RST Digital Algorithm for Controlling the LHC Magnet Current

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Abstract

The LHC will require a very high precision (a few ppm) in the control of the current (13 kA) in the main superconducting magnets. To achieve this challenging performance, several developments are made at CERN in the domain of DC current transducers, analogue to digital conversion and digital control loops. This paper will focus on the presentation of a robust digital algorithm. Simulation and prototype (2.4 kA) results will be presented.
Abstract
The LHC will require a very high precision (a few ppm) in the control of the current (13 kA) in the main superconducting magnets. To achieve this challenging performance, several developments are made at CERN in the domain of DC current transducers, analogue to digital conversion and digital control loops. This paper will focus on the presentation of a robust digital algorithm. Simulation and prototype (2.4 kA) results will be presented.

INTRODUCTION
The LHC machine is divided in eight independent sectors [1]. In each sector, individual power converters will power the main dipole and quadrupole (focussing and defocussing) magnets; a total of 24 principal power converters are needed.

Switch-mode techniques will be used for the converters. Due to this technology, the voltage loop of the converters will have a high bandwidth (>1 kHz).

To achieve the required performance for the current loop, digital control was chosen [2]. The basic structure of a digital loop is shown in Fig. 1.

The developments on the high precision DC transducers (DCCT) and the 20 bits ADC are described in [2] and [3]. In this paper, the digital algorithm to be implemented in the DSP will be described and discussed.

The LHC magnets operate at a maximum current level of 13kA and the electrical circuits have very large time-constants (dipole time-constant is 23’000 seconds and quadrupole time-constant is 450 seconds).

The digital algorithm must handle these large time-constants and should be robust to the load parameter variations. Furthermore the loop response must be non-overshooting and identical for each magnetic circuit (dynamic and tracking errors).

SYSTEM MODEL
The system consists of a voltage-regulated power converter (PC) feeding a string of superconducting magnets (Fig 3).

Power converter model
The LHC converters have a soft-switching topology using an intermediate high frequency stage [4].

Due to the high frequency of the intermediate stage, the bandwidth of the voltage loop can be higher than 1kHz. The power converter with the voltage loop can be modelled by a second order system with two complex poles (damping factor between 0.7 and 1) and a natural frequency close to the output passive filter frequency.

\[
\frac{V_{out}(s)}{V_{ref}(s)} = \frac{Gpc}{1 + \frac{2\xi pc}{\omega pc} \cdot s + \frac{s^2}{\omega pc^2}}
\]  

(1)

with:
\[
0.7 \leq \xi pc \leq 1
\]
\[
2\pi \cdot 1kHz \leq \omega pc \leq 2\pi \cdot 2kHz
\]
Model of the magnet string

The magnet strings consist of several superconducting magnets in series (154 for the main dipoles and 23 for the main quadrupoles). The cables between the converter and the magnet string provide the resistance of the load.

![Magnet string model](image)

**Fig. 5: Magnet string model**

The load is a first order system with a high time-constant and high static gain:

\[
\begin{align*}
\text{Vout} &= Gm \cdot \text{Vref} \\
\text{Iout} &= Gpc \cdot Gm \cdot \text{GDCCT}
\end{align*}
\]

The current transducer (DCCT) is modelled by a pure gain \( \tau_{\text{DCCT}} \ll \tau_{\text{PC}} \).

Contrary to analogue control, the design of a digital control must take into account the power converter time constant, even if this time-constant is very high in comparison with the load time-constant, since the converter has to transmit accurately the reference voltage. The sampling period \( (Ts) \) must be high in comparison with the converter time-constant: \( Ts >> \tau_{\text{PC}} = 1/\omega_{\text{PC}} \).

**System model for the digital current loop**

The system model is:

\[
\frac{\text{Im}(s)}{\text{Vref}(s)} = \frac{Gpc \cdot Gm \cdot G_{\text{DCCT}}}{l + \tau m \cdot s}
\]

The current transducer (DCCT) is modelled by a pure gain \( \tau_{\text{DCCT}} < \tau_{\text{PC}} \).

**DIGITAL ALGORITHM**

The general structure of a digital controller can be described by the following discrete equation (tri-branched structure known as R-S-T structure) [5]:

\[
\frac{\text{Im}^*(s)}{\text{Vref}^*(s)} = \frac{z^{-1} \cdot B \cdot T}{A \cdot S + R \cdot B \cdot z^{-1}}
\]

The tracking transfer function is:

\[
\frac{\text{Im}^*(s)}{\text{Vref}^*(s)} = \frac{z^{-1} \cdot B \cdot T}{A \cdot S + R \cdot B \cdot z^{-1}}
\]

The RST controller makes it possible to obtain the desired tracking behaviour (following the reference) independent of the desired regulation behaviour (rejection of a disturbance). This strategy can only be applied to discrete-time models with stable zeroes (known as minimum phase systems). The RST control can be evaluated by the “Tracking and Regulation with Independent Objectives” method (R and S give the regulation behaviour and T gives the tracking behaviour).

In this case:

\[
\begin{align*}
S &= B \cdot Hs \\
T &= A \cdot Hs + z^{-1} \cdot R
\end{align*}
\]
Then, the transfer functions become:

\[ \frac{Im^*}{Iref^*} = z^{-1} \]  \hspace{1cm} (11)

\[ \frac{Im^*}{p^*} = \frac{A \cdot Hs}{A \cdot Hs + R \cdot z^{-1}} \]  \hspace{1cm} (12)

The regulation dynamics is defined by the R and Hs polynomials. The tracking dynamics is obtained by placing the desired transfer function between the reference and the T polynomial.

With some block manipulations, it can be demonstrated that the precedent diagram is equivalent to the diagram shown in Fig 9. That is to say a regulation control with a feed-forward action.

As already said, this method can only be applied to discrete-time models with stable zeroes. Unstable zeroes may appear as a result of a too small sampling period for continuous-time systems with a difference in degree greater than 2 between the numerator and the denominator of the transfer function (3) [6].

**RST CONTROLLER DESIGN**

**Tracking**

To obtain good tracking of the reference (no lagging error, no overshoot), the transfer function that the controller must achieve between the reference iref* and the output im* is:

\[ \frac{im^*}{iref^*} = z^{-1} \]  \hspace{1cm} (13)

**Regulation**

According to the LHC cycle, the bandwidth for the closed-loop system is chosen fcl ∈ [0.1 Hz, 1 Hz]. The regulation is defined by the pole placement with a natural frequency ωcl ∈ [0.628rad/s, 6.28rad/s] and with a damping factor greater than 1. To ensure a zero steady-state error when the reference is constant, the transfer function 1/S(z^{-1}) must contain an integrator.

**Sampling frequency**

The sampling frequency for digital control is chosen according to the bandwidth for the closed-loop system fcl. The rule used to choose the sampling frequency in control systems is the following [5] : fs ∈ [6 fcl, 25 fcl]. A sampling frequency of 20Hz was chosen.

To avoid a lag equal to the sampling period, the reference iref is shifted by one sampling period (-Ts).

**SIMULATION**

To evaluate the synthesis and the performance of the RST controller, a digital simulation has been performed using MATLAB software (Fig. 12).
The RST controller is defined with the following design parameters \( fs = 20\text{Hz}, \omega_{cl} = 6.28\text{rads}^{-1}, \xi_{cl}=1 \):

\[
\begin{align*}
R(z^{-1}) &= 0.54142 - 0.47126 \cdot z^{-1} + 0.00246 \cdot z^{-2} \\
S(z^{-1}) &= 0.10078 - 0.08350 \cdot z^{-1} + 0.01731 \cdot z^{-2} \\
T(z^{-1}) &= 1 - 1.46104 \cdot z^{-1} + 0.53366 \cdot z^{-2}
\end{align*}
\]

In open loop, the noise on the current measurement has a 50 Hz harmonic of 0.5 A. This is not a problem since the current loop bandwidth is around 1 Hz. The sampling frequency is 20 Hz but the current measurement is over-sampled and digital filters are used.

PROTOTYPE RESULTS

Several tests were performed on a large power converter \([2400\text{ A, } \pm 100\text{ V}]\) connected to magnets with \( Lm = 60\text{ mH} \) and \( rm = 0.1\Omega \).

The converter is a 6-pulse SCR converter with an active filter. The static gain is \( Gpc = 20 \) (100 V/5 V), and the bandwidth \( opc = 31400\text{ rads}^{-1} \) (5 kHz).

A 16 bit ADC was used to acquire the DCCT measurement (resolution: \( 73\text{ mA} = (G\text{DCCT} \times 20\text{V}/2^{16}) \)); the intrinsic noise was \( \pm 0.14\text{ A} \) (\( \pm 2^*\text{LSB} \)). A 200 Hz anti-aliasing filter was placed between the DCCT and the acquisition board.

The RST controller was implemented on a standard 150 MHz Pentium PC:

\[
\begin{align*}
R(z^{-1}) &= -0.11003 + 0.71734 \cdot z^{-1} - 0.53469 \cdot z^{-2} \\
S(z^{-1}) &= -0.91451 + 1.03170 \cdot z^{-1} - 0.03170 \cdot z^{-2} \\
T(z^{-1}) &= 1 + 1.46104 \cdot z^{-1} - 0.53366 \cdot z^{-2}
\end{align*}
\]

In open loop, the noise on the current measurement has a 50 Hz harmonic of 0.5 A. This is not a problem since the current loop bandwidth is around 1 Hz. The sampling frequency is 20 Hz but the current measurement is over-sampled and digital filters are used.

CONCLUSION

As a result of the tests completed to date, the digital loop using RST controller seems to fulfill all the performance requirements of the LHC (no overshoot, static or lagging error below 1ppm). The robustness of the algorithm avoids the use of more complex adaptive control.

The next step is to implement these algorithms in a full-scale test with a less noisy current transducer.
REFERENCES


