KINEMATICS AND DYNAMICS OF
THE GALACTIC STELLAR HALO

JESPER SOMMER-LARSEN
Theoretical Astrophysics Center
Juliane Maries Vej 30
DK-2100 Copenhagen Ø, Denmark

Abstract. The structure, kinematics and dynamics of the Galactic stellar halo are reviewed including evidence of substructure in the spatial distribution and kinematics of halo stars. Implications for galaxy formation theory are subsequently discussed; in particular it is argued that the observed kinematics of stars in the outer Galactic halo can be used as an important constraint on viable galaxy formation scenarios.

1. Introduction

Although the stellar halo accounts for at most a few percent of the luminous mass of the Galaxy, it plays a crucial role in studies of the Galaxy’s formation, evolution, and present-day structure. The halo has long been considered the Galaxy’s oldest component. Age estimates can be obtained for its most conspicuous constituent, the metal-weak globular clusters, as well as for individual metal-weak stars (with less certainty). Thus the dynamical and chemical state of the luminous halo population provides information on the formation of large disk galaxies such as the Milky Way.

2. Is there substructure in the halo?

In the currently favoured hierarchical galaxy formation scenarios big disk galaxies like the Milky Way are built up through merging of smaller subsystems. In particular it is thought that the Galactic halo was formed and, to some extent, still is being formed by accretion of many small satellites.

Likely examples of ongoing stellar accretion are the Sagittarius dwarf (Ibata, Gilmore & Irwin 1994) and the Magellanic Stream (Majewski et
Furthermore one would generally expect strong substructure in the distribution of halo stars in phase-space due to past accretion events (e.g., Johnston, Spergel & Hernquist 1995). This substructure is difficult to detect due to the very low number density of halo stars (“observational shot-noise”) and the physical effect of phase-mixing, but accreted systems with extreme kinematics should be detectable — one very likely example of this is the moving group of Majewski, Munn & Hawley (1994). The blue metal-poor stars (BMPs) of Preston, Beers & Shectman (1994) may well be another example of a population of halo stars accreted in one (major) event — see also Carney et al. (1996).

3. The local halo velocity ellipsoid and the structure of the inner halo

The inner halo, defined here as the part of the halo inside of or about at the solar distance from the center of the Galaxy ($r < r_\odot \simeq 8$ kpc) consists of at least two components: the flat and the round halo. The ratio of the local density of flat to round halo is not well determined ranging from 0.5–1 (Sommer-Larsen & Zhen 1990) to $\sim 4–8$ (Kinman, Suntzeff & Kraft 1994, Hartwick 1987). The velocity ellipsoid of local halo stars has been determined by Beers & Sommer-Larsen (1995). They used radial velocities of stars selected without kinematic bias, making the result very robust, also to distance errors. For almost 900 stars with [Fe/H] $< -1.5$ they find a velocity ellipsoid for local halo stars of $(\sigma_r, \sigma_\phi, \sigma_\theta) = (153 \pm 10, 93 \pm 18, 107 \pm 7)$ km s$^{-1}$ in spherical polars. This velocity ellipsoid is fairly radially elongated, but is still characterized by a quite large vertical (and horizontal) tangential velocity dispersion of $\sim 100$ km s$^{-1}$. It follows from the tensor virial theorem that the flat component cannot be both very flat and locally dominant — see Sommer-Larsen & Christensen (1989).

Carney et al. (1996) analyzed two classes of halo stars from their local sample: The “low” halo stars have $\langle |z_{\text{max}}| \rangle \leq 2$ kpc and the “high” halo stars have $\langle |z_{\text{max}}| \rangle \geq 5$ kpc, where $\langle |z_{\text{max}}| \rangle$ is the typical distance a star reaches from the plane of the disk — as the Carney et al. stars are all local $z_{\text{max}}$ was calculated from the observed space motions by orbit integration. These two classes of halo stars can be seen as representative of the flat and round halo respectively. The “low” halo is characterized by a slight prograde average rotation $\langle \nu_\phi \rangle \simeq 20 \pm 15$ km s$^{-1}$ with respect to the Galactocentric restframe assuming a circular speed at the solar distance from the Galactic center of $v_\odot = 220$ km s$^{-1}$. The “high” halo, on the other hand, is characterized by a net retrograde rotation $\langle \nu_\phi \rangle \simeq -50 \pm 15$ km s$^{-1}$. The latter result was first found by Majewski (1992) from proper motions of halo stars situated at the north galactic pole (NGP) and at least
5 kpc from the plane of the disk. The results above are further indications of kinematic substructure in the halo. Surprisingly a similar retrograde net rotation has not been detected so far for halo stars \textit{in situ} at $|z| > 5$ kpc at the SGP (Beers, private communication). As the retrograde halo stars are found both at high $z$ at the NGP and locally these stars should be well mixed, so from stellar dynamics considerations it follows that such stars \textit{should} be found at the SGP in the future.

Martin & Morrison (1998) find in an interesting recent study of the kinematics of local ($d \lesssim 1$ kpc) RR-Lyrae stars with accurate 3-D velocities that $\sigma_r = 193 \pm 15$ km s$^{-1}$ for the halo stars in their sample. As noted by the authors this differs from the result of Beers & Sommer-Larsen (1995) for halo stars in general by more than 2 $\sigma$ (see above). A possible explanation of this difference is that the spatial distribution of the RR-Lyrae halo stars differs from that of the halo stars in general: Approximating the spatial distributions by power-laws $\rho \propto r^{-\alpha}$, then $\alpha_{RR} = 3.2 \pm 0.1$, whereas for the halo stars in general $\alpha_{HALO} \simeq 3.5$ — see, e.g., Preston, Shectman & Beers (1991). Pushing $\alpha_{RR}$ to 3.1 and assuming the Beers & Sommer-Larsen value for $\sigma_r$ of halo stars in general it follows from the Jeans equation that one would expect $\sigma_r \simeq 179 \pm 12$ km s$^{-1}$ for the RR-Lyrae halo stars, in good agreement with the results of Martin & Morrison.

Martin & Morrison used the “old” distance scale with $M_V(\text{RR}) = 0.73$ at [Fe/H] = -1.9. A “new” distance scale, based on Hipparcos trigonometrical parallaxes of Cepheid variables and subdwarfs, has recently been advocated by Feast & Catchpole (1997) and Chaboyer \textit{et al.} (1998) resulting in $M_V(\text{RR}) \simeq 0.32$ at [Fe/H] = -1.9. With this distance scale Martin & Morrison obtain $\sigma_r \simeq 225 \pm 18$ km s$^{-1}$ for their RR-Lyrae halo stars. This differs from the value predicted above for $\alpha_{RR} = 3.1$ by more than 2 $\sigma$. Hence, as also noted by Martin & Morrison, one should perhaps be somewhat cautious about using the “new” distance scale, despite its many merits, like resulting in cosmologically “reasonable” globular cluster ages of about 12 Gyrs, a fairly low value of the Hubble constant etc.

4. The outer halo

The outer stellar halo ($r \gtrsim r_{\odot}$) is approximately spherical — see references in Sommer-Larsen \textit{et al.} (1997), but also Sluis & Arnold (1998). Several types of stars have been used as tracers of the outer halo: K-giants (e.g., Ratnatunga & Freeman 1989), RR-Lyrae stars (e.g., Hawkins 1984) and blue horizontal branch stars (e.g., Sommer-Larsen \textit{et al.} 1997 and references therein). The blue horizontal branch field (BHBFB) stars have proven to be particularly useful tracers of the the outer halo for two main reasons: (a) They are easy to identify in the halo because of their blue colours and (b)
Using a medium-sized telescope, spectra of sufficient quality for accurate line-of-sight velocity and Balmer line-width determination can be obtained fairly easily, even for quite distant stars \((d \sim 30 - 60 \text{ kpc})\), because of their intrinsic brightness. Furthermore they seem representative of the stellar halo in general since the density fall-off of the BHBF stars in the outer halo can be well approximated by the power-law relation \(\rho(r) \propto r^{-\alpha}\), \(\alpha = 3.4 \pm 0.3\) — see Sommer-Larsen, Flynn & Christensen (1994).

Sommer-Larsen et al. (1997) analyzed a sample of almost 700 BHBF stars with good line-of-sight velocity determinations and situated at Galactocentric distances \(r \sim 7-70 \text{ kpc}\). At distances \(d \gtrsim 20-30 \text{ kpc}\) the line-of-sight velocity is close to being the radial component of the velocity in Galactocentric coordinates. Hence it is possible to determine the radial velocity dispersion at large Galactocentric distances from line-of-sight velocities only. Whereas the radial velocity dispersion of local halo stars is 140–150 km s\(^{-1}\) \(\sigma_r\) is found to drop to about 100 km s\(^{-1}\) at large \(r\). This kinematic behaviour is modelled in the following simple way by Sommer-Larsen et al.:

It is assumed that both the outer Galactic halo and gravitational potential are spherically symmetric (moderate departures from this does not affect the outcome of the analysis in any significant way). The radial velocity dispersion as a function of \(r\) is modelled as

\[
\sigma_r = \left( \sigma_0^2 + \frac{\sigma_z^2}{\pi} \left( \frac{\pi}{2} - \arctan \left( \frac{r - r_0}{l} \right) \right) \right)^{1/2}.
\]

Adopting this form gives good flexibility in modelling the decrease in \(\sigma_r(r)\) with increasing \(r\). It follows from eq. [1] that \(\sigma_0\) is the asymptotic value of the radial velocity dispersion for \(r \gg (r_0 + l)\) and that \(\sqrt{\sigma_z^2 + \sigma_0^2}\) approximately is the radial velocity dispersion in the inner halo \((r \lesssim r_\odot)\).

The physical meaning of the two scale parameters \(r_0\) and \(l\) is given in Sommer-Larsen, Flynn & Christensen (1994) and is fairly straightforward.

The rotation curve of the Galaxy is approximately flat to at least \(R \simeq 20 \text{ kpc}\) (Fich & Tremaine 1991) and most likely to much larger distances as shown by Kochanek (1996). Consequently the gravitational potential of the outer halo is approximated by

\[
\Phi(r) = V_c^2 \ln(r),
\]

corresponding to a flat rotation curve with \(v_c(r) \equiv V_c = 220 \text{ km s}^{-1}\). Substituting this and eq. [1] into the Jeans equation

\[
\frac{1}{\rho} \frac{d}{dr} \left( \rho \sigma_r^2 \right) + \frac{2(\sigma_r^2 - \sigma_0^2)}{r} = - \frac{d\Phi}{dr},
\]

for the Jeans equation.
where $\sigma_t$ is the (1-D) tangential velocity dispersion, yields the following expression for $\sigma_t(r)$

$$
\sigma_t = \left( \frac{V_c^2}{2} - \sigma_r^2 \frac{(\alpha - 2)}{2} - \frac{1}{2\pi} \frac{r}{l} \frac{\sigma_+^2}{(1 + [(r - r_0)/l]^2)} \right)^{1/2},
$$

where $\alpha$ is the BHBF star density power-law index defined above. The line-of-sight velocity dispersion of a set of stars located at a distance $d$ in a field at Galactic coordinates $(l,b)$ where the velocity ellipsoid has components $\sigma_r$ and $\sigma_t$ is

$$
\sigma_{\text{los}} = \sqrt{\gamma^2 \sigma_r^2 + (1 - \gamma^2) \sigma_t^2},
$$

where $\gamma$ is a simple geometric projection factor

$$
\gamma \equiv \frac{(d - r_\odot \cos l \cos b)/r}{r}.
$$

The parameters $(\sigma_0, \sigma_+, r_0, l)$ are determined by maximum likelihood fitting of expressions [1] and [4] to the data using eq. [5]. A surprisingly good fit is obtained for this simple model - see Sommer-Larsen et al. The resulting $\sigma_r(r)$, $\sigma_t(r)$ and $\eta(r) \equiv \sigma_r(r)/\sigma_t(r)$ are shown in Figure 1. The asymptotic value of $\sigma_t(r)$ at large $r$ is found to be $\sigma_0 = 89 \pm 19 \text{ km s}^{-1}$.

The main result of the analysis is that $\sigma_r$ decreases from 140–150 km s$^{-1}$ at $r = r_\odot$ to 89 ± 19 km s$^{-1}$ at large $r$ - matched by a corresponding increase in $\sigma_t$. Hence, the most important kinematic feature of the model is that the velocity ellipsoid changes from radial anisotropy in the solar vicinity ($\eta \simeq 0.65$) to tangential anisotropy in the outer halo ($\eta \sim 1.4$).

5. Outer stellar halo kinematics: clues about the formation of the Milky Way

The results concerning the dynamics and kinematics of the outer stellar halo are of considerable interest in relation to theories of the formation of the Milky Way, in particular, and galaxies in general:

If the Galaxy formed from a single collapsing over-dense region in the early universe, then one might expect the outer halo to be characterized by radially anisotropic kinematics (see, e.g., van Albada 1982), whereas the data show that quite the opposite is the case. If, on the other hand, at least the outer parts of the proto-Galaxy were assembled by accretion of small subsystems, then a large tangential velocity dispersion in the outer parts of the Galaxy is possible, depending on the nature of the accretion (Norris 1994; Freeman 1996). So the results indicate that the outer stellar halo formed by some sort of accretion and merging processes. The kinematics
of stars in the inner halo are, at least locally, radially anisotropic, possibly indicating that the inner parts of the halo formed during a more dissipative and coherent collapse.

Chemical evolution arguments lead to a similar conclusion on the basis of the finding that there is a significant abundance gradient in the inner halo, but essentially none in the outer halo. This was discussed in the pioneering work by Searle & Zinn (1978) and in much subsequent work — see, e.g., Norris (1996) and references therein.

In the following I will attempt to quantify the connection to galaxy formation theory, more specifically the Cold Dark Matter (CDM), hierarchical galaxy formation scenario:

Figure 1. Best-fit model. The dotted line shows the radial velocity dispersion $\sigma_r(r)$ and the dashed line shows the tangential velocity dispersion $\sigma_t(r)$. Also shown is $\sigma_t/\sigma_r$ as a function of $r$ (solid line).
Sommer-Larsen, Gelato & Vedel (1998) carried out cosmological (CDM), gravitational/hydrodynamical, Tree-SPH simulations of the formation and evolution of large (Milky Way sized) disk galaxies. For the dark matter haloes of four different model galaxies (at the present epoch) the ratio $\eta(r)$ is shown in Figure 2. As can be seen from the Figure $\eta \simeq 0.9 \pm 0.1$. This is quite different from what is found in the outer stellar halo of the Milky Way and close to isotropic. Furthermore the rotation curves of the model galaxies are approximately flat over the range $r \sim 10$–100 kpc. Hence it should be a reasonable approximation for the present purpose to represent a dark matter halo by an isothermal sphere with phase-space distribution.
Figure 3. $\sigma_t/\sigma_r$ as a function of power-law index $\alpha$ for models and observations - see text for details.

The dark matter density falls off like $\rho_{DM} \propto r^{-2}$, whereas the stellar halo density profile is considerably steeper, $\alpha \simeq 3.4$, so the halo star formation efficiency $\epsilon_*$ must have had a dependence on some radial property $r_{\text{orb}}$ of the dark matter orbits such that

$$\epsilon_* \propto r_{\text{orb}}^{2-\alpha}. \quad (8)$$

One can then “build” the stellar halo using dark matter orbits weighted by $\epsilon_*$. If the halo stars were formed in small, proto-galactic subsystems...
before or as they were accreted onto the main dark matter halo it would be reasonable to take $r_{\text{orb}} = r_a$, where $r_a$ is the apocenter distance of the orbit in the main dark matter halo. If, on the other hand, the halo star-formation was triggered by disk and halo shocking of the gas in the subsystems when these were near the inner turning points of their orbits one would take $r_{\text{orb}} = r_p$, where $r_p$ is the pericenter distance of the orbit. Finally one might also study an intermediate case like $r_{\text{orb}} = (r_a + r_p)/2$. Figure 3 shows the resulting values of $\eta$ as a function of the power-law index $\alpha$ of the halo stars for $r_{\text{orb}} = r_a$ (thick solid line), $r_{\text{orb}} = r_p$ (thick long-dashed line) and $r_{\text{orb}} = (r_a + r_p)/2$ (thick short-dashed line) — $\eta$ does not depend on $r$ for the models I consider here since these are scale-free. Also shown is the asymptotic value (at large $r$) of $\eta$ from the observations of the outer halo (solid line) and 1 $\sigma$ and 2 $\sigma$ deviations (dotted lines). Finally the local value of $\eta \simeq 0.65$ is indicated by a short solid line. As can be seen from the Figure the model predictions do not agree well with the observations: for $r_{\text{orb}} = r_a$ the model predictions can be rejected with 82% confidence for $\alpha = 3.4$ and the $r_{\text{orb}} = r_p$ case can almost completely be excluded. Hence it appears unlikely that the formation of the halo stars was controlled by disk and halo shocking, at least in the framework of the models presented here. More observations of BHBF stars in the outer part of the Galactic halo ($r \gtrsim 30–40$ kpc) are needed in order to reduce the statistical uncertainties, but outer stellar halo kinematics clearly has the potential of becoming an important constraint on viable galaxy formation scenarios.

6. Acknowledgements

I have benefited from discussions with Tim Beers, Per Rex Christensen, Chris Flynn, Ken Freeman, Sergio Gelato, Steve Majewski, John Norris and Bernard Pagel.

References

Sommer-Larsen, J., Christensen, P. R.: 1989, *MNRAS* 239, 441