source of overdensities on large scales. Rare shell interactions could
produce nongaussian massive galaxies or clusters at low or even high redshift:
above a critical surface density threshold gas cooling would help concentrate
gas and aid collapse. If massive galaxies were discovered at say \( z > 5 \) or a mas-
sive galaxy cluster at \( z > 2 \) this would be another indication that the current
library of cosmological models is inadequate. New data sets such as SDSS and
2DF are urgently needed to verify whether the shape discrepancies in \( P(k) \) will
persist.

7 Summary

If the data are accepted as mostly being free of systematics and ad hoc ad-
ditions to the primordial power spectrum are avoided, there is no acceptable
model for large-scale structure. No one LSS data set can be blamed. Perhaps
it is best to wait for improved data. The Sloan and 2DF surveys are already
acquiring galaxy redshifts. However another philosophy is to search for more
exotic models. Consider for example the primordial isocurvature mode. This
has the advantage of forming primordial black holes of stellar mass, since nor-
malization to large-scale structure and the present spectrum over 10 – 50 Mpc
requires a spectral index that generates nonlinear fluctuations at roughly the
epoch of the quark-hadron phase transition, when the horizon contained ap-
proximately one solar mass \(^{24}\) (hence the primordial black holes may be the
possibly observed MACHOs). The goodness of fit of this model to the combined
CMB/LSS data is similar to that of the ΛCDM model. One cannot distinguish
with current data between an exotic isocurvature model and ΛCDM, although
neither model is satisfactory. To improve on this, clearly something even more
exotic is required. It may be that independent observations will force us in
this direction.

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References

Figure 1: Constraints from LSS and CMB on ΛCDM model
Figure 2: $\Lambda$CDM model with $\Omega_\nu = 0.05$. A blue tilt of the primordial power spectrum ($n = 1.3$) is necessary to counteract the damping of small-scale perturbations by free-streaming of the massive neutrinos, which makes the peak of the model fall even farther below that of the data unless $n > 1$. Even with this best-fit value of $n$, the fit to the data is worse than with no HDM, because CMB observations disfavor such a high value of $n$. 

$\chi^2/\text{dof} = 2.4$

$\Omega_m = 0.4$, $h = 0.7$, $n = 1.3$
Figure 3: ΛCDM model with $\Omega_\nu = 0.1$. The best-fit value of $n$ is now 1.5. The fit to the data worsens as more HDM is added.
Figure 4: Constraints from LSS and CMB on ΛCDM model with a broad enhancement centered at $k = 0.06h^{-1}$Mpc added to the primordial power spectrum.