I

INTRODUCTION

Subject: Gravitational lensing observations - combined

Abstract: The observed gravitational lensing events are consistent with a model in which multiple massive objects are distributed in the foreground. These objects are likely to be dark matter halos, which are concentrated in the central regions of galaxies. The observed lensing events can be used to constrain the mass distribution of these objects and to infer the properties of the underlying dark matter.

Further reading: The paper in press describes these results in more detail.

REFERENCES


SOURCES

Gravitational lensing events on the surface density of subhalos.
In Cooray (1999a; hereafter C99), we studied the former case and suggested the possibility of constraining cosmological parameters based on observed statistics of lensed sources at 850 μm towards a sample of galaxy clusters. This approach is quite similar to the one taken in literature to constrain the present day value of the cosmological constant based on lensed source statistics, such as quasars (e.g., Kochanek 1996; Chiba & Yoshii 1997), radio sources (e.g., Falco et al. 1998; Cooray 1999b) and luminous optical arcs towards clusters (e.g., Bartelmann et al. 1998; C99). Blain (1997) studied gravitational lensing at submm wavelengths, including statistics of lensed submm sources due to foreground galaxies.

Here, we consider the possibility of constraining the background source redshift distribution based on known properties of a sample of submm sources gravitationally lensed by cluster potentials (S98) and an assumed cosmological model. To obtain information on the unlensed sources down to the same flux level, we use the source counts from Barger et al. (1998), Eales et al. (1998), Holland et al. (1998), Hughes et al. (1998) and Smail et al. (1997, 1998b). In § 2 we discuss our calculation and its inputs. In § 3 we present our resulting constraints on the redshift distribution of submm sources and discuss our constraints in the context of current studies on the submm sources and their contribution to the star formation history of the universe. We follow the conventions that the Hubble constant, $H_0$, is $100 h \, \text{km \, s}^{-1} \, \text{Mpc}^{-1}$, the present mean density in the universe in units of the closure density is $\Omega_m$, and the present normalized cosmological constant is $\Omega_{\Lambda}$. In a flat universe, $\Omega_m + \Omega_{\Lambda} = 1$.

2. GRAVITATIONAL LENSING CALCULATION

Our calculation follows C99 in which we calculated the expected number of luminous optical arcs, radio sources and submm galaxies towards galaxy clusters as a function of cosmology. Here, we prescribe the foreground cluster population to be similar to what was observed by S98 and model them as singular isothermal spheres (SIS) with velocity dispersion $\sigma$. In general, SIS models underestimate the number of lensed sources, when compared to complex cluster potentials with substructure (e.g., Bézecourt 1998). This leads to a systematically lower number of lensed sources than expected from true complex potentials and a higher upper limit on the redshift distribution of background sources.

In order to evaluate distances, we use the analytical filled-beam approximation (see, e.g., Fukugita et al. 1992) and calculate the probability, $p(z, \Omega_m, \Omega_{\Lambda})$, for a source at redshift $z$ to be strongly lensed given a set of cosmological parameters $\Omega_m$ and $\Omega_{\Lambda}$. Following C99 (see, also, Cooray, Quashnock & Miller 1999 and Holz, Miller & Quashnock 1999) the number of expected lensed sources, $\bar{N}$, towards the survey volume containing foreground lensing clusters is:

$$\bar{N} = \int F(z) p(z, \Omega_m, \Omega_{\Lambda}) B(f, z) dC(z)$$

$$\equiv \int \sigma(z, \Omega_m, \Omega_{\Lambda}) dC(z)$$

(1)

where $C(z)$ is the redshift distribution of submm sources such that $B(f, z)$ is the fraction of sources with redshifts less than $z$, $\sigma(z, \Omega_m, \Omega_{\Lambda}) dC(z)$ is the magnification bias for submm sources at redshift $z$ with observed flux density at 850 μm of $f$ (see, Kochanek 1991), and $F(z)$ is the effectiveness of clusters at redshifts $z_1$ in producing lensed sources. This nondimensional parameter can be written as (Turner, Ostriker & Gott 1984):

$$F(z_1) = 10 \pi^3 n(z_1) \left( \frac{c}{v} \right)^4 R(z_1)^4$$

(2)

Here, $R(z_1) = c / H_0$, and $n(z_1)$ is the number density of clusters with the velocity dispersion $\sigma$ at redshift $z_1$.

In general, detailed knowledge either on the luminosity function or the flux distribution is required to calculate the magnification bias. However, both these quantities are currently not known for the submm source sample. Instead of individual magnification biases, we use current estimates on the submm sources number counts to obtain an average value. If the number counts of unlensed sources, $n_{ul}$, with flux densities greater than $S_\nu$ towards a given area can be written as $n_{ul} \propto S^{-\alpha}$, then magnification due to gravitational lensing by an amplification $A$ modifies the counts as:

$$n_l \propto A^{-(1+\alpha)} S^{-\alpha}$$

(3)

where $n_l$ is the lensed source counts. The average magnification bias is simply the ratio of lensed to unlensed counts down to a flux density $S_\nu$:

$$\langle B \rangle = \frac{n_l}{n_{ul}} = A^{-(1+\alpha)}$$

(4)

Under the SIS scenario, the probability distribution for amplifications is $P(A) = 2/(A-1)^3$, and the minimum amplifications is $A_{\text{min}} = 2$. The average amplification for a sample of lensed sources is $3$. This average value is consistent with the distribution of amplifications for the submm sources based on detailed modeling of individual cluster potentials: $1.3^{+5.5}_{-0.8}$ (S98). Since none of the observed lensed sources are heavily amplified due to foreground potentials and that the amplification distribution is compatible with the SIS average, our use of SIS model to describe foreground clusters should not affect the results greatly.

Following Smail et al. (1998b), we parameterized submm source counts at 850 μm as:

$$N(> S) = \left( 7.7 \pm 0.9 \right) \times 10^{3} \left( \frac{S}{1 \text{mJy}} \right)^{-(1.1 \pm 0.2)}$$

(5)

where the uncertainties are the 1σ errors. The slope $\alpha = -(1.1 \pm 0.2)$, and thus, the average magnification bias, $\langle B(f, z) \rangle$, ranges from 0.9 to 1.4. This estimate for the magnification bias for 850 μm sources with flux densities in the range of 0.5 to 10 mJy is slightly lower than what was previously considered (e.g., Blain 1997). For the purpose of this calculation, where we are only interested in an upper limit to the redshift distribution, we apply the lowest possible amplification bias to all background sources. This leads to an underestimated lensing rate and an overestimated upper limit on the background source redshift distribution.

Since the redshift distribution of submm sources, $C(z)$, is unknown, we calculate the observed number of lensed sources as a function of $\langle z \rangle$, the effective average redshift under the assumption that all sources are at this redshift:

$$\bar{N} = \langle F(z) \rangle p(z, \Omega_m, \Omega_{\Lambda}) \langle B(f, z) \rangle$$

(6)
This approach is essentially similar to the one taken by Holz et al. (1999) to calculate an upper limit to the redshift distribution of gamma ray bursts based on observed lensing statistics. The assumption of an average redshift is utilized to parameterize our ignorance of the submm source redshift distribution (see, Holz et al. 1998 for a discussion).

The S98 sample contains observations of 7 clusters with an effective total area surveyed of 0.01 deg$^2$. This sample lies in redshifts between 0.19 and 0.4. The highest probability for foreground clusters to lens background sources occurs in the redshift range of 0.2 to 0.7, with some slight dependence on the cosmological parameters (see, e.g., Fig. 2 in Bartelmann et al. 1998). For the purpose of this calculation, we calculate an average $F$ parameter, $\langle F(z_i) \rangle$, based on the number density of clusters in this redshift range. Since the S98 cluster sample contains some of the well known massive clusters, we use a lower limit on the mass distribution determined by the observed velocity dispersions of these seven clusters, and assuming virial theorem for galaxy clusters (see, C99 for further details). The distribution of velocity dispersion for the 7 clusters observed by S98 has a mean of $1150 \pm 310$ km s$^{-1}$. The number density of clusters was calculated based on the Press-Schechter (PS; Press & Schechter 1974) theory with normalization based on the local cluster temperature function. Our PS calculation follows Bahcall & Fan (1998), with normalizations for $\sigma_8$ presented therein. We calculated number density of clusters with virial masses above $7.5 \times 10^{14} M_\odot$, corresponding to the observed velocity dispersion based on virial theorem, as a function of redshift and $\Omega_m$. In order to account for the uncertainty in $\langle F(z_i) \rangle$ due to measurement errors and our assumption of an average value, we allow for an overall uncertainty of 40%.

3. CONSTRAINTS ON THE REDSHIFT DISTRIBUTION

Using observational results from S98, we assume that number of lensed sources towards seven clusters is 10 down to a flux density limit of $\sim$ 6 mJy, thus the lensing rate per cluster down to 6 mJy is about 1.42, which is roughly a factor of 2 higher than the lensing rate for luminous optical arcs with amplifications greater than 10 down to a I band magnitude of 22 (C99; see, also Le Fèvre et al. 1994). Some of the currently presumed lensed submm sources are likely to be cluster member or foreground sources. Several such sources have already been detected within the current S98 sample which have been identified with cluster cD galaxies (Edge et al. 1998) and foreground spiral galaxies at low-redshifts (S98). However, for the purpose of this paper, where we are only interested in a statistical upper limit to average redshift, we can safely take 10 as the observed number of lensed sources. As before, the only effect of such an assumption is to increase the derived upper limit from the true value.

We also ignore biases in the S98 cluster sample and assume it as a random and a fair sample of clusters on the sky. The clusters imaged by S98 are some of the well known massive clusters, towards which the optical lensing rate is somewhat higher than the average value. This is primarily due to the fact that some of these clusters are found with substructures and bimodal mass distributions, producing enhanced potentials for gravitational lensing.

Here again, the systematic bias is such that the observed number of sources is an overestimate of the average number, and the derived upper limit on the background redshift is an overestimate from the true upper limit.

We calculated the expected number of lensed sources down to a flux density of 6 mJy at 850 $\mu$m as a function of redshift for different cosmological models. We vary the redshift of the background sources, assuming that all of them are at the same redshift, and calculate the expected number of lensed sources towards clusters on the whole sky. Based on cluster abundance from PS theory, we convert the number of lensed sources to an average value expected towards seven clusters, such as to mimic S98 observations. We then compare this number to the observed number, and following Cooray et al. (1999) we consider a Bayesian approach to calculate an upper limit to the average redshift of submm sources, under the assumption of a cosmological model. The likelihood $L$ — a function of $\tilde{\Omega}_m$, $\Omega_m$, and $\Omega_\Lambda$ — is the probability of the data, given $\langle z \rangle$, $\Omega_m$, and $\Omega_\Lambda$. The likelihood for n observed sources when $\tilde{N}$ is expected towards seven clusters is given by:

$$\langle L(n) \rangle = \prod_{i=0}^{n} \frac{\Gamma(n + 1)}{\Gamma(i + 1)\Gamma(n - i + 1)} \times \left(1 + \frac{\sigma^2}{\sigma_F^2} \right)^{-n} \tilde{N} \left(\langle z_i \rangle + \frac{n - 1}{\langle z_i \rangle} \right)$$

Here, $n$ is the observed number while $\tilde{N}(\langle z \rangle)$ is the expected number of lensed sources when background sources are at $\langle z \rangle$ when $\Omega_m$ and $\Omega_\Lambda$ is given. We have also taken into account the uncertainty in $\langle F(z_i) \rangle$ by defining $\sigma_F$ to be 0.4, allowing for a 40% uncertainty. This factor is an overall correction to the expected lensing rate, due to a systematic uncertainty in $\langle F(z_i) \rangle$.

In Table 1, we list the derived 68% and 95% upper limits on the redshift distribution of submm sources to produce the current observed statistics. If $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, as suggested by various cosmological probes, including a combined analysis of the Type Ia supernovae at high redshifts (e.g., Riess et al. 1998) and cosmic microwave background anisotropy data (e.g., White 1998; Lineweaver 1998), galaxy cluster evolution (e.g., Bahcall & Fan 1998) and baryonic fraction in galaxy clusters (e.g., Evrard 1997), then the 68% upper limit on the redshift distribution is 3.1. If $\Omega_\Lambda = 0.0$ and the universe is open with $\Omega_m$ of 0.3, then this upper limit increases to 5.2.

Our upper limits on the average redshift can be directly compared to redshift distribution derived by Blair et al. (1998) based on the modelled starformation history using submm and far-infrared number counts and background radiation measurements. The authors derived that 90% of the sources at 850 $\mu$m will lie below the redshift range of 3.8 to 8.2, with median redshift in the range of 2.4 to 4.4. Our average upper limit $\langle z \rangle$ of 3.1 for $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is in agreement with such a distribution. Blair et al. (1998) calculation on the redshift distribution assumes a cosmology of $\Omega_m = 1.0$ and $\Omega_\Lambda = 0.0$. This is the same cosmology for which we have the weakest upper limit with $\langle z \rangle$ of 8.0 at the 68% confidence. The derived redshift limit here also agrees with suggested redshift ranges by S98 using individual colors of plausible identifications.

4. UNCERTAINTIES AND SYSTEMATIC EFFECTS
We have assumed that the S98 sample is a fair sample of galaxy clusters, and have considered it to obtain an average number of observed lensed sources due to foreground galaxy clusters. This assumption is likely to be false given biases and systematic effects in the cluster sample selection. Our treatment of pointed cluster observations as a series of random untargeted observations is likely to create an additional systematic bias, but such a bias is not expected to underestimate the current upper limit. We have also considered a low value for the magnification bias such that the upper limit on background source redshift is overestimated. If, for example, the true magnification bias is 1.4, then the upper limit on \( \langle z \rangle \) decreases to 2.6 from 3.1 in a cosmology of \( \Omega_m = 0.3 \) and \( \Omega \Lambda = 0.7 \).

Other uncertainties include the determination of cluster abundances given systematic and statistical uncertainties involved with the PS calculation, resulting from errors due to \( \sigma_s \) etc. We have tried to compensate for such errors by considering a 40% statistical uncertainty in the derivation of \( \langle F(z) \rangle \). In general, it is likely that we have overestimated the upper limit, since most of the systematic effects tend to bias our results such that we underestimate the expected lensing rate.

5. SUMMARY AND CONCLUSIONS

We have derived upper limits on the redshift distribution of submm sources by comparing statistics of lensed sources towards a sample of galaxy clusters to unlensed sources. Our derived limits depends on cosmology, and if \( \Omega_m = 0.3 \) and \( \Omega \Lambda = 0.7 \), as currently suggested by various cosmological probes, at the 68% level the average redshift of submm sources is less than 3.1. Such an upper limit is consistent with the redshift distribution predicted for submm sources based on starformation models, where starformation history remains constant beyond a redshift of 1.5, using observed far-infrared and submm background radiations. The derived upper limit on the average redshift is also consistent with suggested redshift ranges based on colors of plausible optical identifications for submm sources detected towards cluster potentials.

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REFERENCES


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Table 1

Upper Limit on the Average Redshift \( \langle z \rangle \) for Different Cosmologies.