Gravitation, Thermodynamics, and Quantum Theory

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Abstract

During the past 30 years, research in general relativity has brought to light strong hints of a very deep and fundamental relationship between gravitation, thermodynamics, and quantum theory. The most striking indication of such a relationship comes from black hole thermodynamics, where it appears that certain laws of black hole mechanics are, in fact, simply the ordinary laws of thermodynamics applied to a system containing a black hole. This article will review the present status of black hole thermodynamics and will discuss some of the related unresolved issues concerning gravitation, thermodynamics, and quantum theory.
1 Introduction

At the end of a century—particularly one marking the end of a millennium—it is natural to attempt to take stock of the status of a field of endeavor from as broad a perspective as possible. In the field of physics, two theories—general relativity and quantum mechanics—were developed during the first quarter of the present century. These theories revolutionized the way we think about the physical world. Despite enormous progress during the remainder of this century in exploring their consequences and in the application of these theories to construct successful “standard models” of cosmology and particle physics, at the end of this century we are still struggling to reconcile general relativity and quantum theory at a fundamental level.

The revolutions in physics introduced by general relativity and quantum theory were accompanied by major changes in the language and concepts used to describe the physical world. In general relativity, it is recognized that space and time meld into a single entity—spacetime—and that the structure of spacetime is described by a Lorentz metric, which has a dynamical character. Consequently, simple Newtonian statements such as “two particles are a distance $d$ apart at time $t$” become essentially meaningless in general relativity until one states exactly how $t$ and $d$ are defined for the particular class of spacetime metrics under consideration. Furthermore, concepts such as the “local energy density of the gravitational field” are absent in general relativity. The situation is considerably more radical in quantum theory, where the existence of “objective reality” itself is denied, i.e., observables cannot, in general, consistently be assigned definite values.

I believe that the proper description of quantum phenomena in strong gravitational fields will necessitate revolutionary conceptual changes in our view of the physical world at least comparable to those that occurred in the separate developments of general relativity and quantum theory. At present, the greatest insights into the physical nature of quantum phenomena in strong gravitational fields comes from the analysis of thermodynamic properties associated with black holes. This analysis also provides strong hints that statistical physics may be deeply involved in any fundamental conceptual changes that accompany a proper description of quantum gravitational phenomena.

At the present time, string theory is the most promising candidate for a theory of quantum gravity. One of the greatest successes of string theory to
date has been the calculation of the entropy of certain classes of black holes. However, the formulation of string theory is geared much more towards the calculation of scattering matrices in asymptotically flat spacetimes rather than towards providing a local description of physical phenomena in strong gravitational fields. Within the framework of string theory, it is difficult to imagine even how to pose (no less, how to calculate the answer to) local physical questions like, “What would an observer who falls into a black hole experience as he approaches what corresponds classically to the space-time singularity within the black hole?” Thus, string theory has not yet provided us with new conceptual insights into the physical nature of phenomena occurring in strong gravitational fields that are commensurate with some of its mathematical successes. It may well be that—even assuming it is a correct theory of nature—string theory will bear a relationship to the “ultimate theory of everything” that is similar to the relationship between “old quantum theory” and quantum theory. Therefore, I feel that it is very encouraging that, at the present time, intensive efforts are underway toward providing reformulations of string theory. However, to date, these efforts have mainly been concerned with obtaining a formulation that would unify the (different looking) versions of string theory, rather than achieving new conceptual foundations for describing local quantum phenomena occurring in strong gravitational fields.

Thus, at present, most of the physical insights into quantum phenomena occurring in strong gravitational fields arise from classical and semiclassical analyses of black holes in general relativity. In this article, I will review classical and quantum black hole thermodynamics and then discuss some unresolved issues and puzzles, which may provide some hints as to the new conceptual features that may be present in the quantum description of strong gravitational fields. In the discussion, I will not attempt to provide a balanced account of research in this area, but rather will merely present my own views.

2 Classical black hole mechanics

Undoubtedly, one of the most remarkable and unexpected developments in theoretical physics to have occurred during the latter portion of this century was the discovery of a close relationship between certain laws of black hole physics and the ordinary laws of thermodynamics. It was first pointed out
by Bekenstein [1] that the area nondecrease theorem of classical general relativity [2] is analogous to the ordinary second law of thermodynamics, and he proposed that the area, $A$, of a black hole (times a constant of order unity in Planck units) should be interpreted as its physical entropy. A short time later, Bardeen, Carter, and Hawking [3] extended the analogy between black holes and thermodynamics considerably further by proving black hole analogs of the zeroth and first laws of thermodynamics. In this section, I will give a brief review of the laws of classical black hole mechanics.

First, we review the definition of a black hole and some properties of stationary black holes. In physical terms, a black hole is a region where gravity is so strong that nothing can escape. In order to make this notion precise, one must have in mind a region of spacetime to which one can contemplate escaping. For an asymptotically flat spacetime $(M, g_{ab})$ (representing an isolated system), the asymptotic portion of the spacetime “near infinity” is such a region. The black hole region, $B$, of an asymptotically flat spacetime, $(M, g_{ab})$, is defined as

$$B = M - I^-(I^+)$$

where $I^+$ denotes future null infinity and $I^-$ denotes the chronological past. The event horizon, $H$, of a black hole is defined to be the boundary of $B$. In particular, $H$ is a null hypersurface. Note that the entire future history of the spacetime must be known before the location of $H$ can be determined, i.e., $H$ possesses no distinguished local significance.

If an asymptotically flat spacetime $(M, g_{ab})$ contains a black hole, $B$, then $B$ is said to be stationary if there exists a one-parameter group of isometries on $(M, g_{ab})$ generated by a Killing field $t^a$ which is unit timelike at infinity. The black hole is said to be static if it is stationary and if, in addition, $t^a$ is hypersurface orthogonal. The black hole is said to be axisymmetric if there exists a one parameter group of isometries which correspond to rotations at infinity. A stationary, axisymmetric black hole is said to possess the “$t - \phi$ orthogonality property” if the 2-planes spanned by $t^a$ and the rotational Killing field $\phi^a$ are orthogonal to a family of 2-dimensional surfaces.

A null surface, $K$, whose null generators coincide with the orbits of a one-parameter group of isometries (so that there is a Killing field $\xi^a$ normal to $K$) is called a Killing horizon. There are two independent results (usually referred to as “rigidity theorems”) that show that in wide variety of cases of interest, the event horizon, $H$, of a stationary black hole must be a Killing
horizon. The first, due to Carter [4], states that for a static black hole, the static Killing field \( t^a \) must be normal to the horizon, whereas for a stationary-axisymmetric black hole with the \( t - \phi \) orthogonality property there exists a Killing field \( \xi^a \) of the form

\[
\xi^a = t^a + \Omega \phi^a
\]  

which is normal to the event horizon. The constant \( \Omega \) defined by eq.(2) is called the angular velocity of the horizon. Carter’s result does not rely on any field equations, but leaves open the possibility that there could exist stationary black holes without the above symmetries whose event horizons are not Killing horizons. The second result, due to Hawking [5] (see also [6]), directly proves that in vacuum or electrovac general relativity, the event horizon of any stationary black hole must be a Killing horizon. Consequently, if \( t^a \) fails to be normal to the horizon, then there must exist an additional Killing field, \( \xi^a \), which is normal to the horizon, i.e., a stationary black hole must be nonrotating (from which staticity follows [7], [8]) or axisymmetric (though not necessarily with the \( t - \phi \) orthogonality property). Note that Hawking’s theorem makes no assumptions of symmetries beyond stationarity, but it does rely on the properties of the field equations of general relativity.

Now, let \( \mathcal{K} \) be any Killing horizon (not necessarily required to be the event horizon, \( \mathcal{H} \), of a black hole), with normal Killing field \( \xi^a \). Since \( \nabla^a (\xi^b \xi_b) \) also is normal to \( \mathcal{K} \), these vectors must be proportional at every point on \( \mathcal{K} \). Hence, there exists a function, \( \kappa \), on \( \mathcal{K} \), known as the surface gravity of \( \mathcal{K} \), which is defined by the equation

\[
\nabla^a (\xi^b \xi_b) = -2 \kappa \xi^a
\]  

It follows immediately that \( \kappa \) must be constant along each null geodesic generator of \( \mathcal{K} \), but, in general, \( \kappa \) can vary from generator to generator. It is not difficult to show (see, e.g., [9]) that

\[
\kappa = \lim (V a)
\]  

where \( a \) is the magnitude of the acceleration of the orbits of \( \xi^a \) in the region off of \( \mathcal{K} \) where they are timelike, \( V \equiv (-\xi^a \xi_a)^{1/2} \) is the “redshift factor” of \( \xi^a \), and the limit as one approaches \( \mathcal{K} \) is taken. Equation (4) motivates the terminology “surface gravity”. Note that the surface gravity of a black hole is defined only when it is “in equilibrium”, i.e., stationary, so that its event horizon is a Killing horizon.
In parallel with the two independent “rigidity theorems” mentioned above, there are two independent versions of the zeroth law of black hole mechanics. The first, due to Carter [4] (see also [10]), states that for any black hole which is static or is stationary-axisymmetric with the \( t-\phi \) orthogonality property, the surface gravity \( \kappa \), must be constant over its event horizon \( \mathcal{H} \). This result is purely geometrical, i.e., it involves no use of any field equations. The second, due to Bardeen, Carter, and Hawking [3] states that if Einstein’s equation holds with the matter stress-energy tensor satisfying the dominant energy condition, then \( \kappa \) must be constant on any Killing horizon. Thus, in the second version of the zeroth law, the hypothesis that the \( t-\phi \) orthogonality property holds is eliminated, but use is made of the field equations of general relativity.

A bifurcate Killing horizon is a pair of null surfaces, \( \mathcal{K}_A \) and \( \mathcal{K}_B \), which intersect on a spacelike 2-surface, \( \mathcal{C} \) (called the “bifurcation surface”), such that \( \mathcal{K}_A \) and \( \mathcal{K}_B \) are each Killing horizons with respect to the same Killing field \( \xi^a \). It follows that \( \xi^a \) must vanish on \( \mathcal{C} \); conversely, if a Killing field, \( \xi^a \), vanishes on a two-dimensional spacelike surface, \( \mathcal{C} \), then \( \mathcal{C} \) will be the bifurcation surface of a bifurcate Killing horizon associated with \( \xi^a \) (see [11] for further discussion). An important consequence of the zeroth law is that if \( \kappa \neq 0 \), then in the “maximally extended” spacetime representing a stationary black hole, the event horizon, \( \mathcal{H} \), comprises a branch of a bifurcate Killing horizon [10]. This result is purely geometrical—involving no use of any field equations. As a consequence, the study of stationary black holes which satisfy the zeroth law divides into two cases: “degenerate” black holes (for which, by definition, \( \kappa = 0 \)), and black holes with bifurcate horizons.

The first law of black hole mechanics is simply an identity relating the changes in mass, \( M \), angular momentum, \( J \), and horizon area, \( A \), of a stationary black hole when it is perturbed. To first order, the variations of these quantities in the vacuum case always satisfy

\[
\delta M = \frac{1}{8\pi} \kappa \delta A + \Omega \delta J.
\]

In the original derivation of this law [3], it was required that the perturbation be stationary. Furthermore, the original derivation made use of the detailed form of Einstein’s equation. Subsequently, the derivation has been generalized to hold for non-stationary perturbations [7], [12], provided that the change in area is evaluated at the bifurcation surface, \( \mathcal{C} \), of the unper-
turbed black hole. More significantly, it has been shown [12] that the validity of this law depends only on very general properties of the field equations. Specifically, a version of this law holds for any field equations derived from a diffeomorphism covariant Lagrangian, $L$. Such a Lagrangian can always be written in the form

$$L = L(g_{ab}, R_{abcd}, \nabla_a R_{bced}, \ldots; \psi, \nabla_a \psi, \ldots)$$

(6)

where $\nabla_a$ denotes the derivative operator associated with $g_{ab}$, $R_{abcd}$ denotes the Riemann curvature tensor of $g_{ab}$, and $\psi$ denotes the collection of all matter fields of the theory (with indices suppressed). An arbitrary (but finite) number of derivatives of $R_{abcd}$ and $\psi$ are permitted to appear in $L$. In this more general context, the first law of black hole mechanics is seen to be a direct consequence of an identity holding for the variation of the Noether current. The general form of the first law takes the form

$$\delta M = \frac{\kappa}{2\pi} \delta S_{\text{bh}} + \Omega \delta J + \ldots,$$

(7)

where the “...” denote possible additional contributions from long range matter fields, and where

$$S_{\text{bh}} \equiv -2\pi \int_C \frac{\delta L}{\delta R_{abcd}} n_{ab} n_{cd}.$$  

(8)

Here $n_{ab}$ is the binormal to the bifurcation surface $C$ (normalized so that $n_{ab}n^{ab} = -2$), and the functional derivative is taken by formally viewing the Riemann tensor as a field which is independent of the metric in eq. (6). For the case of vacuum general relativity, where $L = R\sqrt{-g}$, a simple calculation yields

$$S_{\text{bh}} = A/4$$

(9)

and eq.(7) reduces to eq.(5).

As already mentioned at the beginning of this section, the black hole analog of the second law of thermodynamics is the area theorem [2]. This theorem states that if Einstein’s equation holds with matter satisfying the null energy condition (i.e., $T_{ab}k^a k^b \geq 0$ for all null $k^a$), then the surface area, $A$, of the event horizon of a black hole can never decrease with time. In the context of more general theories of gravity, the nondecrease of $S_{\text{bh}}$ also has been shown to hold in a class of higher derivative gravity theories, where
the Lagrangian is a polynomial in the scalar curvature [13], but, unlike the
zeroth and first laws, no general argument for the validity of the second law
of black hole mechanics is known. However, there are some hints that the
nondecrease of $S_{bh}$ may hold in a very general class of theories of gravity
with positive energy properties [14].

Taken together, the zeroth, first, and second laws\(^1\) of black hole mechanics
in general relativity are remarkable mathematical analogs of the correspond-
ing laws in ordinary thermodynamics. It is true that the nature of the proofs
of the laws of black hole mechanics in classical general relativity is strikingly
different from the nature of the arguments normally advanced for the validity
of the ordinary laws of thermodynamics. Nevertheless, as discussed above,
the validity of the laws of black hole mechanics appears to rest upon general
features of the theory (such as general covariance) rather than the detailed
form of Einstein’s equation, in a manner similar to the way the validity of
the ordinary laws of thermodynamics depends only on very general features
of classical and quantum dynamics.

In comparing the laws of black hole mechanics in classical general relativity
with the laws of thermodynamics, the role of energy, $E$, is played by the
mass, $M$, of the black hole; the role of temperature, $T$, is played by a constant
times the surface gravity, $\kappa$, of the black hole; and the role of entropy, $S$, is
played by a constant times the area, $A$, of the black hole. The fact that $E$ and $M$
represent the same physical quantity provides a strong hint that the
mathematical analogy between the laws of black hole mechanics and the laws
of thermodynamics might be of physical significance. However, in classical
general relativity, the physical temperature of a black hole is absolute zero,
so there can be no physical relationship between $T$ and $\kappa$. Consequently, it
also would be inconsistent to assume a physical relationship between $S$ and $A$. As we shall now see, this situation changes dramatically when quantum

\(^1\)It should be noted that I have made no mention of the third law of thermodynamics,
i.e., the “Planck-Nernst theorem”, which states that $S \to 0$ (or a “universal constant”) as
$T \to 0$. The analog of this law fails in black hole mechanics, since there exist “extremal”
black holes of finite $A$ which have $\kappa = 0$. However, I believe that the the “Planck-Nernst
theorem” should not be viewed as a fundamental law of thermodynamics but rather as
a property of the density of states near the ground state in the thermodynamic limit,
which is valid for commonly studied materials. Indeed, examples can be given of ordinary
quantum systems that violate the “Planck-Nernst theorem” in a manner very similar to
the violations of the analog of this law that occur for black holes [15].
effects are taken into account.

3 Quantum black hole thermodynamics

The physical temperature of a black hole is not absolute zero. As a result of quantum particle creation effects [16], a black hole radiates to infinity all species of particles with a perfect black body spectrum, at temperature (in units with $G = c = \hbar = k = 1$)

$$T = \frac{\kappa}{2\pi}.$$  \hfill (10)

Thus, $\kappa/2\pi$ truly is the physical temperature of a black hole, not merely a quantity playing a role mathematically analogous to temperature in the laws of black hole mechanics.

In fact, there are two logically independent results which give rise to the formula (10). Although these results are mathematically very closely related, it is important to distinguish clearly between them. The first result is the original thermal particle creation effect discovered by Hawking [16]. In its most general form, this result may be stated as follows (see [11] for further discussion): Consider a classical spacetime $(M, g_{ab})$ describing a black hole formed by gravitational collapse, such that the black hole “settles down” to a stationary final state. By the zeroth law of black hole mechanics, the surface gravity, $\kappa$, of the black hole final state will be constant over its event horizon. Consider a quantum field propagating in this background spacetime, which is initially in any (non-singular) state. Then, at asymptotically late times, particles of this field will be radiated to infinity as though the black hole were a perfect black body\(^2\) at the Hawking temperature, eq. (10). It should be noted that this result relies only on the analysis of quantum fields in the region exterior to the black hole, and it does not make use of any gravitational field equations.

The second result is the Unruh effect [17] and its generalization to curved spacetime. In its most general form, this result may be stated as follows (see [18], [11] for further discussion): Consider a classical spacetime $(M, g_{ab})$

\(^2\)If the black hole is rotating, the the spectrum seen by an observer at infinity corresponds to what would emerge from a “rotating black body”.

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that contains a bifurcate Killing horizon, $\mathcal{K} = \mathcal{K}_A \cup \mathcal{K}_B$, i.e., there is a one-parameter group of isometries whose associated Killing field, $\xi^a$, is normal to $\mathcal{K}$. Consider a free quantum field on this spacetime. Then there exists at most one globally nonsingular state of the field which is invariant under the isometries. Furthermore, in the “wedges” of the spacetime where the isometries have timelike orbits, this state (if it exists) is a KMS (i.e., thermal equilibrium) state at temperature (10) with respect to the isometries.

Note that in Minkowski spacetime, any one-parameter group of Lorentz boosts has an associated bifurcate Killing horizon, comprised by two intersecting null planes. The unique, globally nonsingular state which is invariant under these isometries is simply the usual (“inertial”) vacuum state, $|0\rangle$. In the “right and left wedges” of Minkowski spacetime defined by the Killing horizon, the orbits of the Lorentz boost isometries are timelike, and, indeed, these orbits correspond to worldlines of uniformly accelerating observers. If we normalize the boost Killing field, $b^a$, so that Killing time equals proper time on an orbit with acceleration $a$, then the surface gravity of the Killing horizon is $\kappa = a$. An observer following this orbit would naturally use $b^a$ to define a notion of “time translation symmetry”. Consequently, when the field is in the inertial vacuum state, a uniformly accelerating observer would describe the field as being in a thermal equilibrium state at temperature

$$T = \frac{a}{2\pi}$$

as originally found by Unruh [17].

Although there is a close mathematical relationship between the two results described above, it should be emphasized these results refer to different states of the quantum field. In the Hawking effect, the asymptotic final state of the quantum field is a state in which the modes of the quantum field that appear to a distant observer to have propagated from the black hole region of the spacetime are thermally populated at temperature (10), but the modes which appear to have propagated in from infinity are unpopulated. This state (usually referred to as the “Unruh vacuum”) would be singular on the white hole horizon in the analytically continued spacetime containing a bifurcate Killing horizon. On the other hand, in the Unruh effect and its generalization to curved spacetimes, the state in question (usually referred to as the “Hartle-Hawking vacuum”) is globally nonsingular, and all modes
of the quantum field in the “left and right wedges” are thermally populated.\(^3\)

It also should be emphasized that in the Hawking effect, the temperature (10) represents the temperature as measured by an observer near infinity. For any observer following an orbit of the Killing field, \(\xi^a\), normal to the horizon, the locally measured temperature of the modes which appear to have propagated from the direction of the black hole is given by

\[
T = \frac{\kappa}{2\pi V},
\]

where \(V = (-\xi^a\xi_a)^{1/2}\). In other words, the locally measured temperature of the Hawking radiation follows the Tolman law. Now, as one approaches the horizon of the black hole, the modes which appear to have propagated from the black hole dominate over the modes which appear to have propagated from infinity. Taking eq.(4) into account, we see that \(T \to a/2\pi\) as the black hole horizon, \(\mathcal{H}\), is approached, i.e., in this limit eq.(12) corresponds to the flat spacetime Unruh effect.

Equation (12) shows that when quantum effects are taken into account, a black hole is surrounded by a “thermal atmosphere” whose local temperature as measured by observers following orbits of \(\xi^a\) becomes divergent as one approaches the horizon. As we shall see explicitly below, this thermal atmosphere produces important physical effects on quasi-stationary bodies near the black hole. On the other hand, for a macroscopic black hole, observers who freely fall into the black hole would not notice any important quantum effects as they approach and cross the horizon.

The fact that \(\kappa/2\pi\) truly represents the physical temperature of a black hole provides extremely strong evidence that the laws of black hole mechanics are not merely mathematical analogs of the laws of thermodynamics, but rather that they in fact are the ordinary laws of thermodynamics applied to black holes. If so, then \(A/4\) must represent the physical entropy of a black hole in general relativity. What is the evidence that this is the case?

Although quantum effects on matter fields outside of a black hole were fully taken into account in the derivation of the Hawking effect, quantum effects of the gravitational field itself were not, i.e., the Hawking effect is derived in the context of semiclassical gravity, where the effects of gravitation

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\(^3\)The state in which none of the modes in the region exterior to the black hole are populated is usually referred to as the “Boulware vacuum”. The Boulware vacuum is singular on both the black hole and white hole horizons.
are still represented by a classical spacetime. As discussed further below, a proper accounting of the quantum degrees of freedom of the gravitational field itself undoubtedly would have to be done in order to understand the origin of the entropy of a black hole. Nevertheless, as I shall now describe, even in the context of semiclassical gravity, I believe that there are compelling arguments that $A/4$ must represent the physical entropy of a black hole.

Even within the semi-classical approximation, conservation of energy requires that an isolated black hole must lose mass in order to compensate for the energy radiated to infinity by the particle creation process. If one equates the rate of mass loss of the black hole to the energy flux at infinity due to particle creation, one arrives at the startling conclusion that an isolated black hole will radiate away all of its mass within a finite time. During this process of black hole “evaporation”, $A$ will decrease, in violation of the second law of black hole mechanics. Such an area decrease can occur because the expected stress-energy tensor of quantum matter does not satisfy the null energy condition—even for matter for which this condition holds classically—in violation of a key hypothesis of the area theorem. Thus, it is clear that the second law of black hole mechanics must fail when quantum effects are taken into account.

On the other hand, there is a serious difficulty with the ordinary second law of thermodynamics when black holes are present: One can simply take some ordinary matter and drop it into a black hole, where, classically at least, it will disappear into a spacetime singularity. In this latter process, one loses the entropy initially present in the matter, but no compensating gain of ordinary entropy occurs, so the total entropy, $S$, of matter in the universe decreases.

Note, however, that in the black hole evaporation process, although $A$ decreases, there is significant amount of ordinary entropy generated outside the black hole due to particle creation. Similarly, when ordinary matter (with positive energy) is dropped into a black hole, although $S$ decreases, by the first law of black hole mechanics, there will necessarily be an increase in $A$. These considerations motivated the following proposal [1], [19]. Perhaps in any process, the total generalized entropy, $S'$, never decreases

$$\Delta S' \geq 0,$$

where $S'$ is defined by

$$S' \equiv S + A/4.$$
It is not difficult to see that the generalized second law holds for an isolated black hole radiating into otherwise empty space. However, it is not immediately obvious that it holds if one carefully lowers a box containing matter with entropy $S$ and energy $E$ toward a black hole. Classically, if one lowers the box sufficiently close to the horizon before dropping it in, one can make the increase in $A$ as small as one likes while still getting rid of all of the entropy, $S$, originally in the box. However, it is here that the quantum “thermal atmosphere” surrounding the black hole comes into play. The temperature gradient in the thermal atmosphere (see eq.(12)) implies that there is a pressure gradient and, consequently, a buoyancy force on the box. As a result of this buoyancy force, the optimal place to drop the box into the black hole is no longer the horizon but rather the “floating point” of the box, where its weight is equal to the weight of the displaced thermal atmosphere. The minimum area increase given to the black hole in the process is no longer zero, but rather it turns out to be an amount just sufficient to prevent any violation of the generalized second law from occurring [20]. A number of other analyses [21], [22], [23] also have given strong support to validity of the generalized second law.

The generalized entropy (14) and the generalized second law (13) have obvious interpretations: Presumably, for a system containing a black hole, $S'$ is nothing more than the “true total entropy” of the complete system, and (13) is then nothing more than the “ordinary second law” for this system. If so, then $A/4$ truly is the physical entropy of a black hole.

I believe that the above semi-classical considerations make a compelling case for the merger of the laws of black hole mechanics with the laws of thermodynamics. However, if one is to obtain a deeper understanding of why $A/4$ represents the entropy of a black hole in general relativity, it clearly will be necessary to go beyond semi-classical considerations and attain an understanding of the quantum dynamical degrees of freedom of a black hole. Thus, one would like to calculate the entropy of a black hole directly from a quantum theory of gravity. There have been many attempts to do so, most of which fall within the following categories: (i) Calculations that are mathematically equivalent to the classical calculation described in the previous section. (ii) Calculations that ascribe a preferred local significance to the horizon. (iii) State counting calculations of configurations that can be associated with black holes.

The most prominent of the calculations in category (i) is the derivation of
black hole entropy in Euclidean quantum gravity, originally given by Gibbons and Hawking [24]. Here one starts with a formal, functional integral expression for the partition function in Euclidean quantum gravity and evaluates it for a black hole in the “zero loop” (i.e., classical) approximation. As shown in [25], the mathematical steps in this procedure are in direct correspondence with the purely classical determination of the entropy from the form of the first law of black hole mechanics. Thus, although this derivation gives some intriguing glimpses into possible deep relationships between black hole thermodynamics and Euclidean quantum gravity, the Euclidean derivation does not appear to provide any more insight than the classical derivation into accounting for the quantum degrees of freedom that are responsible for black hole entropy. Similar remarks apply to a number of other entropy calculations that also can be shown to be equivalent to the classical derivation (see [26]).

Within category (ii), a key approach has been to attribute the entropy of the black hole to “entanglement entropy” resulting from quantum field correlations between the exterior and interior of the black hole (see, in particular, [27]). As a result of these correlations, the state of the field when restricted to the exterior of the black hole is mixed, and its von Neumann entropy, \(-\text{tr}[\hat{\rho}\ln\hat{\rho}]\), would diverge in the absence of a short distance cutoff. If one now inserts a short distance cutoff of the order of the Planck scale, one obtains a von Neumann entropy of the order of the horizon area, \(A\). A closely related idea is to attribute the entropy of the black hole to the ordinary entropy of its thermal atmosphere (see, in particular, [28]). Since \(T\) diverges near the horizon in the manner specified by eq.(12), the entropy of the thermal atmosphere diverges, but if one puts in a Planck scale cutoff, one gets an entropy of order \(A\). Indeed, this calculation is really the same as the entanglement entropy calculation, since the state of a quantum field outside of the black hole at late times is thermal, so its von Neumann entropy is equal to its thermodynamic entropy.

These and other approaches in category (ii) provide a natural way of accounting for why the entropy of a black hole is proportional to its surface area, although the constant of proportionality typically depends upon a cutoff or other free parameter and is not calculable. However, it is far from clear why the black hole horizon should be singled out for such special treatment of the quantum degrees of freedom in its vicinity, since, for example, similar quantum field correlations will exist across any other null surface. Indeed, as
discussed further at the end of the next section, it is particularly puzzling why
the local degrees of freedom associated with the horizon should be singled
out since, as already noted above, the black hole horizon at a given time
is defined in terms of the entire future history of the spacetime and thus
has no distinguished local significance. Finally, for approaches in category
(ii) that do not make use of the gravitational field equations—such as the
ones described above—it is difficult to see how one would obtain a black hole
entropy proportional to eq.(8) (rather than proportional to $A$) in a more
general theory of gravity.

By far, the most successful calculations of black hole entropy to date are
ones in category (iii) that obtain the entropy of certain extremal and nearly
extremal black holes in string theory. It is believed that at “low energies”,
string theory should reduce to a 10-dimensional supergravity theory. If one
treats this supergravity theory as a classical theory involving a spacetime
metric, $g_{ab}$, and other classical fields, one can find solutions describing black
holes. On the other hand, one also can consider a “weak coupling” limit of
string theory, wherein the states are treated perturbatively about a back-
ground, flat spacetime. In the weak coupling limit, there is no literal notion
of a black hole, just as there is no notion of a black hole in linearized gen-
eral relativity. Nevertheless, certain weak coupling states can be identified
with certain black hole solutions of the low energy limit of the theory by
a correspondence of their energy and charges. (Here, it is necessary to in-
troduce “D-branes” into string perturbation theory in order to obtain weak
coupling states with the desired charges.) Now, the weak coupling states are,
in essence, ordinary quantum dynamical degrees of freedom in a flat back-
ground spacetime, so their entropy can be computed by the usual methods of
flat spacetime statistical physics. Remarkably, for certain classes of extremal
and nearly extremal black holes, the ordinary entropy of the weak coupling
states agrees exactly with the expression for $A/4$ for the corresponding clas-
sical black hole states; see [29] for a review of these results.

Since the formula for entropy has a nontrivial functional dependence on
energy and charges, it is hard to imagine that this agreement between the
ordinary entropy of the weak coupling states and black hole entropy could be
the result of a random coincidence. Furthermore, for low energy scattering,
the absorption/emission coefficients (“gray body factors”) of the correspond-
ing weak coupling states and black holes also agree [30]. This suggests that
there may be a close physical association between the weak coupling states
and black holes, and that the dynamical degrees of freedom of the weak coupling states are likely to at least be closely related to the dynamical degrees of freedom responsible for black hole entropy. However, it seems hard to imagine that the weak coupling states could be giving an accurate picture of the local physics occurring near (and within) the region classically described as a black hole. Thus, it seems likely that in order to attain additional new conceptual insights into the nature of black hole entropy in string theory, further significant progress will have to be made toward obtaining a proper local description of strong gravitational field phenomena.

4 Some unresolved issues and puzzles

I believe that the results described in the previous two sections provide a remarkably compelling case that black holes are localized thermal equilibrium states of the quantum gravitational field. Although none of the above results on black hole thermodynamics have been subject to any experimental or observational tests, the theoretical foundation of black hole thermodynamics is sufficiently firm that I feel that it provides a solid basis for further research and speculation on the nature of quantum gravitational phenomena. Indeed, it is my hope that black hole thermodynamics will provide us with some of the additional key insights that we will need in order to gain a deeper understanding of quantum gravitational phenomena. In this section, I will raise and discuss four major, unresolved issues in quantum gravitational physics that black hole thermodynamics may help shed light upon.

I. What is the nature of singularities in quantum gravity?

The singularity theorems of classical general relativity assert that in a wide variety of circumstances, singularities must occur in the sense that spacetime cannot be geodesically complete. However, classical general relativity should break down prior to the formation of a singularity. One possibility is that in quantum gravity, these singularities will be “smoothed over”. However, it also is possible that at least some aspects of the singularities of classical general relativity are true features of nature, and will remain present in quantum gravitational physics.

Black hole thermodynamics provides a strong argument that the singularity inside of a black hole in classical general relativity will remain present in at least some form in quantum gravity. In classical general relativity,
the matter responsible for the formation of the black hole propagates into a singularity in the deep interior of the black hole. Suppose that the matter which forms the black hole possesses quantum correlations with matter that remains far outside of the black hole. Then it is hard to imagine how these correlations could be restored during the process of black hole evaporation; indeed, if anything, the Hawking process should itself create additional correlations between the exterior and interior of the black hole as it evaporates (see [11] for further discussion). However, if these correlations are not restored, then by the time that the black hole has evaporated completely, an initial pure state will have evolved to a mixed state, i.e., “information” will have been lost. In the semiclassical picture, such information loss does occur and is ascribable to the propagation of the quantum correlations into the singularity within the black hole. If pure states continue to evolve to mixed states in a fully quantum treatment of the gravitational field, then at least the aspect of the classical singularity as a place where “information can get lost” must remain present in quantum gravity. This issue is frequently referred to as the “black hole information paradox”, and its resolution would tell us a great deal about the existence and nature of singularities in quantum gravity.

II. Is there a relationship between singularities and the second law?

The usual arguments for the validity of the second law of thermodynamics rest upon having very “special” (i.e., low entropy) initial conditions. Such special initial conditions in systems that we presently observe trace back to even more special conditions at the (classically singular) big bang origin of the universe. Thus, the validity of the second law of thermodynamics appears to be intimately related to the nature of the initial singularity [31]. On the other hand, the arguments leading to the area increase theorem for black holes in classical general relativity would yield an area decrease theorem if applied to white holes. Thus, the applicability (or, at least, the relevance) of the second law of black hole mechanics appears to rest upon the fact that black holes can occur in nature but white holes do not. This, again, could be viewed as a statement about the types of singularities that can occur in nature [31]. If, as argued here, the laws of black hole mechanics are the laws of thermodynamics applied to a system containing a black hole, then it seems hard to avoid the conclusion that a close relationship must exist between the second law of thermodynamics and the nature of what we classically describe
as singularities.

III. Are statistical probabilities truly distinct from quantum probabilities?

Even in classical physics, probabilities come into play in statistical physics as ensembles representing our ignorance of the exact state of the system. On the other hand, in quantum physics, probabilities enter in a much more fundamental way: Even if the state of a system is known exactly, one can only assign a probability distribution to the value of observables. In quantum statistical physics, probabilities enter in both of these ways, and it would seem that these two ways should be logically distinguishable. However, density matrices have the odd feature of entering quantum statistical physics in two mathematically equivalent ways: (i) as an exact description of a particular (mixed) quantum state, and (ii) as a statistical ensemble of a collection of pure quantum states. In particular, one may choose to view a thermal density matrix either as a single, definite (mixed) state of the quantum system, or as a statistical ensemble of pure states. In the former case, the probability distribution for the values of observables would be viewed as entirely quantum in origin, whereas in the latter case, it would be viewed as partly statistical and partly quantum in origin; indeed, for certain observables (such as the energy of the system), the probabilities in the second case would be viewed as entirely statistical in origin. The Unruh effect puts this fact into a new light: When a quantum field is in the ordinary vacuum state, $|0\rangle$, it is in a pure state, so the probability distribution for any observable would naturally be viewed by an inertial observer to be entirely quantum in origin. On the other hand, for an accelerating observer, the field is in a thermal state at temperature $(11)$, and the probability distribution for “energy” (conjugate to the notion of time translation used by the accelerating observer) would naturally be viewed as entirely statistical in origin. Although there are no physical or mathematical inconsistencies associated with these differing viewpoints, they seem to suggest that there may be some deep connections between quantum probabilities and statistical probabilities; see [32] for further exploration of these ideas.

IV. What is the definition/meaning of entropy in general relativity?

The issue of how to assign entropy to the gravitational field has been raised and discussed in the literature (see, in particular, [31]), although it seems clear that a fully quantum treatment of the degrees of freedom of the gravitational field will be essential for this issue to be resolved. However,
as I will emphasize below, even the definition and meaning of the entropy of “ordinary matter” in general relativity raises serious issues of principle, which have largely been ignored to date.

First, it should be noted that underlying the usual notion of entropy for an “ordinary system” is the presence of a well defined notion of “time translations”, which are symmetries of the dynamics. The total energy of the system is then well defined and conserved. The definition and meaning of the usual notion of entropy for classical systems is then predicated on the assumption that generic dynamical orbits “sample” the entire energy shell, spending “equal times in equal volumes”; a similar assumption underlies the notion of entropy for quantum systems (see [14] for further discussion). Now, an appropriate notion of “time translations” is present when one considers dynamics on a background spacetime whose metric possesses a suitable one-parameter group of isometries, and when the Hamiltonian of the system is invariant under these isometries. However, such a structure is absent in general relativity, where no background metric is present. The absence of any “rigid” time translation structure can be viewed as being responsible for making notions like the “energy density of the gravitational field” ill defined in general relativity. Notions like the “entropy density of the gravitational field” are not likely to fare any better. It may still be possible to use structures like asymptotic time translations to define the notion of the total entropy of an (asymptotically flat) isolated system. (As is well known, total energy can be defined for such systems.) However, for a closed universe, it seems highly questionable that any meaningful notion will exist for the “total entropy of the universe” (including gravitational entropy).

The comments in the previous paragraph refer to serious difficulties in defining the notions of gravitational entropy and total entropy in general relativity. However, as I now shall explain, even in the context of quantum field theory on a background spacetime possessing a time translation symmetry—so that the “rigid” structure needed to define the usual notion of entropy of matter is present—there are strong hints from black hole thermodynamics that even our present understanding of the meaning of the “ordinary entropy”

4Furthermore, it is clear that gross violations of any sort of “ergodic behavior” occur in classical general relativity on account of the irreversible tendency for gravitational collapse to produce singularities, from which one cannot then evolve back to uncollapsed states—although the semiclassical process of black hole evaporation suggests the possibility that ergodic behavior could be restored in quantum gravity.
of matter is inadequate.

Consider the “thermal atmosphere” of a black hole. As discussed in Section 3 above, since the locally measured temperature is given by eq.(12), if we try to compute its ordinary entropy, a new ultraviolet catastrophe occurs: The entropy is infinite unless we put in a cutoff on the contribution from short wavelength modes.\(^5\) As already noted in Section 3, if we insert a cutoff of the order of the Planck scale, then the thermal atmosphere contributes an entropy of order the area, \(A\), of the horizon (in Planck units). Note that the bulk of the entropy of the thermal atmosphere is highly localized in a “skin” surrounding the horizon, whose thickness is of order of the Planck length. The presence of this thermal atmosphere poses the following puzzle:

**Puzzle:** *What is the “physical entropy” of the thermal atmosphere?*

One possibility is that the thermal atmosphere should be assigned an entropy of order the area of the horizon, as indicated above. As discussed in Section 3, this would then account (in order of magnitude) for the entropy of black holes. However, this also would mean that there would be no room left to assign entropy to any “internal degrees of freedom” of the black hole, i.e., all of the entropy of a black hole would be viewed as residing in a Planck scale skin surrounding the horizon. To examine the implications of this view in a more graphic manner, consider the collapse of a very massive spherical shell of matter, say of mass \(M = 10^{11}M_\odot\). Then, as the shell crosses its Schwarzschild radius, \(R \sim 3 \times 10^{11}\) km, the spacetime curvature outside of the shell is much smaller than that at the surface of the Earth, and it will take more than another week before the shell collapses to a singularity. An unsophisticated observer riding on the shell would have no idea that any doom awaits him, and he would notice nothing of any significance occurring as the Schwarzschild radius is crossed. Nevertheless, within a time of order the Planck time after crossing of the Schwarzschild radius, the “skin” of thermal atmosphere surrounding the newly formed black hole will come to equili-

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\(^5\)Since a field has infinitely many degrees of freedom, it threatens to make an infinite contribution to entropy. The old ultraviolet catastrophe—which plagued physics at the turn of the previous century—was resolved by quantum theory, which, in effect, provides a cutoff on the entropy contribution of modes with energy greater than \(kT\), so that, at any \(T\), only finitely many degrees of freedom are relevant. The new ultraviolet catastrophe arises because, on account of arbitrarily large redshifts, there now are infinitely many modes with energy less than \(kT\). To cure it, it is necessary to have an additional cutoff (presumably arising from quantum gravity) on short wavelength modes.
rium with respect to the notion of time translation symmetry for the static Schwarzschild exterior. Thus, if entropy is to be assigned to the thermal atmosphere as above, then the degrees of freedom of the thermal atmosphere—which previously were viewed as irrelevant vacuum fluctuations making no contribution to entropy—suddenly become “activated” by the passage of the shell for the purpose of counting their entropy. A momentous change in the entropy of matter in the universe has occurred, and all of this entropy increase is localized near the Schwarzschild radius of the shell, but the observer riding the shell sees nothing.\(^6\)

Another possibility is that the infinite (prior to the imposition of a cut-off) entropy of the thermal atmosphere is simply another infinity of quantum field theory that needs to be properly “renormalized”; when a proper renormalization has been done, the thermal atmosphere will make a negligible contribution to the total entropy. This view would leave room to attribute black hole entropy to “internal degrees of freedom of the black hole”, and would avoid the difficulties indicated in the previous paragraph. However, it raises serious new difficulties of its own. Consider a black hole enclosed in a reflecting cavity which has come to equilibrium with its Hawking radiation. Surely, far from the black hole, the entropy of the thermal radiation in the cavity should not be “renormalized away”. But this radiation is part of the thermal atmosphere of the black hole. Thus, one would have to postulate that at some distance from the black hole, the renormalization effects begin to become important. In order to avoid the difficulties of the previous paragraph, this would have to occur at a distance much larger than the Planck length. But, then, what happens to the entropy in a box of ordinary thermal matter as it is slowly lowered toward the black hole. By the time it reaches its “floating point”, its contents are indistinguishable from the thermal atmosphere. Thus, if the floating point is close enough to the black hole for the renormalization to have occurred, the entropy in the box must have disappeared, despite the fact that an observer inside the box still sees it filled with thermal radiation. Furthermore, if one lowers (or, more accurately, pushes)

\(^6\)Similarly, if the entropy of the thermal atmosphere is to be taken seriously, then it would seem that during a period of uniform acceleration, an observer in Minkowski spacetime should assign an infinite entropy (since the horizon area is infinite) to a Planck sized neighborhood of a pair of intersecting null planes lying at a distance \(c^2/a\) from him. Observers near these null planes presumably would be quite surprised by the assignment of a huge entropy density to an ordinary, empty region of Minkowski spacetime.
an empty box to the same distance from the black hole, it will have an en-
tropy less than the box filled with radiation. Therefore, the empty box would
have to be assigned a negative entropy.

I believe that the above puzzle suggests that we presently lack the proper
conceptual framework with which to think about entropy in the context of
general relativity. In any case, it is my belief that the resolution of the above
issues will occupy researchers well into the next century, if not well into the
next millenium.

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