Ultra-high-energy cosmic ray acceleration by relativistic blast waves

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ABSTRACT
We consider the acceleration of charged particles at the ultra-relativistic shocks, with Lorentz factors \(\Gamma_s \gg 1\) relative to the upstream medium, arising in relativistic fireball models of gamma-ray bursts (GRBs). We show that for Fermi-type shock acceleration, particles initially isotropic in the upstream medium can gain a factor of order \(\Gamma_s^2\) in energy in the first shock crossing cycle, but that the energy gain factor for subsequent shock crossing cycles is only of order 2, because for realistic deflection processes particles do not have time to re-isotropise upstream before recrossing the shock.

We evaluate the maximum energy attainable and the efficiency of this process, and show that for a GRB fireball expanding into a typical interstellar medium, these exclude the production of ultra-high-energy cosmic rays (UHECRs), with energies in the range \(10^{18.5} - 10^{20.5}\) eV, by the blast wave. We propose, however, that in the context of neutron star binaries as the progenitors of GRBs, relativistic ions from the pulsar wind bubbles produced by these systems could be accelerated by the blast wave. We show that if the known binary pulsars are typical, the maximum energy, efficiency, and spectrum in this case can account for the observed population of UHECRs.

Key words: cosmic rays – acceleration of particles – shock waves – gamma-rays: bursts – binaries: close – pulsars: general.

1 INTRODUCTION
The observation of some hundred cosmic ray events to date with energy above the Greisen–Zatsepin–Kuz’min cutoff, \(E_{\text{GZK}} \simeq 10^{18.5}\) eV, (e.g. Takeda et al. 1998, and references therein) has sparked renewed interest in their origin. Ultra-high-energy cosmic rays (UHECRs), with energies in the range \(E \sim 10^{18.5} - 10^{20.5}\) eV, are generally believed to be extragalactic in origin, based on their harder spectrum, isotropic arrival directions on the sky and the fact that they are not confined by the Galactic magnetic field. One class of models involves continuous production of UHECRs at large shocks, such as those associated with powerful radio galaxies, or with ongoing large-scale structure formation in clusters of galaxies (e.g. Norman, Melrose & Achterberg 1995, and references therein). A major difficulty with these models is that the number of possible sources within the volume contributing to the flux above \(E_{\text{GZK}}\), which has a radius \(D_{\text{max}} \sim 50 - 100\) Mpc, is too small to explain the observed number of independent events.

An alternative model (Waxman 1995a; Vietri 1995; Milgrom & Usov 1995) considers impulsive production in the sources of gamma-ray bursts (GRBs). Observations of X-ray and optical afterglows of GRBs (van Paradijs et al. 1997; Metzger et al. 1997) have confirmed the cosmological origin of the phenomenon; the energy associated with each event is then thought to be \(E_{\text{GRB}} \simeq 10^{51} - 10^{53}\) erg. Waxman (1995a) and Vietri (1995) noted that assuming comparable efficiencies for gamma-ray and UHECR production, the estimated GRB rate, \(Q_{\text{GRB}} \sim 10^{-8}\) Mpc\(^{-3}\) yr\(^{-1}\), implies a flux of UHECRs reaching Earth from within \(D_{\text{max}}\) remarkably similar to the one observed. Another, perhaps more compelling, argument is that the dispersion in UHECR arrival times due to small-angle deflections in the intergalactic magnetic field implies that at any one time, enough GRB sources contribute to the UHECR flux to account for the observed number of independent arrival directions (Miralda-Escudé & Waxman 1996; Achterberg et al., in preparation).

Relativistic fireball models for GRBs involve an ultrarelativistic blast wave with Lorentz factor \(\Gamma_s \approx 10^2 - 10^3\) bounding the fireball (Rees & Meszáros 1992), and internal mildly relativistic shocks (\(\Gamma_s \approx 2 - 10\)) due to unsteady outflows (Rees & Meszáros 1994). The quenching of interstellar scintillation observed in GRB radio afterglows (Frail et al. 1997) confirms the relativistic expansion of the source. Vietri (1995) proposed that these relativistic shocks can be sites of efficient particle acceleration up to energies exceeding \(10^{20}\) eV. In particular, he argued that an ultra-relativistic
shock with Lorentz factor $\Gamma_s$ will lead to an energy gain per crossing cycle $E_t / E_i \approx \Gamma_s^2$, where $E_i$ and $E_t$ are the initial and final particle energies. If such an energy gain could be obtained repeatedly at the ultra-relativistic blast wave, UHECR energies would be reached in only a few cycles. In this letter, we first consider the Fermi acceleration process at ultra-relativistic shocks in some detail; apart from the recent simulations by Bednarz and Ostrowski (1998), earlier calculations of such relativistic shock acceleration have concentrated on mildly relativistic shocks ($\Gamma \lesssim 10$). We distinguish between the energy gain in the initial and subsequent shock crossing cycles, and show that for physically realistic particle deflection processes upstream, the latter yield only an energy gain $E_t / E_i \sim 2$. We estimate the time scale for the acceleration process, and examine the maximum energy attainable, as well as the global energetics, for UHECR production scenarios at relativistic blast waves, first in a typical interstellar medium, and then in the plausible environment of a pulsar wind bubble.

2 ULTRA-RELATIVISTIC SHOCK ACCELERATION

We consider an ultra-relativistic shock of Lorentz factor $\Gamma_s \gg 1$ relative to the upstream medium. Assuming the fluid is weakly magnetised, the shock jump conditions then imply that the shock velocity relative to the downstream medium reduces to $c/3$, while the relative Lorentz factor of the downstream and upstream media satisfies $\Gamma_t = \Gamma_u/\sqrt{2}$ (e.g. Blandford & McKee 1976).

2.1 Energy gain: initial vs. repeated crossings

At non-relativistic shocks, the standard scenario of particle acceleration (Krymski 1977; Bell 1978; Axford, Leer & Skadron 1978; Blandford & Ostriker 1978) assumes that the acceleration (Krymski 1977; Bell 1978; Axford, Leer & Skadron 1978) assumes that the acceleration process begins and ends in the upstream and downstream respectively. Here $\beta_s$ is the velocity of the downstream medium (in units of the speed of light) relative to upstream, $\Gamma_s$ is the associated Lorentz factor, and the quantities $\mu_{-u}$ and $\mu_{-u}'$ measure the cosine of the angle between the particle velocity and the shock normal, when the particle crosses the shock into the downstream and upstream respectively. We use the convention that the shock normal points into the upstream medium, so that $\mu_{-u} > 0$. Primed and unprimed quantities are respectively measured in the downstream and upstream rest frames. The only assumption is that scattering is elastic in the local fluid frame.

Kinematics require that $1 \geq \mu_{-u} > \beta_s = 1/3$, so that the factor $(1 + \beta_s \mu_{-u})$ in (1) is of order unity. If $\mu_{-u}$ is more or less isotropically distributed, as would be the case for a population of relativistic particles already present in the undisturbed upstream medium, the factor $(1 - \beta_s \mu_{-u})$ is also of order unity, and energy gains $E_t / E_i \sim \Gamma_s^2$ can be achieved, as envisioned by Vietri (1995). A similar conclusion holds for particles which are initially non-relativistic upstream.

For all but the initial shock crossing into the downstream medium, however, the distribution in $\mu_{-u}$ for a relativistic shock will be highly anisotropic (Peacock 1981). For an ultra-relativistic particle with Lorentz factor $\gamma \gg \Gamma_s$, the Lorentz transformation of the incident angle reduces to $\mu' \approx (2 - \Gamma_s^2 \theta^2)/(2 + \Gamma_s^2 \theta^2)$ when $\theta \equiv \cos^{-1} \mu \ll 1$. The kinematic condition $\mu_{-u} > 1/3$ is thus equivalent to $\theta_{-u} < 1/\Gamma_s$, which defines the shock ‘loss cone’ in the upstream frame. We show below that for physically realistic deflection processes upstream, the particle cannot be deflected very far beyond this loss cone before the shock overtakes it, so that the angle $\theta_{-u} \sim 1/\Gamma_s$ as well. This severely limits the energy gain attainable for all but the initial shock crossing; when $\theta_{-u} \ll 1$, the downstream energy gain in (1) reduces to

$$\frac{E_t}{E_i} = \frac{1 + \beta_s \mu_{-u}}{1 + \beta_s \mu_{-u}'} \approx 2 + \frac{\Gamma_s^2 \theta^2_{-u}}{2 + \Gamma_s^2 \theta^2_{-u}};$$

which is of order unity if $\theta_{-u} \sim 1/\Gamma_s$.

2.2 Upstream dynamics

We consider two mechanisms for the upstream deflection of particles needed to allow repeated shock crossings: regular deflection by a large-scale magnetic field, and scattering by small-scale magnetic fluctuations.

2.2.1 Regular deflection

We first examine the case where the magnetic field may be considered uniform over the region sampled by the particle in its excursion upstream. We take the shock velocity to define the $z$-direction, and without loss of generality assume that the magnetic field lies in the $x-z$ plane, $B = (B_z, 0, B_t)$, with $qB_z \geq 0$.

The equation of motion for the velocity $v/c$ of an ultra-relativistic particle of charge $q$ can be solved approximately by expanding $\beta_z \approx \mu \approx 1 - \frac{1}{2} \theta^2$ to lowest order in $\beta_z$ and $\beta_y$, as $\theta^2 \approx \beta_y^2 + \beta_z^2 = O(1/\Gamma_s^2)$ throughout the particle’s orbit upstream. Then as long as $B_z \gg B_t / \Gamma_s$, which will be the case for all but a fraction $\sim 1/\Gamma_s^2$ of possible magnetic field orientations, we can ignore the terms in $B_t$ in the equation of motion. If we denote by $i$ and $f$ the ingress and egress particle velocities, we find that $\beta_y = \mu_{-u} \beta_z$ and

$$\beta_y = -\frac{1}{2} \beta_{y0} + \sqrt{\frac{3}{\Gamma_s^2 - 3}} \beta_{\parallel 0} - 3 \frac{3}{4} \beta_{\perp 0}.$$ (3)

Since $\beta_{\parallel 0}^2 + \beta_{\perp 0}^2 < 1/\Gamma_s^2$, eq. (3) implies

$$1 < \Gamma_s \theta_{-u} \leq 2 \iff \frac{1}{3} > \mu_{-u} \geq \frac{1}{3}.$$ (4)

The energy gain ratio can then be obtained by substituting $\theta_{-u}^2 \approx \beta_{\parallel 0}^2 + \beta_{\perp 0}^2$ in (2). It has its maximum at $\beta_{\parallel 0} = 0$ and $\Gamma_s \beta_{\perp 0} \approx -0.27$, where $E_t / E_i \approx 2.62$. 
2.2.2 Direction-angle scattering

We now consider the case where the magnetic field upstream is not uniform but irregular. In the simplest such model, the field consists of randomly oriented magnetic cells with field amplitude $B$ and radius (correlation length) $\ell_c$. If the particle’s gyration radius, $r_g \equiv E/qB$, satisfies $r_g/\Gamma_s \ll \ell_c$, it will be deflected sufficiently to recross the shock within a single cell, and the regular deflection regime above will apply. In the opposite case, $r_g/\Gamma_s \gg \ell_c$, the particle’s momentum direction will diffuse in time, with angular diffusion coefficient $D_\theta = c\ell_c/(3\Gamma_s^2)$ (Achterberg et al., in preparation).

In this scattering regime, a particle initially crossing the shock at an angle $\theta_{u \rightarrow d}$ will, after a time $t$, be at an average distance upstream relative to the shock given by

$$\langle z(t) \rangle - z_u(t) \approx ct \left( \frac{1}{2 \Gamma_s^2} - \frac{\theta_{u \rightarrow d}^2}{2} - D_\theta t \right), \quad (5)$$

where we have used the fact that $\theta = O(1/\Gamma_s)$ and expanded to lowest order in $1/\Gamma_s$. Defining the typical upstream residence time $t_u$ as the solution of $\langle z(t_u) \rangle = z_d(t_u)$, the typical downstream recrossing angle may be estimated as

$$\langle \theta_{u \rightarrow d} \rangle \approx \langle \theta_d(t_u) \rangle \approx \frac{2}{\Gamma_s^2} - \theta_{u \rightarrow d}^2. \quad (6)$$

The typical direction angle upon recrossing the shock, $\langle \theta_{u \rightarrow d} \rangle$, is thus again of order $1/\Gamma_s$. Substituting (6) into the energy gain formula (2) shows that the energy gain typically reaches its largest values when $\theta_{u \rightarrow d} = 0$, where $E_d/E_i \approx 2$. These results are approximate, rather than exact averages, in that they neglect the correlation between individual shock recrossing times and crossing angles.

2.3 Acceleration time scale

As the energy gain per shock crossing cycle, $\Delta E \equiv E_d - E_i$, is comparable to the initial energy $E_i$, the acceleration time scale will be of order the cycle time, which is the sum of the typical upstream and downstream residence times $t_u$ and $t_d$. In the case of deflection by a uniform magnetic field, the upstream residence time is the time required for the particle to be deflected by an angle of order $1/\Gamma_s$ [eq. (3)], i.e. $t_u \sim \Gamma_s \omega_{\perp}^{-1} \equiv E/(q\Gamma_s B_\perp c)$. The downstream residence time will depend on the downstream scattering process, but if we assume Bohm diffusion, it can be roughly estimated as the gyrot ime, i.e. $t_d' \sim \omega_{\perp}^{-1} \equiv E/(qB'c)$. If the downstream magnetic field is simply due to the compression resulting from the shock jump conditions, $B' \approx B_\perp \equiv \sqrt{\Gamma_s} B_\perp$, then taking into account the Lorentz transformation of the energy for a typical particle emerging upstream ($E' \sim E/\Gamma_s$) and the transformation of the time interval between two events occurring at the shock ($t_d' \approx t_d/\Gamma_s$), one finds that $t_d \sim t_d'$. Moreover, turbulence downstream could amplify the magnetic field well above the value resulting from simple compression; assuming it reaches equipartition with the thermal pressure downstream, one can show that in this case $B' \sim (c/v_A) \Gamma_s B_\perp$, where $v_A$ is the upstream Alfven speed. This yields a correspondingly shorter downstream residence time:

$$t_d \sim \frac{E}{q^2\Gamma_s \sqrt{4\pi \rho}}, \quad (7)$$

where $\rho$ is the upstream mass density.

The full shock crossing cycle time is thus dominated by $t_u$ in all cases of interest. In the case of scattering upstream, where $r_g \gg \ell_c$, solving (5) yields a typical upstream residence time $t_u \sim r_g^2/(c^2 \ell_c^2)$. This may be combined with the regular deflection case in the single expression

$$t_u \sim \frac{E}{q^2\Gamma_s B_\perp} \times \max \left( 1, \frac{r_g}{\Gamma_s \ell_c} \right). \quad (8)$$

For a given upstream magnetic field amplitude $B$, the regular deflection regime thus constitutes an absolute lower limit on the acceleration time for repeated shock crossings.

3 FIREBALLS IN THE GENERAL ISM

We now review the properties of the relativistic blast wave driven into the surrounding interstellar medium in fireball models of gamma-ray bursts. After an acceleration phase, an initially radiation-dominated fireball enters a relativistic ‘free expansion’ phase, driving a blast wave with the approximately constant Lorentz factor $\Gamma_s \approx \sqrt{2} \eta$, where $\eta \equiv E_{GRB}/(M c^2)$, $M$ being the baryonic mass in which the fireball energy $E_{GRB}$ is initially deposited (Piran, Shemi & Narayan 1993; Mészáros, Laguna & Rees 1993). The parameter $\eta$ is generally assumed to lie in the range $10^{-3}-10^3$.

If cooling of the shocked material can be ignored, this is followed by an adiabatic deceleration stage in which the blast wave Lorentz factor decreases with radius $R_s$ as $\Gamma_s \propto R_s^{-3/2}$ (Blandford & McKee 1976). The transition between the free expansion and deceleration phases occurs roughly at the radius $R_d$ at which the energy in the swept-up surrounding material becomes comparable to $E_{GRB}$:

$$R_d \approx \left( \frac{3}{4\pi \eta^3 e} \right)^{1/3}, \quad (9)$$

where $e$ is the total energy density of the surrounding material; for a typical interstellar medium of number density $n$, this is simply the rest-mass energy $e \approx n m_p c^2$. The subsequent evolution of $\Gamma_s$, is then given by

$$\Gamma_s(R_s) \approx \sqrt{2} \eta \left( \frac{R_s}{R_d} \right)^{-3/2} \approx \left( \frac{3 E_{GRB}}{2\pi \eta^3 e} \right)^{1/2} R_s^{-3/2}, \quad (10)$$

and thus once $R_s > R_d$ the behaviour of $\Gamma_s$ is independent of the value of $\eta$.

We now consider various scenarios for the acceleration of UHECRs by the relativistic blast wave of a fireball expanding into the general interstellar medium.

3.1 Maximum energy for Fermi acceleration

In order for particles to be accelerated beyond the initial boost by shock acceleration of the Fermi type, repeated shock crossings must occur. The acceleration time, which we argued above is dominated by the upstream residence time $t_u$, must therefore be shorter than the age of the blast wave measured in the upstream frame, $R_u/c$. Using (8), the maximum energy for which this can be the case is

$$E Q B \Gamma_s \times \max \left( R_u, \sqrt{\ell_c R_u} \right). \quad (11)$$

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The most favourable regime energetically is that of regular\footnote{Note that the maximum energy (11) is larger by a factor $\Gamma_0$ than that resulting from a simple geometrical comparison of the gyration radius with $R_\ast$. This is due to the fact that a particle typically only executes a fraction $\sim \Gamma_0^{-1}$ of a Larmor orbit upstream before recrossing the shock.}\footnote{The behaviour of $\Gamma_0$ as a function of $R_\ast$ implies that the highest energy in (11) is reached at the transition between the free expansion and deceleration phases, $R_\ast \approx R_d$. Using the definition (9) of $R_d$, we obtain:

$$E_5 \times 10^{15} \frac{Z}{B_{-6}} \frac{E_{52}^{1/3} h_3^{1/3} n_0^{-1/3}}{E_{GJ1/2}^{1/3}} \text{eV},$$

(12)

for ions of charge $q = Ze$, where $B_{-6} \equiv B / (\text{10}^{-6} \text{G})$, $E_{52} \equiv E_{\text{GCR}} / (10^{52} \text{erg})$, $n_0 \equiv n / (1 \text{ cm}^{-3})$. Given typical interstellar magnetic fields of a few microgauss and the weak dependence on the other parameters, this rules out the production of UHECRs by Fermi acceleration at the unmodified external blast waves of relativistic fireballs.}

3.2 Initial boost of galactic cosmic rays

3.2.1 Maximum energy

One remaining possibility for blast wave acceleration of UHECRs is that they result from the initial shock crossing cycle energy boost of a pre-existing upstream population of relativistic particles. This is, in essence, the first of the two acceleration mechanisms considered by Vietri (1995).

This initial boost requires only the time $t_4$ for the particle to be scattered downstream, which if equipartition fields are generated can be considerably shorter than $t_\omega$, yielding a correspondingly higher maximum energy. Requiring the downstream half-cycle time (7) for a particle of final energy $E$ to be shorter than the age of the fireball, we obtain:

$$E_7 \times 10^{20} \frac{Z}{B_{-6}} \frac{E_{52}^{1/3} h_3^{1/3} n_0^{1/6}}{E_{GJ1/2}^{1/3}} \text{eV}.$$

(13)

This process can thus attain UHECR energies, provided particles with sufficient initial energy to be boosted in this range are present upstream. To reach $10^{20} \text{eV}$, this requires relativistic particles with energy above $\sim 10^{14} \eta_3^{-2} \text{eV}$.\footnote{We will assume the known binary millisecond pulsar systems PSR 1913+16, PSR 1534+12, and PSR 2127+11C to be typical of GRB progenitors. The spiral-in times of these binaries due to gravitational radiation are $\tau \sim 3 \times 10^7 \text{yr}$ (Narayan et al. 1992), while the periods and period derivatives of the millisecond pulsars observed in these systems yield spin-down luminosities $\sim 10^{33} \text{erg s}^{-1}$ and characteristic times $\sim 10^8 \text{yr}$ (Taylor, Manchester & Lyne 1993). Thus these pulsars will continuously inject relativistic particles into the surrounding medium over the lifetime of the system, with an approximately constant wind luminosity.}

3.2.2 Global energetics

The energy invested in boosting pre-existing relativistic particles is of order $\Gamma_0^2 e\epsilon_{\text{CR}}$ per unit volume swept up by the blast wave, where $\epsilon_{\text{CR}}$ is the upstream energy density of relativistic particles. Meanwhile, the blast wave expends an energy $\sim \Gamma_0^2 e\epsilon_{\text{CR}}$ per unit volume in shock-heating the surrounding medium, which has upstream energy density $e \approx nm_p c^2$. Thus the fraction of the fireball’s energy that can go into boosting cosmic rays is of order $f \sim \epsilon_{\text{CR}}/e$. Taking the interstellar medium values in our Galaxy to be typical, $\epsilon_{\text{CR}} \sim 1 \text{ eV cm}^{-3}$ and $e \simeq 10^8 n_0 e\text{V cm}^{-3}$, we see that this mechanism can only have a very low efficiency, $f \sim 10^{-9}$.

The efficient yield of UHECRs required in the GRB hypothesis, f0.1 (Waxman 1995a; Vietri 1995), could only be obtained if a large fraction of the surrounding medium’s energy density was in relativistic particles. This is in fact not implausible in the neutron star binary merger scenario for GRBs, as we now argue.

4 FIREBALLS IN PULSAR WIND BUBBLES

The surrounding medium in which the relativistic fireball explodes is probably one modified by the activity of the progenitor system prior to the burst event. In the neutron star binary merger scenario for GRBs, these progenitors are identified with the binary pulsar systems observed in our own Galaxy (Narayan, Paczynski & Piran 1992). The pulsar in these systems can be expected to emit a relativistic wind, which over the lifetime of the binary can fill a large surrounding volume with shocked relativistic plasma. While this plasma will probably be predominantly in the form of electron-positron pairs created in the pulsar magnetosphere, it has been argued that pulsar winds must also contain ions in order to account for the accelerated pair spectrum observed in the Crab Nebula (Hoshino et al. 1992). The Crab Nebula’s wisps provide additional observational evidence for this ion component (Gallant & Arons 1994).

4.1 Maximum energy for boosted ions

We will assume the known binary millisecond pulsar systems PSR 1913+16, PSR 1534+12, and PSR 2127+11C to be typical of GRB progenitors. The spiral-in times of these binaries due to gravitational radiation are $\tau \sim 3 \times 10^7 \text{yr}$ (Narayan et al. 1992), while the periods and period derivatives of the millisecond pulsars observed in these systems yield spin-down luminosities $\sim 10^{33} \text{erg s}^{-1}$ and characteristic times $\sim 10^8 \text{yr}$ (Taylor, Manchester & Lyne 1993). Thus these pulsars will continuously inject relativistic particles into the surrounding medium over the lifetime of the system, with an approximately constant wind luminosity.

The relativistic flow of the pulsar wind will be thermalised in a termination shock, and this shocked material will form a relativistic plasma bubble in the interstellar medium. At the ages involved, we expect this pulsar wind bubble to be approximately isobaric and in pressure equilibrium with the surrounding medium. Once their directed kinetic energy is randomised by the shock, the pairs may suffer energy losses by synchrotron radiation over time, but these are negligible for the ions, which can only suffer adiabatic expansion losses. In an isobaric bubble these losses are negligible, so that the ions approximately conserve their post-shock energy throughout the bubble.\footnote{We expect the ions, although a minor component of the pulsar wind particles by number (see, e.g., Michel 1991). We expect the ions, although a minor component of the pulsar wind particles by number ($h_i \ll h_e$), to form the dominant component by mass ($m_i h_i/m_e$), so that the fraction of the wind energy carried by them, $\xi \equiv [1 + h_i m_i/(m_e h_i)]^{-1}$, will be close to unity (Hoshino et al. 1992). We will also assume that the ions carry the magnetospheric return current, so that $h_i \sim 1/Z$ (Gallant & Arons 1994). This then determines the typical energy, $m_i \gamma_i c^2$, of the thermalised post-shock ions.}
The relativistic plasma, while the pulsar wind bubble radius energy spectrum centered around Although the upstream ions have a relatively narrow energy spectrum, the relativistic fireball will decelerate well of relativistic particles upstream, the average energy gain of the pulsar wind bubble at $(1)$ for boosted particles satisfies $\frac{d}{dT} I_n^\gamma = - \frac{1}{\Gamma_n^2}$, the exact value depending on the typical angle $\langle \mu_n^\gamma \rangle$ of those particles recrossing upstream. Taking a factor $2\Gamma_n^2$ to be representative, the typical energy of the ions boosted in the free expansion phase of the blast wave is

$$E_{\max} \approx 2 \eta^2 \mu_n^\gamma c^2 \approx 10^{20} \eta^3 Z \xi E_{33}^{1/2} \text{ eV},$$

where $E_{33} \equiv E / (10^{33} \text{ erg s}^{-1})$. UHECR energies are thus naturally obtained in this scenario, for typical values of the pulsar wind parameters, as long as $\eta 10^3$.

4.2 Spectrum and efficiency

Although the upstream ions have a relatively narrow energy spectrum centered around $\mu_n^\gamma c^2$, the boosted ions will have a broader distribution in energies provided the blast wave decelerates within the pulsar wind bubble, so that the boosting factor $\Gamma_n^2$ is not constant. The deceleration radius $R_d$ is given by $(9)$, with $\epsilon$ the energy density of the relativistic plasma, while the pulsar wind bubble radius $R_b$ is obtained by equating the volume integral of $\epsilon$ with the total energy output of the pulsar. It follows that

$$\frac{R_d}{R_b} \approx 0.3 \xi E_{52}^{-2/3} E_{33}^{-1/3} \tau_7^{-1/3},$$

where $\tau \equiv \tau_7 \times 10^7 \text{ yr}$ is the age of the system. Thus for typical parameters, the relativistic fireball will decelerate well within the pulsar wind bubble.

Since $E \propto \Gamma_n^2 \propto R_c^{3}$ in the deceleration phase, and the number of ions swept up by the blast wave scales as $dN \propto R_c^2 dR_c$, the spectrum of the boosted ions will be

$$\frac{dN}{dE} \propto E^{-2}.$$  

(16)

This is consistent with the observed UHECR spectrum (Waxman 1995b). It can be shown that the same spectral index follows more generally if the ion density is not uniform, as assumed here, but obeys a power law in radius.

The lower bound of the spectrum $(16)$ follows from the boosting factor $\Gamma_n^2$ of the blast wave when it reaches the edge of the pulsar wind bubble at $R_b$, and is

$$E_{\min} \approx 3 \times 10^{18} Z \xi \mu E_{52}^{1/3} \tau_7^{-1/2} \text{ eV}.$$  

(17)

Thus not only can a large fraction of the fireball energy go into boosted ions, but these ions have a power-law spectrum extending more or less exactly over the energy range where the extragalactic UHECR component is observed.

5 SUMMARY

In this letter, we examined Fermi-type acceleration at an ultra-relativistic shock, and found that particles with initially isotropic momenta upstream can increase their energy by a factor of order $\Gamma_n^2$ in the initial shock crossing cycle. In all subsequent shock crossing cycles, however, we showed that the particle energy typically only doubles. This is due to the fact that particles do not have time to re-isotropize upstream before being overtaken by the shock, which we demonstrated in the specific cases of a large-scale ordered magnetic field and of small-scale magnetic fluctuations.

We argued that the maximum energy that can be reached by repeated shock crossings at the unmodified relativistic blast wave from a GRB fireball is well below the UHECR range. Pre-existing relativistic particles of sufficient energy can nonetheless be boosted to UHECR energies in the first shock crossing cycle. For a fireball expanding into a typical interstellar medium, where galactic cosmic rays provide the seed particles, we showed, however, that this process is too inefficient to account for UHECR production.

We proposed that the blast wave instead expands into a pulsar wind bubble produced by the progenitor system, a plausible hypothesis in the neutron star binary merger scenario for GRBs. We showed that for parameters typical of the neutron star binary systems observed in our Galaxy, relativistic ions in the pulsar wind bubble can be efficiently boosted by the blast wave to energies exceeding $10^{20}$ eV. We argued that these boosted ions would have an $E^{-2}$ spectrum, extending down in energy to about $10^{18.5}$ eV.

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