Thermal Properties of Two-Dimensional Advection Dominated Accretion Flow

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ABSTRACT

We study the thermal structure of the widely adopted two-dimensional advection dominated accretion flow (ADAF) of Narayan & Yi (1995a). The critical radius for a given mass accretion rate, outside of which the optically thin hot solutions do not exist in the equatorial plane, agrees with one-dimensional study. However, we find that, even within the critical radius, there always exists a conical region of the flow, around the pole, which cannot maintain the assumed high electron temperature, regardless of the mass accretion rate, in the absence of radiative heating. This could lead to torus-like advection inflow shape since, in general, the ions too will cool down. We also find that Compton preheating is generally important and, if the radiative efficiency, defined as the luminosity output divided by the mass accretion rate times the velocity of light squared, is above \( \sim 0.02 \), the polar region of the flow is preheated above the virial temperature by Compton heating and it may result in time-dependent behaviour or outflow while accretion continues in the equatorial plane. Thus, under most relevant circumstances, ADAF solutions may be expected to be accompanied by polar outflow winds. While preheating instabilities exist in ADAF, as for spherical flows, the former are to some extent protected by their characteristically higher densities and higher cooling rates, which reduce their susceptibility to Compton driven overheating.

Subject headings: accretion, black hole, X-ray sources, QSO’s

1. Introduction

Depending on the angular momentum it carries and how the angular momentum is dissipated, gas accretes onto compact objects in various ways. If the gas contains very little angular momentum or bulk flow, the flow becomes spherical (Hoyle & Lyttleton 1939; Bondi & Hoyle 1944; Bondi 1952; Loeb & Laor 1992; Foglizzo & Ruffert 1997; Nio, Matsuda, & Fukue 1998). If it has enough angular momentum and if the angular momentum is efficiently removed by some mechanism, the flow flattens to a disk shape (Pringle & Rees 1972; Shakura & Sunyaev 1973).
Until recently, only the extreme of these two types of solutions have been studied (see Chakrabarti 1996a,b for history and unified scheme of accretion solutions and Park & Ostriker 1998 for review on general accretion flow).

Spherical accretion has the merit of being simple, and therefore can be accurately calculated. It generally has the infall velocity close to the free-fall value, and most of the gas energy generated is lost into the hole in the case of accretion onto black holes, making the radiation efficiency, $e$, of the accretion, a self consistently calculable quantity depending primarily on the entropy at large radii and the accretion rate. The efficiency can be very low or significantly high (Shapiro 1973a,b, 1974; Mészáros 1975; Park & Ostriker 1989; Park 1990a,b). In the opposite limiting case accretion proceeds in the form of thin disk when the gas has enough angular momentum and cools efficiently. It has a fixed and high radiation efficiency, $e \sim 0.1$, and does not emit high energy photons due to the low temperature of the gas. Although the thin disk accretion has been successfully applied to many astronomical sources (see Pringle 1981 and Frank, King, & Raine 1992 for reviews), its radiation spectrum is incompatible with certain kinds of celestial sources believed to be powered by accretion.

There are also other types of accretion disk solutions beyond the cool thin disk. Shapiro, Lightman, & Eardley (1976) showed that geometrically and optically thin, high-temperature accretion disk can exist in which ions and electrons are weakly coupled and have different temperatures. Although it was more successful in explaining high-energy sources like Cyg X-1, it proved to be unstable on a thermal time scale (Pringle 1976; Piran 1978; Park 1995). Seminal works on geometrically thick accretion disks appeared a few years later (Jaroszyński, Abramowicz, & Paczyński 1980; Paczyński & Wiita 1980). Abramowicz et al. (1988) further combined these works with ‘α-viscosity’ model to find a so called ‘slim disk’ solution. It is a geometrically and optically thick accretion disk solution with a significant fraction of the gas energy being transported via advection, but treated in one-dimensional framework. These slim disk solutions exist only when the mass accretion rate is near or above the Eddington mass accretion rate defined as

$$\dot{M}_{\text{Edd}} = \frac{L_E}{c^2} = 2.19 \times 10^{-9} \, M_\odot \, \text{yr}^{-1}$$

where $L_E$ is the Eddington luminosity and the numerical value is for the pure hydrogen. (In the case of accretion onto neutron stars or thin disk accretion, the Eddington accretion rate has sometimes been defined as $e^{-1}L_E/c^2$ since $e$ is almost a fixed value. However, in the type of accretion in which $e$ is not known a priori, this definition would be confusing and misleading because even when $\dot{M} \sim \dot{M}_{\text{Edd}}$, the luminosity from the accretion can be much smaller than $L_E$. This choice of definition makes $\dot{M}_{\text{Edd}}$ in this work 10 times smaller than $\dot{M}_{\text{Edd}}$ in NY3.) Slim disks have a radiation spectrum similar to that of the thin disks.

Only recently, a new type of accretion solution has been found that stands somewhere between the spherical and thin disk accretion (Narayan & Yi 1994, 1995a,b, hereafter NY1, NY2, and NY3, respectively, and NY collectively; Abramowicz et al. 1995; see Narayan, Mahadevan, & Quataert 1998 for review and references). These solutions are called ‘advection dominated
accretion flow’ (ADAF) because most of the gas energy is advected with the flow and ultimately into the hole due to relatively large infall velocity. This type of accretion is possible at sufficiently low mass accretion rate so that radiative cooling is inefficient and ions can be kept near virial temperature. The isodensity contours of self-similar ADAF changes from that of a sphere to a somewhat flattened spheroid, depending on the parameters chosen (NY2), but in general the flow looks more like the spherical or spheroidal limit rather than the disk-like solutions. It is in a way a reminiscence of ion-torus model (Rees et al. 1982).

ADAF has a number of distinct and desirable properties. First, like spherical accretion, the radiation efficiency can span a large range and is generally much smaller than $\sim 0.1$. Second, the electron temperature is much higher than that in thin disk accretion and, therefore, can produce high-energy photons. Third, the self-similar form of the solutions makes calculation of various properties very easy (Spruit et al. 1987; NY1). These merits have led to many successful applications of ADAF to various accretion powered sources that are difficult to model by the standard thin disk solutions: Sgr A$^*$ (Narayan, Yi, & Mahadevan 1995; Mahadevan, Narayan, & Krollik 1997; Mannoto, Mineshige, & Kusunose 1997; Narayan et al. 1998), NGC 4258 (Lasota et al. 1996), soft X-ray transient sources (Narayan, McClintock, & Yi 1996; Narayan, Barret, & McClintock 1997), low-luminosity galactic nuclei (Di Matteo & Fabian 1997; Mahadevan 1997), torque-reversing X-ray pulsars (Yi & Wheeler 1998), and X-ray background (Yi & Boughn 1998).

However, most elaborate works on ADAF have been based on one-dimensional height-integrated equations (Narayan, Kato, & Honma 1997; Chen, Abramowicz, & Lasota 1997; Nakamura et al. 1997; Gammie & Popham 1998; Popham & Gammie 1998; Chakrabarti 1996a) even though the desirable properties depend explicitly on the two-dimensional characteristics of the flow. The two-dimensional nature of ADAF has been explored only in self-similar form (NY2; Xu & Chen 1997) or by numerical simulations of adiabatic, inviscid flow very close to the hole (Molteni, Lanzafame, & Chakrabarti 1994; Ryu et al. 1995; Chen et al. 1997; Igumenshchev & Beloborodov 1997). When treated as a height-integrated disk instead of two-dimensional flow, locally produced photons are assumed to escape the flow without affecting other parts of the flow, which is only true when the flow is truly disk-like. Also, parts of the thermal and dynamical structure of the two-dimensional flow can be widely different from those of the averaged disk, under general accretion conditions, because of the flow structure in the vertical direction.

So, in this work, we study the thermal properties of the ADAF based on the two-dimensional self-similar flow solutions of NY2. One important difference between this work and previous studies is the allowance for the preheating of the flow at large radii by photons produced at the inner, hotter part of the flow. When the luminosity is high enough, this can substantially change the dynamics of the ADAF leading, quite possibly, to some kind of time-dependent behaviour or outflow.
2. Thermal Properties of Advection Dominated Accretion Flows

In generic accretion flow, the thermal state of the gas at a given point is determined by the balance between various heating and cooling processes. Gas can be heated by $PdV$ work, viscous dissipation, and the interaction with radiation. It cools mostly by emission of radiation. However, heating and cooling are not always balanced, and the extra energy gain or loss will be stored as internal energy and carried with the flow. This advection of internal energy mostly acts as loss of gas energy at a fixed point in the accretion flow and sometimes is called advective cooling.

The advection-dominated accretion flow solutions of NY refer to a family of solutions where gas at a given point is heated by viscous dissipation and cooled mostly by the advection with negligible radiative cooling. However, the definition of “advective cooling” in NY contains the $PdV$ adiabatic heating term. So one should be careful when advective cooling is used in thermal balance calculations. For example, gas of an adiabatic index, $\gamma = 5/3$, and temperature profile, $T \propto r^{-1}$, has zero advective cooling because $PdV$ heating exactly balances the internal energy advection. If $\gamma < 5/3$, the adiabatic heating alone cannot maintain $T \propto r^{-1}$ profile and viscous dissipation provides the additional heating needed to maintain $r^{-1}$ profile.

To facilitate further discussions, we define five time scales relevant in general accretion flow:

- the inflow time scale
  \[ t_{\text{flow}} = r/v_r, \]  
  \[ (2) \]

- the advective cooling time scale
  \[ t_{\text{adv}} = \frac{\varepsilon}{q_{\text{adv}}}, \]  
  \[ (3) \]

- the viscous heating time scale
  \[ t_{\text{vis}} = \frac{\varepsilon}{q_{\text{vis}}}, \]  
  \[ (4) \]

- the radiative heating time scale
  \[ t_H = \frac{\varepsilon}{H}, \]  
  \[ (5) \]

- and the radiative cooling time scale
  \[ t_C = \frac{\varepsilon}{C}, \]  
  \[ (6) \]

where $r$ is the radius, $v_r$ the infall velocity, $\varepsilon$ the internal energy of gas, $q_{\text{adv}}$ the cooling rate due to advection, $q_{\text{vis}}$ the viscous heating rate, $H$ the radiative heating rate, and $C$ the radiative cooling rate, all per unit volume. In self-similar solutions, $t_{\text{flow}} = t_{\text{adv}}/\epsilon \simeq t_{\text{adv}}$ and $t_{\text{adv}} = f^{-1}t_{\text{vis}} = (1 - f)f^{-1}t_C$ where $f$ is the ratio between the advective cooling and the total cooling rates and $\epsilon = (5/3 - \gamma)/(\gamma - 1)$; (NY). When the flow is advection dominated, $f = 1$ and $t_{\text{flow}} \simeq t_{\text{adv}} \simeq t_{\text{vis}} \ll t_C$. 


3. Cooling in Two-Dimensional ADAF

The self-similar ADAF is, like the Bondi solution, very attractive due to its simplicity. Most physical quantities are simple power-laws of radius and can be readily calculated. However, this also implies that the solution may not be appropriate everywhere for any real accretion flow.

The simplicity of ADAF self-similar solution is based on the simple energy balance equation. The assumption of a constant ratio between the viscous heating, advective cooling, and radiative cooling, $1 : f : 1 - f$, maintains the self-similarity of the solutions, yet puts severe restrictions on cooling processes. Should the ratio be constant in radius, the energy equation would be satisfied at only one point. NY overcome this problem by finding a semi-self consistent variable $f$ for applications to real accretion flows. However, this is only possible when a positive $f$ can be found: the cooling should be smaller than the heating. Solutions do not exist, if positive $f$ is not possible.

Cooling may be so strong that the flow can not be kept at a high temperature (Rees et al. 1982). This leads to the critical mass accretion $\dot{M}_{\text{crit}}$ for a given radius above which the optically thin hot solutions do not exist (Abramowicz et al. 1995; NY3). And for a given $\dot{m}$, hot solutions exist only within some critical radius, $r_{\text{crit}}$.

First, we will study similar issues in two-dimensional ADAF. The viscous heating per unit volume in self-similar ADAF is simply

$$q_{\text{vis}}^+ = f^{-1} q_{\text{adv}}^- = \frac{3}{2} \epsilon \rho |v_r| c_s^2,$$

where $\rho(r, \vartheta)$ is the gas density, $v_r$ the radial velocity, and $c_s$ the isothermal sound speed. The composite formula for atomic cooling and non-relativistic bremsstrahlung (Stellingwerf & Buff 1982; Nobili, Turolla, & Zampieri 1991) extended to relativistic bremsstrahlung is used for cooling,

$$C = \sigma_T \alpha_f m_e c^2 n_i^2 \left\{ \lambda_{\text{br}}(T_e) + 6.0 \times 10^{-22} \theta_e^{-1/2} \right\}^{-1} + \left( \frac{\theta_e}{4.82 \times 10^{-6}} \right)^{-12},$$

where $\sigma_T$ is the Thomson cross section, $\alpha_f$ the fine-structure constant, $n_i$ the number density of ions, and $\theta_e \equiv k T_e / m_e c^2$. The relativistic bremsstrahlung rate is (Svensson 1982; Stepney & Guilbert 1983; NY3)

$$\lambda_{\text{br}} = \left( \frac{n_e}{n_i} \right) \left( \sum_i Z_i^2 \right) F_{\text{ei}}(\theta_e) + \left( \frac{n_e}{n_i} \right)^2 F_{\text{ee}}(\theta_e),$$

where

$$F_{\text{ei}} = 4 \left( \frac{2}{\pi^3} \right)^{1/2} \theta_e^{1/2} (1 + 1.781 \theta_e^{1.34}) \quad \text{for } \theta_e < 1$$

$$= \frac{9}{2\pi} \theta_e \left[ \ln(1.123 \theta_e + 0.48) + 1.5 \right] \quad \text{for } \theta_e > 1$$

$$F_{\text{ee}} = \frac{5}{6\pi^{3/2}} \left( 44 - 3 \theta_e^2 \right) \theta_e^{3/2} (1 + 1.1 \theta_e + \theta_e^2 - 1.25 \theta_e^{5/2}) \quad \text{for } \theta_e < 1$$

$$= \frac{9}{\pi} \theta_e \left[ \ln(1.123 \theta_e) + 1.2746 \right] \quad \text{for } \theta_e > 1,$$
and $Z_i$ is the charge of ions. The unknown electron temperature is assumed to be described by the fitting formula $T_e = T_i T_a / (T_i + T_a)$ with $T_a \approx 10^9$ K and $(kT_i/m_p c^2) = c_s^2 (\vartheta) (r_s/r)^{-1}/4$ with $r_s \equiv 2GM/c^2$, which approximates the electron temperature profile in ADAF (NY3).

The ratio $f$ is calculated from the definition,

$$1 - f \equiv \frac{C}{q_{vis}}$$

in the $(r, \vartheta)$ plane. The two-dimensional structure of the flow is basically determined by one parameter $\epsilon' \equiv \epsilon/f$ (see NY2). Contours of $f = 0.9$ (innermost curve), 0.8, 0.7, 0.6, 0.5, 0.3, 0.2, 0.1, and 0.0 (outermost curve) for ADAF with $\dot{m} = 0.03$, $\epsilon' \equiv \epsilon/f = 1.0$, and $\alpha = 0.1$ are shown in Figure 1. The heavily shaded region above the contours represents $f < 0$, i.e., the region where hot electrons can not exist due to cooling. The one-dimensional calculation of NY3 (Fig. 3) shows that $f = 0.5$ flow has $r_{crit} \sim 10^4 r_s$, which agrees well with $r_{crit} \simeq 1.1 \times 10^4 r_s$ for $\vartheta = 0$ in Figure 1.

Note that there is a roughly cone-shaped shaded region around the pole where the hot solution is not possible. For $\vartheta \ll 1$, this region extends down to $\sim r_s$. This is mainly due to the slow infall velocity near the pole (NY2) to which the advective cooling is proportional. Since the viscous heating and radiative cooling are proportional to the advective cooling in self-similar ADAF, a small infall velocity near the pole implies a small advective cooling, and hence small viscous heating and radiative cooling. But even atomic plus bremsstrahlung cooling can have a far higher cooling rate than is assumed in ADAF; therefore the flow cannot maintain the high temperature in such regions. Normally, electrons in the region would cool down to $\sim 10^4$ K. To first order, the dynamics of the flow would not be affected as long as the Coulomb coupling is weak and the ion temperature is near virial. But the low temperature of the electrons for the gas near the polar axis will make the coupling stronger [energy exchange rate $\propto T_e^{-1/2}$] and it is possible that the ion temperature would consequently drop below the virial value, resulting in the collapse of the polar region. In fact, as we shall show in Park & Ostriker (1999), this is the typical case. The possibility of the collapse of the polar region of the flow even at high temperature (for different reasons) is also discussed by Blandford & Begelman (1998). This will create a funnel around the polar axis (Fig. 1) and the flow will look more like torus than spheroid (Rees et al. 1982; Paczyński 1998).

4. Comptonization

When the accretion flow is quasi-spherical as in ADAF, whatever photons are produced at smaller radii inevitably interact with the outer part of the flow on their way out. In generic conditions, Compton scattering of photons off electrons is usually the most important interaction. They can either heat or cool the gas depending on the the spectrum of the radiation and the temperature of the gas. High energy photons heat the electrons while low energy ones cool them. However, the flow at large radius is generally heated because most photons are produced at hotter inner regions, and this is called preheating (Ostriker et al. 1976).
The importance of preheating on the flow can be estimated by comparing the relevant time scales. If $t_{\text{vis}} \ll t_H$, preheating can be safely ignored. However, if there exists a region where $t_H < t_{\text{vis}}$, then preheating dominates over the viscous heating or the advective cooling. Since radiative fluxes are, in the lowest approximation, quadratic in the flow rate (Shapiro 1973a,b), whereas advective heating/cooling terms are linear in the flow rate, preheating will become significant for high enough accretion rates. In the spherical case we found that the solutions are significantly altered for the mass accretion rate $\dot{m} \equiv \dot{M}/\dot{M}_{\text{Edd}} > 10^{-1.5}$ and the luminosity $l \equiv L/L_{\text{Edd}} > 10^{-7.5}$, with no high-temperature solutions possible having $l > 0.03$ and $e > 3 \times 10^{-4}$ due to the preheating instability (Park 1990a,b). Since the density of the ADAF is higher than that in spherical accretion flow for the same mass accretion rate due to the small infall velocity, we expect that this limit will occur for higher values of $e$ and $l$.

In ADAF, all physical quantities are products of a radial part, a function of radius $r$ only, with an angular part, a function of spherical polar angle $\vartheta$ only. So

$$t_{\text{vis}} = \frac{1}{c} \Omega_K^{-1}(r) v^{-1}(\vartheta),$$

(13)

where the radial infall velocity is defined as $v_r = r \Omega_K(r) v(\vartheta)$ with $\Omega_K(r) \equiv (GM/r^3)^{1/2}$ and

$$t_H = \left( \frac{m_e c^2}{4 k T_X} \right) \left( \frac{L_E}{L_X} \right) \frac{3 c_s^2(\vartheta) r}{2 c_s(\vartheta)},$$

(14)

where $m_e$ is the electron mass, $c$ the speed of light, $T_X$ the Compton temperature of the radiation defined as the energy-weighted mean of photon energy averaged over the photon spectral number density, $4 k T_X \equiv < (h \nu)^2 > / < h \nu >$, $L_X$ the luminosity of the Comptonizing radiation, $L_E$ the Eddington luminosity, $c_s(\vartheta)$ the isothermal sound speed divided by the Keplerian velocity $c_s(r, \vartheta) \equiv (p/\rho)^{1/2} = r \Omega_K(r) c_s(\vartheta)$, $p$ the total pressure, and $\rho$ the gas density (Levich & Syunyaev 1971). The inequality $t_H < t_{\text{vis}}$ now reduces to

$$v(\vartheta) c_s^2(\vartheta) \left( \frac{r_s}{r} \right)^{1/2} < \frac{2}{3} \left( \frac{4 k T_X}{m_e c^2} \right) \left( \frac{L_X}{L_E} \right).$$

(15)

We take the case of the flow $L_X/L_E = 10^{-3}$, $4 T_X = 10^9$ K, and $e' = 0.1$ as an example (NY3). Equation (15) represents the region above the solid curve in Figure 2, in which preheating is the dominant heating process and should not be ignored in the thermal balance equation. In advection dominated flow, $t_{\text{flow}} \simeq t_{\text{adv}} \simeq t_{\text{vis}}$ and, therefore, $t_H < t_{\text{flow}}$. The flow is not adiabatic anymore and dynamics of the flow will be significantly altered.

Similarly, there could be many soft photons, thereby lowering the radiation temperature $T_X \ll T_e$, and the flow would be cooled by Compton scattering. The condition for this is

$$v(\vartheta) c_s^2(\vartheta) \left( \frac{r_s}{r} \right)^{1/2} < \frac{2}{3} \left( \frac{4 k T_e}{m_e c^2} \right) \left( \frac{L_X}{L_E} \right),$$

(16)

which corresponds to the region above the dotted curve in Figure 2 for the same flow parameters.
For the flow near the equator, Compton heating or cooling may be ignored within some radius. But above the equatorial plane at large radius or around the pole, Compton heating or cooling can be more important than the viscous heating. It is quite possible that Compton preheating would heat the flow near the pole to a high temperature flow, that would otherwise cool down. Or if there are abundant soft photons, the flow would cool due to the Compton cooling. This result is independent of the mass accretion rate and depends only on the Comptonizing luminosity. Esin (1997) similarly found the importance of Compton cooling (called ‘non-local cooling’) under certain conditions in a careful one-dimensional analysis, whereas Compton heating was found to be generally unimportant. The main reason for this discrepancy is that different radiation temperature, i.e., spectrum, is assumed and that physical parameters of height-integrated flow can be widely different from those of two-dimensional flow, especially near the polar axis (NY2).

However, a real ADAF could be far more complicated than this. Firstly, the radiation temperature $T_X$ can vary from place to place. It should correctly represent the spectrum of the radiation field at a given position, which is the sum of the radiation transferred from other regions of the flow and that produced locally. Secondly, the luminosity $L_X$ is also a function of position, which is again related to the density and temperature of the gas. Thirdly, the gas temperature is determined by the amount of Compton heating, therefore the radiation temperature and the radiation energy density. Hence, more accurate analysis requires simultaneously solving the energy equations for gas and radiation. This has been done only for spherical accretion flow (Park 1990a,b; Nobili, Turolla, & Zampieri 1991; Mason & Turolla 1992).

5. Preheating limit

If preheating increases above some critical value, it can affect the accretion flow more dramatically than just changing the thermal balance. In Bondi-type spherical accretion flow, Ostriker et al. (1976) found that too much preheating changes the dynamics of the flow around the sonic radius and would disrupt the steady flow. This results in various time-dependent behaviour in the flow and in the outcoming radiation (Cowie, Ostriker, & Stark 1978; Ciotti & Ostriker 1997).

Our next question is whether there is a similar preheating limit for ADAF. Since ADAF is self-similar, it does not have an accretion radius or outer boundary. So we have to look at the temperature structure in preheated ADAF. We will use simplified thermal balance equation to solve for the temperature for the ADAF with strong preheating luminosity.

In ADAF, the cooling rate is assumed to be a constant fraction of the advective cooling to obtain self-similar forms of solutions. This simplification may not be too bad, if the solution is taken as height-integrated (or angle-averaged) form. But in two-dimensional ADAF, the flow time approaches infinity along the polar direction and is an order-of-magnitude larger than the usual free-fall time along the equatorial direction (NY2). There is no radial advection along the pole, and
we expect the Compton heating to be balanced by the radiative cooling. Similarly we apply the thermal balance equation to all parts of the flow to estimate the preheating effect, which is valid when \( t_H < t_{vis} \) (Fig. 2). The temperature is determined by requiring \( H(T_{eq}; r, \vartheta) = C(T_{eq}; r, \vartheta) \). When atomic line cooling and bremsstrahlung are the dominant processes, \( T_{eq}(r) \) has the form that outside some radius \( r_\ast \), \( T(r) \approx 10^4 \) K and at \( r_\ast \), \( T(r) \) jumps to \( T_\ast \approx 2 \times 10^6 \) K at which temperature the bremsstrahlung cooling rate becomes comparable to the peak of the atomic line cooling (Buff & McCray 1974). In this domain there is a classical phase change and the temperature suddenly jumps, because there is no stable equilibrium between \( \sim 10^4 \) K and \( T_\ast \). If we further assume that the luminosity profile \( L_X(r) \) is constant in \( r \), the temperature profile inside \( r_\ast \) is simply \( T(r) = T_\ast (r/r_\ast)^{-1} \) if the gas cools only by non-relativistic bremsstrahlung.

At the transition radius \( r_\ast \), Compton heating is equal to the peak of the cooling curve by definition (eq. 8),

\[
\frac{4k(T_X - T_v)}{m_e c^2} \frac{l}{m} [n(\vartheta)]^{-1} \left( \frac{r_\ast}{r_\ast} \right)^{-1/2} = 8.8 \times 10^{-5},
\]

(17)

where the gas number density is defined as \( n(r, \vartheta) = n(\vartheta)(M/4\pi c m_p r_\ast^2)(r/r_\ast)^{-3/2} \). Since the temperature of the electrons is not too different from \( \sim 10^5 \) K at the inner part of the flow where most of the radiation is produced, we take \( T_X \approx (10^9/4) \) K \( \gg T_v(r) \). This choice of \( T_X \) means that the radiation should contain enough hot photons to heat the gas. If there are too many soft photons, e.g., synchrotron photons, \( T_X \) could be lower.

The accretion flow would be disrupted, if the flow is heated above the virial temperature \( T_{vir} \), defined as \( (5/2)k T_{vir} \equiv GM_m/r \). The flow temperature suddenly jumps from \( 10^4 \) K to \( T_\ast \) at radius \( r_\ast \), and the inflow would stop or reverse (i.e., become outflow) if \( T_\ast > T_{vir}(r_\ast) \). The condition \( T_\ast > T_{vir}(r_\ast) \) is equivalent to \( r_\ast > r_v \approx 9.4 \times 10^3 r_\ast \) (eq. 17) since \( r_v \) is defined as \( T_{vir}(r_v) = T_\ast \).

For the efficiency \( e = l/\dot{m} = 0.022 \) flow with \( \epsilon' = 1.0 \) and \( 4kT_\ast/m_e c^2 = 0.1 \), \( r_\ast \) as a function of \( \vartheta \) is shown in Figure 3 as a solid curve, and \( r_v \) as a dotted one. The part of the flow between \( r_\ast \) and \( r_v \) with \( r_\ast > r_v \) is overheated and the steady inflow is not possible whereas the part of the flow with \( r_\ast < r_v \) can accrete normally. The flow in regions A and C of Figure 3 has low temperature \( \sim 10^4 \) K and is stable. The flow in regions B and D are Compton heated above \( T_\ast \) with the flow in region B being unstable, while that in region D being stable. So it would be possible that the flow accretes along the equatorial plane while there is outflow along the pole due to preheating. Blandford & Begelman (1998) also propose advection dominated inflow-outflow via different mechanism: Part of the conservative flow can have positive energy due to the energy flux transported by the viscous torque (see also NY2).

The critical efficiency \( e_{cr} \) above which this overheating starts to occur in any part of the flow corresponds to \( r_\ast|_{\vartheta=0} = r_v \) since preheating is most effective in the polar direction due to lower infall velocity and lower density. Figure 4 shows \( e_{cr} \) determined for \( 4kT_X/m_e c^2 = 0.1 \) as squares. Depending on the value of \( \epsilon' \), any flow with efficiency above \( \sim 0.02 \) will suffer preheating instability.
at or near the pole, and can develop the time-dependent behaviour or outflow as in spherical case (Cowie, Ostriker, & Stark 1978). This value of $e_{cr} \sim 0.02$ corresponds to $l \approx 10^{-2.5}$ and $\dot{m} \approx 0.2$ in NY3 solutions, and is about 50 times larger than that in the spherical accretion flow. Since the radiation temperature assumed here is similar to that of the self-consistent spherical flow (Park 1990a), it is mainly due to the difference in density of the accretion flow. The gas density of ADAF is on average $30 \sim 100$ times greater for the same total mass accretion rate due to the smaller infall velocity than that in the spherical accretion flow which is almost freely falling (Park 1990a; NY2). This provides a major advantage for ADAF (in comparison with spherical flow): higher luminosities and efficiencies should be possible.

6. Luminosity from ADAF

One of the unique difference between ADAF (or spherical accretion) onto black holes and thin disk accretion is that its radiation efficiency cannot be assumed a priori but must be determined self consistently. In thin disk accretion, whatever the energy input to the flow, it is locally radiated away because of the long inflow time, and its radiation efficiency is essentially determined by the position of inner edge of the disk. However, accretion flow with significant radial velocity can carry the energy into the hole as well as radiates away. So the outcoming radiation can be either significant or negligible depending on the dynamics and thermal structure of the flow (see Park & Ostriker 1998 for review).

Here, we estimate how much outgoing radiation is produced self consistently by self-similar ADAF. The amount of radiative cooling in ADAF is always assumed to be some fraction $(1 - f)$ of the viscous heating. The remaining fraction $f$ is advected with the flow and not radiated. Hence the radiative luminosity would be $(1 - f)$ times the sum of all viscous heating (Eq. 7) over all radii and angles,

$$L_{rad} = \frac{2}{3} \epsilon'(1 - f) \int_{r_{in}}^{r_{out}} dr \int_0^\pi 2\pi d\theta |v_r| \rho c_s^2 r.$$  

Since the parameter $f$ is not determined a priori for self-similar ADAF, we take $f = 0$ to estimate the maximum luminosity that can be produced. Substituting physical quantities of NY2 yields the dimensionless maximum luminosity as a function of $\epsilon'$. Since the right-hand side of the equation (18) is proportional to $\dot{M}$, the maximum efficiency $e_{max}$ rather than the maximum luminosity is determined. We assume $r_{in} = 3r_s$, and the values of $e_{max}$ for each $\epsilon'$ are shown in Figure 4 as circles. Comparison with the critical efficiency, $e_{cr}$, shows that the flow with $\epsilon' \gtrsim 0.3$ has a possibility of preheating disruption. Higher $\epsilon'$ flows are more vulnerable to preheating because they have higher infall velocity and pressure, therefore, higher viscous heating and radiation output.

The exact value of $e_{max}$ may differ somewhat from the values above, because simple thermal balance equation is not always valid. In reality, the local cooling rate can be larger than the viscous heating because of the additional $PdV$ work. A very good example is the spherical accretion flow
without viscous dissipation. The viscous heating is zero, yet the gas is heated by compression due to gravity and radiates (Shapiro 1973a,b; Park 1990a,b). Therefore, $e_{\text{max}}$ can be higher.

7. Self Consistent Flows

So far we have discussed the cases where gas near the polar axis or at large radius is cooled in the absence of preheating, or it can be heated too much and disrupted by preheating. However, it is also possible that the flow can be maintained at high temperature by preheating without being disrupted. In spherical accretion flow, there exist two branches of solutions for certain mass accretion rate (Park 1990a,b; see Park & Ostriker 1998 for references): In the lower luminosity branch, gas is cooled down to $\sim 10^4$ K and thus, not much energy is released through radiation. In the other, higher luminosity branch, gas at large radius is Compton heated by hot radiation produced in the inner region, and a higher radiation efficiency is achieved. We do find that this is also true for ADAF. For some mass accretion rates, there exist hot solutions self-consistently maintained by Compton preheating, whereas the flow would cool down to thin disk in the absence of preheating (Park & Ostriker 1999).

8. Summary

We have studied the thermal properties of the self-similar, two-dimensional advection dominated accretion flow (NY2) with special consideration given to the radiative cooling and heating. We find that

1. A hot solution is possible only within some critical radius for a given mass accretion rate in the equatorial plane, confirming the one-dimensional analysis (NY3). Also, for any mass accretion rate, a roughly conical region around the pole cannot maintain high-temperature electrons in the absence of radiative heating with the collapsed region shrinking as $\dot{m}$ is reduced. If ions become coupled to the now cold electrons, (as seems likely) an empty funnel around the polar axis will be created.

2. Part of the flow at large radii or above the equatorial plane should be affected through Compton heating or cooling by photons produced at smaller radii, if the luminosity is high enough. If the radiation efficiency of the accretion is above $\sim 0.02$, and the outcoming radiation has mean photon energy comparable to the electron temperature of the inner region, preheating due to the inverse Compton scattering would overheat the polar region of the flow, and may create time-dependent behaviour or outflow while accretion still goes on in the equatorial directions. For NY3 solutions, these phenomena should begin to occur for luminosities $l \gtrsim 10^{-2.5}$ and accretion rate $\dot{m} \gtrsim 0.2$ in Eddington units.

3. The role of Compton preheating is quite intriguing in ADAF as in spherical flows. On the
one hand it can have the attractive feature of driving polar winds whenever the total luminosity is above some rather low bound. But it may also allow another branch of solutions making the original ADAF solutions more viable for higher mass accretion rates than those for which solutions were valid. The detailed calculation of this new branch of solutions will be given in Park & Ostriker (1999).

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Fig. 1.— Contours of $f = 0.9$ (innermost curve), 0.8, 0.7, 0.6, 0.5, 0.3, 0.2, 0.1, and 0.0 (outermost curve) for $\dot{m} = 0.03$, $\epsilon' \equiv \epsilon/f = 1.0$, and $\alpha = 0.1$. Heavily shaded region above the contours has $f < 0$, i.e., hot electrons can not exist due to excessive cooling.
Fig. 2.— In the region above the solid curve the Compton heating time scale is shorter than the viscous heating time scale and, approximately, the inflow time scale for $L_X/L_E = 10^{-3}$, $4T_X = 10^9$ K, and $\epsilon' = 0.1$ flow. Above the small dotted curve, the Compton cooling time scale is shorter than the viscous time scale if $T_X \ll T_e$. Radius $r$ and disk height $z$ are in units of Schwarzschild radius $r_s$. 
Fig. 3.— For $e = 0.022$ flow with $\epsilon = 1.0$ and $4kT_x/m_e c^2 = 0.1$, $r_*$ is shown as a solid curve, and $r_v$ as a dotted one. In regions A and C the flow is cool ($T \simeq 10^4$ K); while in B and D it is hot ($T > T_*$). In region B the temperature is above the virial temperature, the energy is positive and infall would be impossible.
Fig. 4.— The critical efficiency $e_{cr}$ (squares) at which overheating starts to occur for $4kT_X/m_e c^2 = 0.1$ as a function of the parameter $\epsilon'$. The maximum radiation efficiency $e_{max}$ (circles) by assuming all viscous heating being converted to radiation.