Measurement of the Proton and Deuteron Spin Structure Functions g3 and g2

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Abstract

We have measured the spin structure functions $g_2^p$ and $g_2^d$ and the virtual photon asymmetries $A_2^p$ and $A_2^d$ over the kinematic range $0.02 \leq x \leq 0.8$ and $1.0 \leq Q^2 \leq 30$ (GeV/c)$^2$ by scattering 38.8 GeV longitudinally polarized electrons from transversely polarized NH$_3$ and $^6$LiD targets. The absolute value of $A_2$ is significantly smaller than the $\sqrt{x}$ positivity limit over the measured range, while $g_2$ is consistent with the twist-2 Wandzura-Wilczek calculation. We obtain results for the twist-3 reduced matrix elements $d_0^p$, $d_0^d$ and $d_0^n$. The Burkhardt-Cottingham sum rule integral $\int g_2(x)dx$ is reported for the range $0.02 \leq x \leq 0.8$.

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The deep inelastic spin structure functions of the nucleons, \(g_1\) and \(g_2\), depend on the spin distribution of the partons and their correlations. The function \(g_1\) can be primarily understood in terms of the quark parton model (QPM) and perturbative QCD with higher twist terms at low \(Q^2\). There exists no such picture for \(g_2\). Feynman[1] claimed that the transverse structure function \(g_T = g_1 + g_2\) had a simple parton interpretation in terms of the transverse polarization of the quark spins which is proportional to quark masses. However, \(g_2\) is sensitive to higher twist effects such as quark-gluon correlations\([2]\) and is not easily interpreted in pQCD where such effects are not included. By interpreting \(g_2\) using the operator product expansion (OPE) \([2, 3]\), it is possible to study contributions to the nucleon spin structure beyond the simple QPM. The virtual photon-nucleon asymmetry \(A_2\) is proportional to \(g_T/F_2\) where \(F_2\) is the unpolarized structure function.

The structure function \(g_2\) can be written \([4]\):

\[
g_2(x, Q^2) = g_2^\text{WW}(x, Q^2) + g_2(x, Q^2)
\]

where

\[
g_2^\text{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy,
\]

\[
g_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y} \left( \frac{m^2}{M^2} h_T(y, Q^2) + \xi(y, Q^2) \right) \frac{dy}{y}
\]

where \(x\) is the Bjorken scaling variable and \(Q^2\) is the absolute value of the virtual photon four-momentum squared. The twist-2 term \(g_2^\text{WW}\) was derived by Wandzura and Wilczek \([5]\) and depends only on the well-measured \(g_1\) \([6, 7, 8, 9, 10, 11]\). The function \(h_T(x, Q^2)\) is an additional twist-2 contribution \([12, 4]\) that depends on the transverse polarization density in the nucleon. The \(h_T\) contribution to \(g_2\), is suppressed by the ratio of the quark-to-nucleon mass \([12]\) and is thus small for up and down quarks and will be neglected in this analysis \([13]\). The twist-3 part, \(\xi\), comes from quark-gluon correlations and is the main focus of our study.

The OPE allows us to write the hadronic matrix element in deep inelastic scattering in terms of a series of renormalized operators of increasing twist \([2, 3]\). The moments of \(g_1\) and \(g_2\) at fixed \(Q^2\) can be related to the twist-2 and twist-3 reduced matrix elements, \(a_n\) and \(d_n\), and higher twist terms which are suppressed by powers of \(1/Q\). Neglecting quark mass terms:

\[
\int_0^1 x^n g_1(x, Q^2) dx = \frac{a_n}{2} + O(M^2/Q^2), \quad n = 0, 2, 4, ...
\]

\[
\int_0^1 x^n g_2(x, Q^2) dx = \frac{n}{n + 1} \left( \frac{d_n - a_n}{2} \right) + O(M^2/Q^2), \quad n = 2, 4, ...
\]

In these integrals the contribution of \(d_n\) is not suppressed relative to the twist-2 contribution and thus can be easily extracted. Neglecting \((1/Q)\) terms, the \(d_n\) matrix elements can be written as:

\[
d_n = \frac{2(n + 1)}{n} \int_0^1 x^n g_2(x, Q^2) dx
\]

3
and thus measures deviations of $g_2$ from the twist-2 $g_2^{WW}$ term.

The Burkhardt-Cottingham sum rule\cite{14} for $g_2$ at large $Q^2$,

$$\int_0^1 g_2(x) \, dx = 0 \label{eq:4},$$

was derived from virtual Compton scattering dispersion relations. It does not follow from the OPE since the $n = 0$ sum rule is not defined for $g_2$. Its validity depends on the lack of singularities for $g_2$ at $x = 0$. The Efremov-Leader-Teryaev (ELT) sum rule\cite{15} involves the valence quark contributions to $g_1$ and $g_2$:

$$\int_0^1 x g_1'(x) + 2 g_2'(x) \, dx = 0 \label{eq:5}.$$  

Assuming that the sea quarks are the same in protons and neutrons, the sum rule takes a form $\int_0^1 x g_1'(x) - 2 g_2'(x) = 0$ that we can apply to our data.

Measurements of $g_2$ and $A_2$ exist for the proton and deuteron \cite{7, 16, 17, 18}, as well as for the neutron \cite{8, 19}. In this Letter, we report new measurements of $g_2$ and $A_2$ for the proton and deuteron made during experiment E155 at SLAC.

A 38.80 GeV, 120 Hz electron beam with a longitudinal polarization of $(81.3 \pm 2.0)\%$ struck transversely polarized NH$_3$ or LiD\cite{20} targets. The beam helicity direction was randomly chosen pulse-by-pulse. Scattered electrons were detected in three independent spectrometers centered at 2.75°, 5.5°, and 10.5°. The two small angle spectrometers were the same as in SLAC E154 \cite{9}, while the large angle spectrometer was new for this experiment. It consisted of a single dipole magnet and two quadrupoles, and covered a momentum range from 7 to 20 GeV, and scattering angle range from 9.6° to 12.5° with a maximum solid angle of 1.5 mrad at 8 GeV. Electrons were separated from a much larger flux of pions using a gas Čerenkov counter and a segmented electromagnetic calorimeter. Further information on the technique can be found in references \cite{7, 9, 10}.

The measured counting rate asymmetries from the two beam helicities were corrected for beam polarization, target polarization, tracking efficiencies, pion and charge symmetric backgrounds, and radiative effects. Uncertainties in the radiative corrections were estimated by varying the input models over a range consistent with the measured data. The deuteron data were extracted from the LiD results by applying a correction for both the lithium and deuterium nuclear wave functions with $^6\text{Li} \sim \alpha + d$\cite{20}. The structure function $g_1(x, Q^2)$ and the virtual photon absorption asymmetry $A_2(x, Q^2)$ are usually determined from the two measurable asymmetries, $A_\perp(E, x, Q^2)$ (dominant contribution) and $A_\parallel(E, x, Q^2)$ (small contribution), corresponding to transverse and longitudinal target polarization with respect to the incoming electron beam helicity. Because in this experiment these asymmetries were measured at two different beam energies (38.8 and 48.3 GeV respectively), we instead chose to determine $g_2$ and $A_2(x, Q^2)$ from $A_\perp$ (dominant contribution) and $A_1$ (small contribution) using:

$$g_2(x, Q^2) = \frac{F_2(x, Q^2)}{2x \gamma (1 + R(x, Q^2))} \left[ A_\perp(E, x, Q^2)/d + A_1(x, Q^2)(\zeta - \gamma) \right] \label{eq:6}$$

$$A_2(x, Q^2) = A_\perp(E, x, Q^2)/d + \zeta A_1(x, Q^2) \label{eq:7}.$$
where $\zeta = \eta(1+\epsilon)/(2\epsilon)$, $\eta = \epsilon \sqrt{Q^2}/(E - E')$, $E$ and $E'$ are the incident and scattered electron energies, $\gamma = 2Mx/\sqrt{Q^2}$, $d = (1 + E'/E)\sqrt{2\epsilon(1+\epsilon)/(1+\epsilon R)}$, and $\epsilon^{-1} = 1 + 2[1 + \gamma^2] \tan^2(\theta/2)$. We used a new $Q^2$-dependent parameterization of $A_1$ using existing data [6, 7] and data from this experiment[10]. The NMC fit to $F_2(x, Q^2)[21]$ and the new SLAC fit to $\bar{R}(x, Q^2) = \sigma_L/\sigma_T$ [22] were used.

Results for $A_2$ and $xg_2$ for the three spectrometers are given in Table 1 with statistical errors. The systematic errors were negligible by comparison. The data cover the kinematic range $0.02 \leq x \leq 0.8$ and $1.0 \leq Q^2 \leq 30$ (GeV/c)$^2$ with an average $Q^2$ of 5 (GeV/c)$^2$. Figure 1 shows the values of $A_2$ as a function of $Q^2$ for several values of $x$ along with results from E143[7]. There is no evidence of a $Q^2$ dependence for $A_2$ or $xg_2$ (not shown) within the experimental errors so the data from all spectrometers were averaged. These averaged results for $A_2$ and $xg_2$ are shown at the bottom of Table 1 and $A_2$ is presented in Fig. 2 along with the $\sqrt{R}$ positivity limit and data from previous experiments. The data are in good agreement with the previous measurements and improve the accuracy for the deuteron. The combined results are significantly smaller than the positivity limit over most of the measured range. $A_2$ is consistent with $A_2^{WW}$ calculated from $g_2^{WW}$ and $A_2^{\pi}$ is significantly larger than zero around $x \sim 0.2$. Results for $xg_2$ are shown in Fig. 3 along with the twist-2 component, $xg_2^{WW}$ calculated using our new parameterization of the $A_1$ data. The combined SLAC data agrees with $g_2^{WW}$ with a $\chi^2/$(dof) of 1.3 and 0.9 for proton and deuteron respectively for 17 degrees of freedom. The comparison with $g_2 = 0$ has similar agreement with $\chi^2/$(dof) of 1.5 and 0.9 respectively. Also shown is the bag model calculation of Stratmann[23]. A recent Chiral Soliton Model calculation[24] (not shown) also agrees with the data.

We used Eq. 3 to calculate the matrix elements $d_n$ assuming that $\bar{g}_2$ is independent of $Q^2$ in the measured region. This is not unreasonable since $d_n$ is supposed to depend logarithmically on $Q^2[2]$. The part of the integral for $x$ below the measured region was assumed to be zero because of the $x^n$ suppression. For $x \geq 0.8$ we used $\bar{g}_2 \propto (1 - x)^m$ where $m=2$ or 3, normalized to the data for $x \geq 0.5$. Because $\bar{g}_2$ is small at high $x$, the contribution was negligible for both cases. We obtain values of $d_2^p = 0.005 \pm 0.008$ and $d_2^n = 0.008 \pm 0.005$ at an average $Q^2$ of 5 (GeV/c)$^2$. We combined these results with those from SLAC experiments on the neutron (E142[8] and E154[19]) and proton and deuteron (E143[7]) and obtained average values $d_2^p = 0.007 \pm 0.004$ and $d_2^n = 0.004 \pm 0.010$.

Figure 4 shows the experimental values of $d_2$ for proton and neutron with their error, plotted along with theoretical calculations using Bag Models (Song[12], Stratmann[23], and Ji[25]); QCD sum rules ( Stein[26], BBK[27], Ehrnberger[28]); and Lattice QCD[29]. The results are compatible with all the models within the still large errors except for the proton lattice calculation.

We evaluated the Burkhardt-Cottingham integral (Eq. 4) in the measured region of $0.02 \leq x \leq 0.8$ at $Q^2 = 5$(GeV/c)$^2$ by assuming that $\bar{g}_2$ is independent of $Q^2$ and thus that all the $Q^2$ dependence of $g_2$ is in $g_2^{WW}$. The results for the proton and deuteron are -0.022 ± 0.071 and 0.023 ± 0.044 respectively. Averaging with the E143 results which cover a slightly more restrictive $x$ range gives -0.015 ± 0.029 and 0.010 ± 0.039. All of these integrals are consistent with the Burkhardt-Cottingham sum rule prediction of zero. However, this does not represent a conclusive test of the sum rule because the behavior of $g_2$ as $x \to 0$ is not
known. We evaluated the ELT integral, Eq. 5, using our data in the measured region. The result at $Q^2 = 5\text{ (GeV}/c)^2$ is $-0.015 \pm 0.036$, which is consistent with the expected value of zero. Including the E143 $g_2$ data [7] improves the accuracy to $0.003 \pm 0.022$. Again the extrapolation to $x=0$ is not known, but in this case the contribution is suppressed by a factor of $x$.

In summary, we have presented a new measurement of $A_2$ and $g_2$ for the proton and deuteron in the kinematic range $0.02 \leq x \leq 0.8$ and $1.0 \leq Q^2 \leq 30 \text{ (GeV}/c)^2$. Our results for $A_2$ are significantly smaller than the $\sqrt{R}$ positivity limit over most of the measured range and data for $g_2$ are consistent with the twist-2 $g_2^{WW}$ prediction. The values obtained for the twist-3 matrix element $d_2$ from this measurement and the SLAC average are also consistent with zero. Future measurements at SLAC and Jefferson National Laboratory will significantly reduce the errors and enable us to make more conclusive statements about the higher twist content of the nucleon.

We wish to thank the personnel of the SLAC accelerator department for their efforts which resulted in the successful completion of the E155 experiment. We would also like to thank J. Ralston for useful discussions and guidance. This work was supported by the Department of Energy; by the National Science Foundation; by the Kent State University Research Council (GCP); and by the Centre National de la Recherche Scientifique and the Commissariat a l’Energie Atomique (French groups).

References

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Table 1: Results for $A_2$ and $xg_2$ with statistical errors for proton and deuteron at the measured $x$ and $Q^2$ [(GeV/c)^2] for the three spectrometers with E=38.8 GeV.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$(&lt; Q^2 &gt;)$</th>
<th>$A_2^p$</th>
<th>$xg_2^p$</th>
<th>$A_2^d$</th>
<th>$xg_2^d$</th>
</tr>
</thead>
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<tr>
<td>0.022</td>
<td>1.15</td>
<td>$0.149\pm0.111$</td>
<td>$0.439\pm0.335$</td>
<td>$-0.036\pm0.074$</td>
<td>$-0.103\pm0.212$</td>
</tr>
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<td>0.026</td>
<td>1.32</td>
<td>$-0.020\pm0.032$</td>
<td>$-0.069\pm0.088$</td>
<td>$-0.023\pm0.021$</td>
<td>$0.060\pm0.056$</td>
</tr>
<tr>
<td>0.039</td>
<td>1.56</td>
<td>$-0.034\pm0.025$</td>
<td>$-0.090\pm0.057$</td>
<td>$0.023\pm0.017$</td>
<td>$0.043\pm0.035$</td>
</tr>
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<td>1.94</td>
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<td>$0.012\pm0.021$</td>
<td>$0.013\pm0.034$</td>
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<td>0.099</td>
<td>2.34</td>
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<td>$-0.020\pm0.054$</td>
<td>$0.040\pm0.031$</td>
<td>$0.041\pm0.034$</td>
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$\theta \approx 5.5^\circ$

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$\theta \approx 10.5^\circ$

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AVERAGE

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Figure 1: $A_2$ for the proton and deuteron as a function of $Q^2$ for selected values of $x$. Data are for this experiment (solid) and E143[7](open). The errors are statistical; the systematic errors are negligible. The Bag Model calculation of Stratmann[23] is also shown.
Figure 2: The asymmetries $A_2$ for proton and deuteron for this experiment (E155) with data from all spectrometers averaged (Table 1). The errors are statistical; the systematic errors are negligible. Also shown are the data from SLAC E143 \cite{7} and SMC\cite{6}. Our $A_2^{WW}$ calculation is shown as the solid line and the $\sqrt{R}$ positivity limit is shown as the dotted curve, evaluated at the average $Q^2$ for this experiment at each $x$. 
Figure 3: The structure function $xg_2$ for all spectrometers combined and data from E143[7]. The errors are statistical; the systematic errors are negligible. Also shown is our twist-2 $g_2^{WW}$ at the average $Q^2$ of this experiment at each value of $x$ and the calculations of Stratmann [23] and Song [12].
Figure 4: The twist-3 matrix element $d_2$ for the proton and neutron from the combined data from SLAC experiments E142 [8], E143 [7], E154 [19], and E155 (Data). Also shown are theoretical models from left to right: Bag Models [12, 23, 25], QCD Sum Rules [26, 27, 28], lattice QCD [29], and Chiral Soliton Model [24]. The shaded region indicates the experimental errors.